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Local quantum thermometry using Unruh-DeWitt detectors

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Resumen

En este trabajo, proponemos una definición operacional de la temperatura local de un campo cuántico usando detectores Unruh-DeWitt, de forma similar a la empleada en los efectos Unruh y Hawking. Con esta definición, un sistema cuántico inhomogéneo en equilibrio puede tener diferentes temperaturas locales, en analogía con el teorema de Tolman-Ehrenfest en relatividad general. Hemos estudiado la distribución de la temperatura local en el estado fundamental de un sistema fermiónico con términos de hopping en un espacio curvo. La temperatura observada tiende a cero conforme el acoplo termómetro-sistema, g, disminuye. Además, para valores pequeños pero finitos de g, mostramos que el producto de la temperatura local observada y el logaritmo de la velocidad local de la luz es aproximadamente constante. Nuestras predicciones son susceptibles de comprobación en sistemas de átomos ultrafríos.

Abstract

We propose an operational definition for the local temperature of a quantum field employing Unruh-DeWitt detectors, as used in the study of the Unruh and Hawking effects. With this definition, an inhomogeneous quantum system in equilibrium can have different local temperatures, in analogy with the Tolman-Ehrenfest theorem from general relativity. We have studied the local temperature distribution on the ground state of hopping fermionic systems on a curved background. The observed temperature tends to zero as the thermometer-system coupling, g, vanishes. Yet, for small but finite values of g, we show that the product of the observed local temperature and the logarithm of the local speed of light is approximately constant. Our predictions should be testable on ultracold atomic systems.

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Nomenclature

Symbols

g	Thermometer-system coupling
$|g_{00}(x)|^{1/2}$	Inhomogeneous time-lapse function
L	Number of sites of the fermionic chain
n_0	Long-term time average of of the expected value of the thermometer occupation
t_i	Hopping amplitudes

Acronyms / Abbreviations

GS	Ground states
OBC	Open boundary conditions
PBC	Periodic boundary conditions
SQD	Single qubit detector

1 Introduction

Recently, quantum simulators built upon ultracold atomic gases [1] have been designed in order to explore the very interesting interplay between quantum mechanics and curved space-time [2], including the effects of dimensionality [3] or unusual topology [4]. Moreover, a detailed proposal for a quantum simulator to explore Unruh physics in cold atoms has been put forward [5]. The idea behind all the proposed quantum simulators on curved space-times [1–5] is the following: a static metric with an inhomogeneous time-lapse function $|g_{00}(x)|^{1/2}$ for fermionic systems can be simulated by tuning the local hopping amplitudes between the cells of an optical lattice. This relation can be also understood in reverse: an inhomogeneity in the hopping amplitudes may be read as a non-trivial space-time metric. This idea has sparked interest in the low energy states of these inhomogeneous spin chains and fermionic hopping models, which can be understood as dynamics on a curved metric. For example, it has been shown how a modulation of the metric can give rise to ground states (GS) which present extremely long-range correlations, such as the *rainbow state* [6, 7]. The entanglement entropy of the GS of local quantum systems usually follows the *area law* [8, 9], but in a curved metric we can have a strong violation, with a volumetric growth of the block entropies. This led to a thermal interpretation of the rainbow state [10], which can be viewed as a thermo-field double. Thus we see that, in some situations, it makes sense to attach a non-zero temperature to a quantum ground state.

One of the most surprising results in thermodynamics on curved space-times was stated by Richard Tolman and Paul Ehrenfest in 1930 [11, 12]: the temperature of an equilibrium system in a static space-time may vary from point to point, and it is inversely proportional to the local lapse function,

$$T(x) \cdot |g_{00}(x)|^{1/2} = \text{const.} \tag{1}$$

The result is of thermodynamical nature, and can be proved without any assumptions on the dynamics [13]. It can be applied to the study of the *Unruh effect*: an accelerated observer travelling through a Minkowski vacuum must feel a thermal bath of particles at a temperature proportional to its acceleration [14, 15]. Due to the principle of equivalence, such an observer can be considered to be at rest in *Rindler space-time*, which is characterized by a lapse function which increases linearly with the distance to a *horizon*, $|g_{00}(x)|^{1/2} \propto x$. Then, the Tolman-Ehrenfest theorem predicts that the local temperature

must decay as the inverse of that same distance, $T(x) \propto x^{-1}$ [16, 17][1]. It is relevant to notice that the Unruh effect is defined in an operational way [12, 15]: an *Unruh-DeWitt* detector is defined as a simple quantum system with a local monopolar interaction with the field. The temperature will manifest itself in the quantum fluctuations within the detector[2].

A great amount of theoretical work has been devoted to the *locality-of-temperature* problem, i.e., to find under which conditions a subsystem of a global system at temperature T can be considered to be again in a thermal state at the same temperature [22–24]. In general terms, the answer is that this is possible when a certain measure of the energy contained in the correlations is lower than the physical temperature, T. Thus, this work will explore the opposite limit, when $T = 0$, so quantum correlations can create non-trivial local thermal effects. Thus, one may ask how small a thermometer can be in order to make sensible measurements. Quantum thermometry is indeed an area undergoing a rapid growth. The idea of using a single qubit as a thermometer has been put forward recently by several groups [25–28]. In this case, the fluctuations in the temperature estimate should be taken into account [29–31], which are expected to follow the Landau relation, $\Delta T \sim T^2/C$, where C is the heat capacity of the system.

This work proposes to explore local quantum thermometry on the ground state of inhomogeneous free fermionic Hamiltonians by observing the quantum fluctuations of a single-qubit Unruh-DeWitt detector, locally linked to our system. The long term average of the occupation provides an estimate of the local temperature, while its frequency dependence provides further information about the system. We show that, for finite couplings between the detector and the system, the observed local temperature and the time-lapse are related via a modification of the Tolman-Ehrenfest relation. Nonetheless, when the coupling tends to zero, the temperature vanishes, at it should on a ground state. The reason to employ free systems is that we will focus on the interplay between geometry

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[1] There are some important caveats when attempting to apply the Tolman-Ehrenfest theorem to the Unruh effect, see [16, 17].

[2] The Unruh effect predicts moreover that the quantum fluctuations of such a detector will follow a Fermi-Dirac distribution for a free fermionic field and Bose-Einstein for a free bosonic field, as long as the spatial dimension is odd, or the opposite if it is even [12]. The Unruh effect has not been properly measured in the laboratory yet [18], although there are some promising directions [19]. Nonetheless, a relevant indirect test was highlighted by Bell and Leinaas in 1983 [20, 21], related to the Sokolov-Ternov effect: if an electron beam is accelerated in a synchrotron under a strong magnetic field it will self-polarize, but this polarization will not be complete. This can be explained in two ways, leading to the same predictions. The most usual is using the relativistic transformations of the interaction between the magnetic field and the spin. But, alternatively, we can consider the electron spin as an Unruh-DeWitt detector in a thermal bath due to its acceleration as it moves on curved paths through the Minkowski vacuum.

and thermal effects, and we leave the effects of interaction for further work. Notice that, despite of our use of the Unruh-DeWitt detector, our measurement does not bear relation to the Unruh effect.

There have been other proposals to define effective and local temperatures for non-equilibrium and/or inhomogeneous systems in the literature. Some of the most relevant are based either on the fluctuation-dissipation theorem [32–34] or the connection to a thermal bath with a vanishing heat flow [35–40]. We will comment on the relation to our approach at the end of this work.

This work is organized as follows. In section 2 we describe our physical model, an Unruh-DeWitt detector locally attached to the ground state of a fermionic system on a curved background. The methodological issues are discussed in section 3, and the numerical results are shown in section 4. Section 5 is devoted to a variational general study of the physics of single-qubit detectors in interaction with free fermionic systems. Finally, in section 6 we present a summary of the conclusions and suggestions for further work.
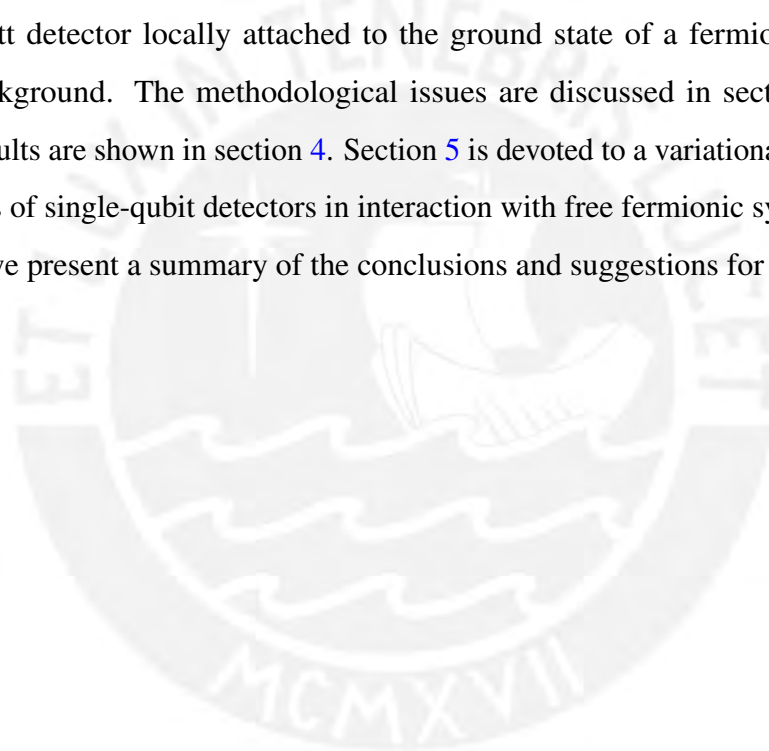
2   Unruh-DeWitt Thermometry

Let us consider a system of spinless fermions on L sites characterized by a Hamiltonian H_S, and let $c_i^\dagger$ denote the creation operator at site i. We introduce a new site, the Unruh-DeWitt detector or thermometer, with label 0 and a chemical potential $\mu > 0$, whose Hamiltonian is:

$$H_D = \mu c_0^\dagger c_0. \tag{2}$$

Let $H_0 \equiv H_S + H_D$, and let us cool the system into its ground state, which will contain $L/2$ fermions (half-filling) in the system while the detector will be empty. We now quench the system by attaching the detector to site p of the system via an interaction term of the form

$$H_I = g\left(c_0^\dagger c_p + h.c.\right), \tag{3}$$

where g is a (small) coupling constant. The total Hamiltonian of the system is now given by

$$H = H_0 + H_I = H_S + H_D + H_I, \tag{4}$$

see figure 1 for an illustration. If the detector is in the extreme, the system presents some similarity to a Kondo lattice, but it remains a pure hopping Hamiltonian, non-interacting.
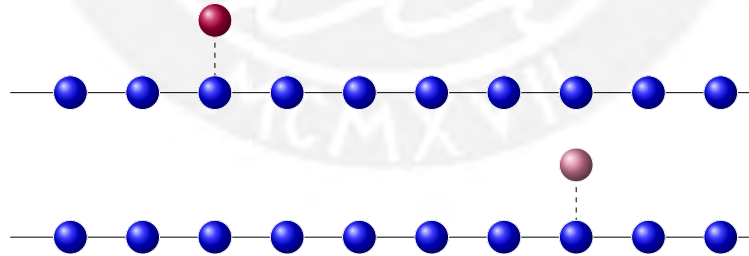

Figure 1: We set up a fermionic chain with L sites in its ground state (blue), plus a *thermometer site* or Unruh-DeWitt detector (red), initially empty and uncoupled. At time $t = 0$ we establish a local hopping between them (dashed line), and trace the evolution of the expected occupation of the thermometer as a function of time, $n_0(t)$. Since the occupied and empty states have different energies, we can *infer* a temperature from the long term behavior of $n_0(t)$. The inferred temperature may depend on the position of the thermometer, as illustrated in the panels.

After the quench, we observe that the expected value of the occupation of the detector is a function of time, $\langle n_0(t) \rangle$, and we can define n_0 to be its long-term time average,

$$n_0 \equiv \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^\tau dt \, \langle n_0(t) \rangle, \tag{5}$$

if this limit exits. Since the energy difference between the empty and the occupied states of the detector is μ, which we assume to be sufficiently above the Fermi energy of the system, we can give a thermal interpretation to that magnitude:

$$n_0 \equiv \frac{1}{1 + \exp(\beta \mu)}, \tag{6}$$

from which we can infer a local temperature $T = 1/\beta$, associated to site p. If the energy provided by the coupling, $\sim g$, is small, we can assume that we are not perturbing the system noticeably and, therefore, we are measuring an intrinsic property of the quantum system. Of course, the proper value of the temperature should always be taken as $g \to 0$. For finite values of g, we will speak of *observed* values of the local temperature.

Notice that this procedure bears a strong similarity to the operational definition of the Unruh temperature [15], the main difference being that our detector is at rest.

3   Computing the thermometer occupation

For concreteness, let us consider our system to be a 1D free fermion lattice with L sites and a position-dependent hopping amplitude:

$$H_S = -\sum_i t_i c_i^\dagger c_{i+1} + h.c. \, , \tag{7}$$

where the t_i are the hopping amplitudes, encoding the geometry. If they are slowly varying, they can be understood as a local time-lapse function $|g_{00}(x)|^{1/2}$ of a static metric [2]:

$$ds^2 = -t^2(x)dt^2 + dx^2, \tag{8}$$

where we assume $x_i = i\Delta x$ and $t(x_i) \approx t_i/\Delta x$. We can also think of $t(x)$ as a local speed of light in an optical metric. Notice that the restriction to a 1D non-interacting system is only made for convenience. An important property of the Hamiltonian (7) is that its single-particle spectrum presents particle-hole symmetry. Thus, for the ground state at half-filling, the particle density $\langle n_i \rangle = 1/2$ is always homogeneous.

Let us compute the local temperature defined by Eq. (6). Before the quench the Hamiltonian is $H_0 = H_S + H_D$, and after the quench it is $H = H_0 + H_I$. Both Hamiltonians are free, thus their eigenstates can be obtained in terms of single-body energies and orbitals:

$$H_0 \rightarrow \quad \left\{ \varepsilon_k, \quad b_k^\dagger = \sum_i B_{ki} c_i^\dagger \right\}, \tag{9}$$

$$H \rightarrow \quad \left\{ \eta_l, \quad d_l^\dagger = \sum_i D_{li} c_i^\dagger \right\}. \tag{10}$$

The linear transformations among the single-body orbitals $b_k^\dagger$, $d_k^\dagger$ and $c_i^\dagger$ are all unitary. Furthermore, we define

$$d_l^\dagger \equiv \sum_k U_{lk} b_k^\dagger = \sum_{k,i} D_{li} \bar{B}_{ik} b_k^\dagger. \tag{11}$$

The initial state is the ground state of H_0:

$$|\Psi_0\rangle = \prod_{k \in K} b_k^\dagger |0\rangle \, , \tag{12}$$

where K is the set of occupied levels in the initial system, i.e.: those whose energy $\varepsilon_k < 0$ (we will assume it to be non-degenerate, so there are no zero modes). Let us express the time evolution in the Heisenberg image, making the operators evolve. Thus, we need to obtain

$$n_0(t) \equiv \langle \Psi_0 | c_0^\dagger(t) c_0(t) | \Psi_0 \rangle. \tag{13}$$

The orbitals of H evolve as $d_k^\dagger(t) = d_k^\dagger e^{-i\eta_k t}$, where $d_k^\dagger(0) = d_k^\dagger$. The evolution of the on-site $c_i^\dagger(t)$ operators is given by:

$$c_0^\dagger(t) = \sum_l \bar{D}_{0l} d_l^\dagger e^{-i\eta_k t}. \tag{14}$$

Putting all together, we obtain

$$n_0(t) = \sum_{l,l'} \bar{D}_{0l} D_{0l'} e^{i\eta_l t} e^{-i\eta_{l'} t} \langle \Psi_0 | d_l^\dagger d_{l'} | \Psi_0 \rangle \tag{15}$$

$$= \sum_{l,l'} \bar{D}_{0l} D_{0l'} e^{-i(\eta_l - \eta_{l'})t} \sum_{k \in K} U_{lk} \bar{U}_{l'k}. \tag{16}$$

From here we read that the Fourier transform of the temporal fluctuations of the detector occupation, $\hat{n}_0(\omega)$, has peaks at frequencies $\omega_{ll'} \equiv \eta_l - \eta_{l'}$:

$$\hat{n}_0(\omega) = \sum_{l,l'} W_{ll'} \, \delta(\omega - \omega_{ll'}), \tag{17}$$

with weights given by the expression:

$$W_{ll'} = \sum_{k \in K} \bar{D}_{0l} D_{0l'} U_{lk} \bar{U}_{l'k}. \tag{18}$$

Assuming that the η_l are all different, we can read the expression for the long-term average of the expectation value of the occupation, (5), as the zero-frequency component:

$$n_0 = \sum_l W_{ll} = \sum_l \sum_{k \in K} |D_{0l}|^2 |U_{lk}|^2. \tag{19}$$

For a finite system, expression (19) always makes sense and converges to the long term average of the occupation as long as there are no degeneracies in the single-particle spectrum of H, $\{\eta_l\}$. A relevant question in practice is what does *long term* mean exactly.

The answer is: long enough for all non-zero frequencies in expression (17) to average out, which will require a time inversely proportional to the slowest non-zero value of $\eta_l - \eta_{l'}$.

4 Numerical Results

We have performed numerical simulations in order to explore the relation between the local temperature, the thermometer occupation and the local properties of the state. In all cases, unless otherwise specified, we choose the thermometer chemical potential $\mu = 0.5$ and $g = 0.1$.

In figure 2 we show the time evolution of the expected value of the occupation of the thermometer $\langle n_0(t) \rangle$ when it is attached to different sites of a $L = 500$ fermionic Rindler-like chain with couplings of the form $t_i = t_0 + i\Delta t$ ($t_0 = 0.6$ and $\Delta t = 0.4$) and open boundaries. Notice that the different values of the long-time average are easy to spot from the beginning, and rather marked. The periodic bursts are related to the time taken by the perturbation created by the quench to bounce back at the boundaries and return.
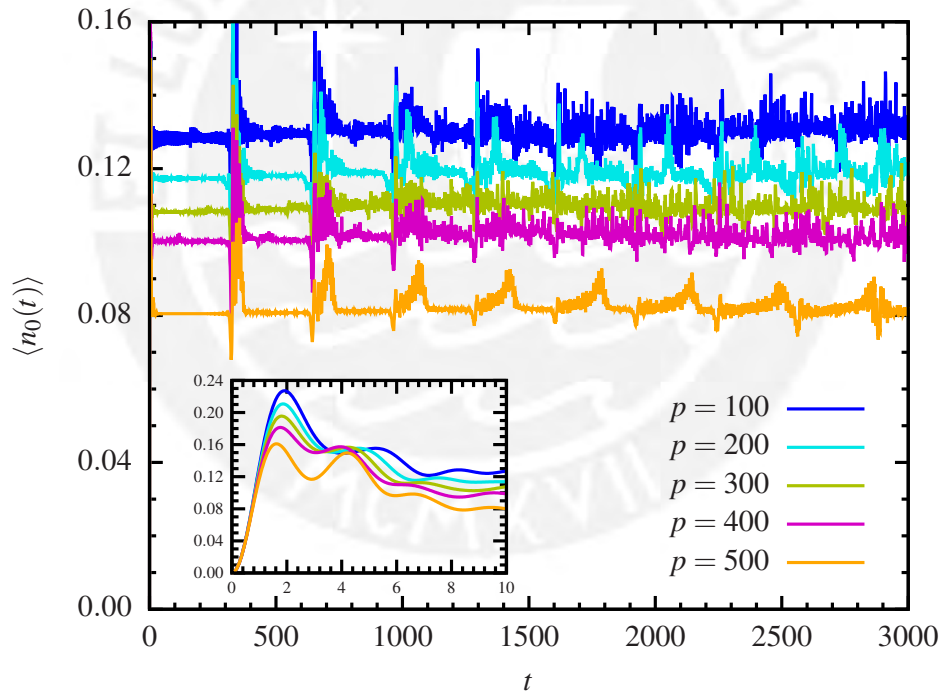

Figure 2: Time evolution of the expected value of $\langle n_0(t) \rangle$, i.e., the occupation of the thermometer site, for a fermionic inhomogeneous hopping model with Rindler-like metric and $L = 500$ and couplings $t_i = 0.6 + 0.4(i/L)$, using $\mu = 0.55$ and $g = 0.6$. The inset shows the same values for shorter times. Notice that the initial values, $\langle n_0(0) \rangle = 0$ in all cases, but it jumps to a high level in a very short time.

Figure 3 shows the inverse of the long term average of the occupation of the thermometer when attached at different sites, $n_0(x)^{-1}$, obtained using Eq. (19), for different background geometries, which we will describe from top to bottom. (A) A constant

hopping term, $t_i = 1$, both with open and periodic boundary conditions (OBC and PBC) for a system with $L = 500$. For PBC, the occupation is homogeneous due to the translation invariance. For OBC, the average value of n_0^{-1} is the same as for PBC, but we observe large fluctuations due to the boundaries. (B) Rindler chain,

$$t_i = i\Delta t, \tag{20}$$

with fixed Δt and open boundaries. The left extreme of the system, $t \sim 0$, behaves similarly to a horizon. We use also $L = 500$, $\Delta t = 0.005$, 0.01 and 0.02, and $g = 0.1$. In this case, the result is more surprising: we observe that $n_0(x)^{-1} \sim x$, in similarity to the growth of the hopping term. Thus, we can assert our main conjecture:

$$n_0(x)^{-1} \sim t(x), \tag{21}$$

where the proportionality constant between them may depend on the parameters of the thermometer, g and μ. This expression, nonetheless, is only approximate. Moreover, the local occupation of the thermometer presents strong parity oscillations. (C) Rainbow chain,

$$t_i = \alpha^{|i-L/2|}, \tag{22}$$

with $\alpha \in (0, 1]$, i.e., the hoppings fall exponentially from the center. The ground state of this system presents volumetric growth of the entanglement [6, 7, 10], and can be interpreted as a thermo-field state. In this case, using $L = 40$ and $\alpha = 0.9$ and 0.7, we also observe the conjectured form (21) to hold approximately. In this case, no parity oscillations appear, but we can see that the occupation saturates when we move away from the center. (D) Sinusoidal chain,

$$t_i = 1 + A\sin(2\pi i/L), \tag{23}$$

which we explore for $L = 500$ and $A = 0.5$ and 1. The first case follows our conjectured form (21) very accurately. The second, $A = 1$ presents a horizon at $i = 3L/4$, and around its neighbourhood our conjecture is less accurate.

The local relation between occupation n_0 and hopping t is further explored in the top panel of figure 4, which plots $g^2 n_0$ vs t for different Rindler systems, varying g, and the
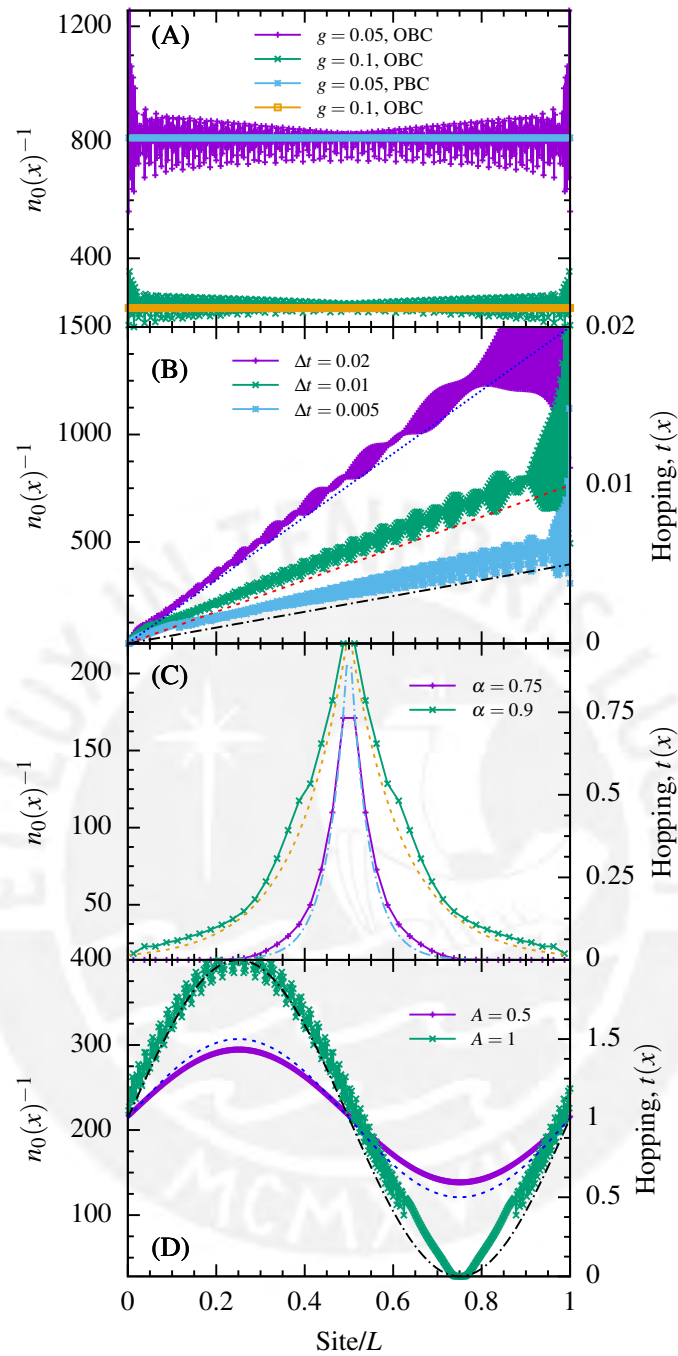
Figure 3: Inverse of the average local thermometer occupation, $n_0(x)^{-1}$ for different background geometries. When suitable, the hopping distribution is shown in dotted lines. (A) Homogeneous system with open and periodic boundary conditions (OBC and PBC) and two different values of g, for a system with $L = 500$ sites and $\mu = 0.5$. (B) Rindler geometry, Eq. (20) with $L = 500$, $\mu = 0.5$ and $g = 0.1$, using $\Delta t = 0.005, 0.01$ and 0.02. (C) Rainbow geometry, Eq. (22), with $L = 40$, $\mu = 0.5$, $g = 0.1$ and two values of $\alpha = 0.9$ and 0.7. (D) Sinusoidal geometry, Eq. (23), with $L = 500$, $\mu = 0.5$, $g = 0.1$ and $t_0 = 1$, with two different amplitudes: $A = 1$ and $A = 1/2$.

sine and rainbow systems. The data seem to collapse to a straight line, which amounts to an improvement of our previous relation (21) to

$$n_0(x)^{-1} \sim t(x)/g^2. \tag{24}$$

The physical reason for the g^2 factor will be explained in the next section. Furthermore, assuming Eq. (24) to be true, we may also conjecture that the local inverse temperature $\beta(x) \equiv T^{-1}(x)$ will behave like

$$\beta(x) \sim \log(t(x)/g^2), \tag{25}$$

and this expression is tested in figure 4, which shows the local hopping in the horizontal axis, in logarithmic scale, and $\beta(x)$ in the vertical one, for most of the systems used in figure 3, using always $g = 0.1$. For large t the relation between β and t is shown to be approximately logarithmic, and for the whole range of values considered they seem to collapse to a single curve. The effect of varying g on the inverse temperature is shown in the inset: it amounts to a vertical additive shift, as it should be apparent from Eq. (25).

The fluctuations in the thermometer occupation can be analysed beyond their long-term average value. The full spectral decomposition of $\langle n_0(t) \rangle$ can be studied using Eq. (17). In figure 5 we show the frequency decomposition of the quantum noise on the thermometer, $|\langle \hat{n}_0(\omega) \rangle|$ for a Rindler system with $t_i = i/L$ and $L = 500$, $g = 0.1$ and $\mu = 0.5$, when the Unruh-DeWitt detector is placed at different sites. Notice that the central peak, which corresponds to the long-term average n_0, is relatively isolated. The active frequencies correspond to a block which gets broader as we move away from the horizon.

For comparison, the inset of figure 5 shows the same spectral decomposition $|\langle \hat{n}_0(\omega) \rangle|$ for the quantum noise of the detector at any point of a homogeneous system. The shape is rather similar to the response functions for Rindler space: the isolated central peak plus the continuous block of frequencies.
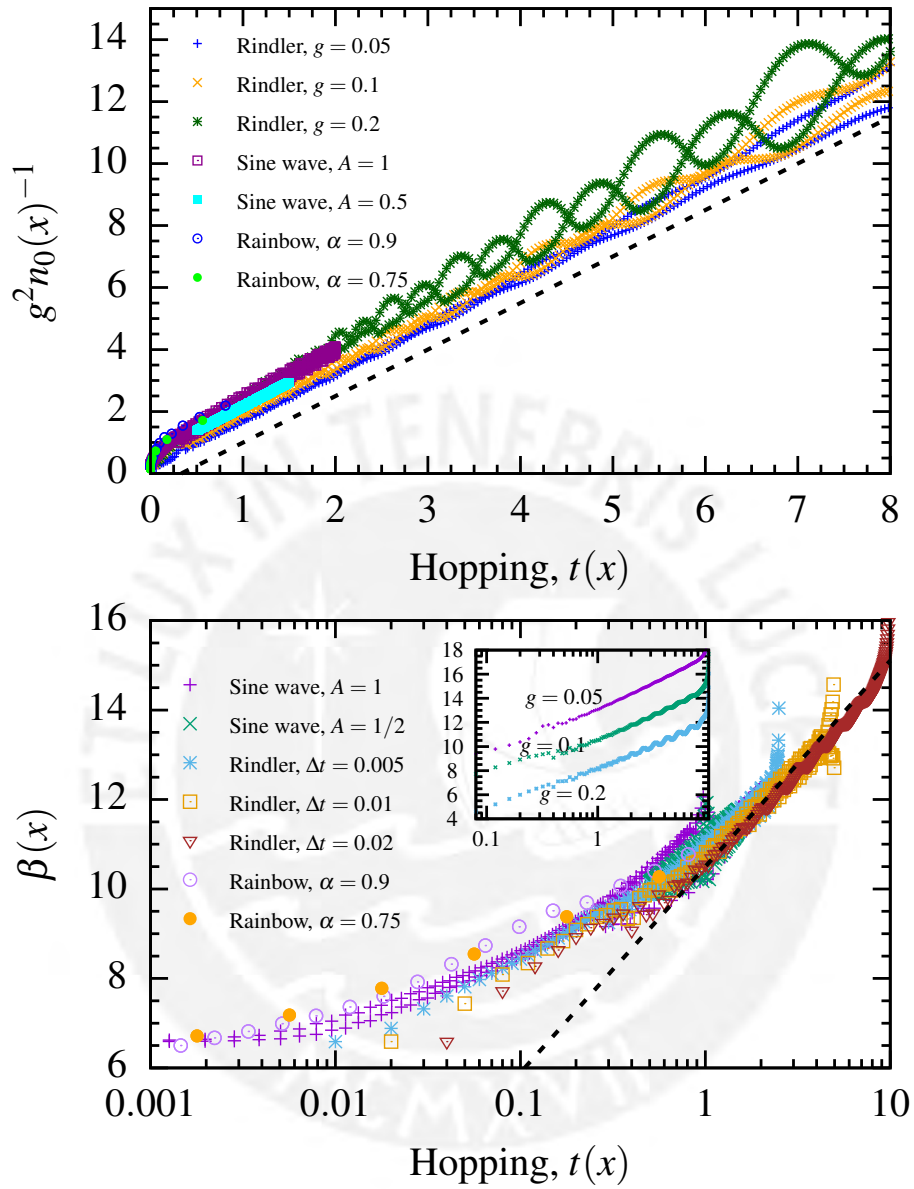
Figure 4: Top: Plot of $g^2 n_0(x)^{-1}$ versus the hopping $t(x)$, for different geometries and values of g, showing the approximate linear relationship. Concretely: Rindler system with $\Delta t = 0.02$ and $g = 0.05$, 0.1 and 0.2, and the rainbow and sinusoidal systems shown in figure 3. The dotted straight line has slope 1.618 and is slightly shifted for clarity. Bottom: Local inverse temperature, $\beta(x)$ plotted against the local hopping $t(x)$ for some of the systems shown in figure 3, always using $g = 0.1$. The hopping axis is shown in log-scale, in order to highlight the nearly logarithmic behavior of the relation between $\beta(x)$ and $t(x)$ for large t, see Eq. (25). Notice the approximate data collapse to a single curve. The dotted line is $10 + 2\log(t)$. Inset: effect of g, shown plotting also β vs t in logarithmic scale, for a Rindler system with $\Delta t = 0.02$ and 500 sites, for three values of g: 0.05, 0.1 and 0.2.
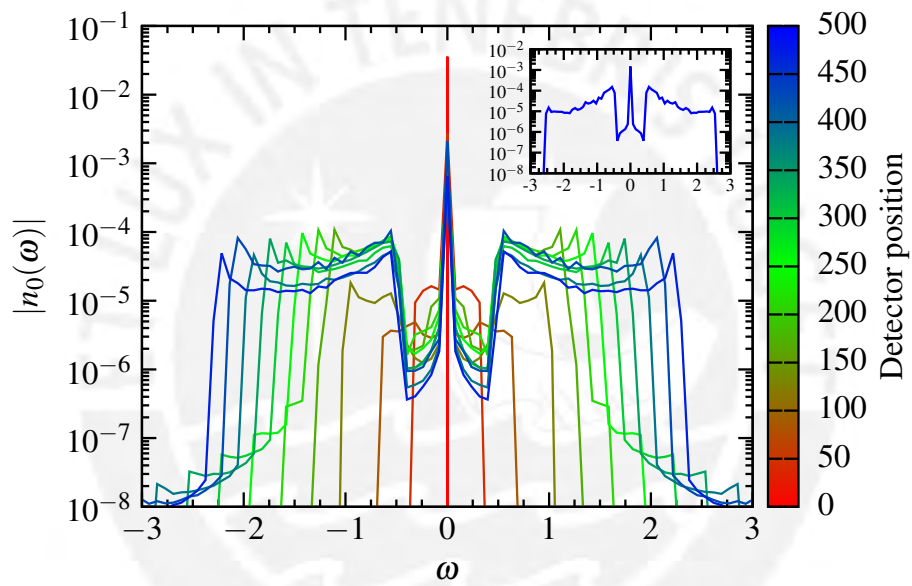
Figure 5: Spectral decomposition of the thermometer noise, $|\langle \hat{n}_0(\omega) \rangle|$ when the detector is placed at different positions over a Rindler system with $L = 500$. Inset: spectral decomposition of the thermometer noise when the detector is placed on a homogeneous system.

5   Single Qubit Detectors

Let us discuss how the the single-body spectrum of a free fermionic system changes when a new site is attached to site p, as shown in Eq. (3), which we will call a *single qubit detector* (SQD), see figure 1 for an illustration. Let the unperturbed system be characterized by a set of single-body orbitals $\left\{ \psi_i^k \right\}$, with energies E_k.

A simple yet very accurate study can be done using a two-level variational approach, in which each deformed single-body state is obtained minimizing the energy within the subspace spanned by the original orbital and the state localized in the new site. For each unperturbed orbital, k, we propose an ansatz of this form:

$$|\Psi\rangle_k = \alpha_k |1\rangle_k \otimes |0\rangle_D + \beta_k |0\rangle_k \otimes |1\rangle_D, \tag{26}$$

where $\{ |0\rangle_k, |1\rangle_k \}$ denote the states in which mode k is either empty or occupied, and the same reads for $\{ |0\rangle_D, |1\rangle_D \}$ and the detector. The effective Hamiltonian of this two-level system can be written as:

$$H_{eff} = \begin{pmatrix} E_k & g\psi_p^k \\ \bar{g}\bar{\psi}_p^k & \mu \end{pmatrix}. \tag{27}$$

Notice that only ψ_p^k is relevant in this approach. The energy shift for the orbital will be given by

$$\tilde{E}_k = \frac{1}{2} \left(E_k + \mu \pm \sqrt{(E_k - \mu)^2 + 4g^2 |\psi_p^k|^2} \right) \tag{28}$$

$$\approx E_k + \frac{g^2 |\psi_p^k|^2}{E_k - \mu}. \tag{29}$$

Notice that the expression presents a pole at $E_k = \mu$, although we will stay safe: μ is always chosen to be sufficiently above the Fermi energy, which is zero in our case. Correspondingly, the probability of finding the fermion in the new site is now

$$|\beta_k|^2 \approx \frac{(\tilde{E}_k - E_k)^2}{(\tilde{E}_k - E_k)^2 + g^2 |\psi_p^k|^2} \approx \frac{g^2 |\psi_p^k|^2}{(E_k - \mu)^2}. \tag{30}$$

The astonishing validity of this approximation can be seen in figure 6, where we compare the exact and the two-level variational results with the exact calculation.
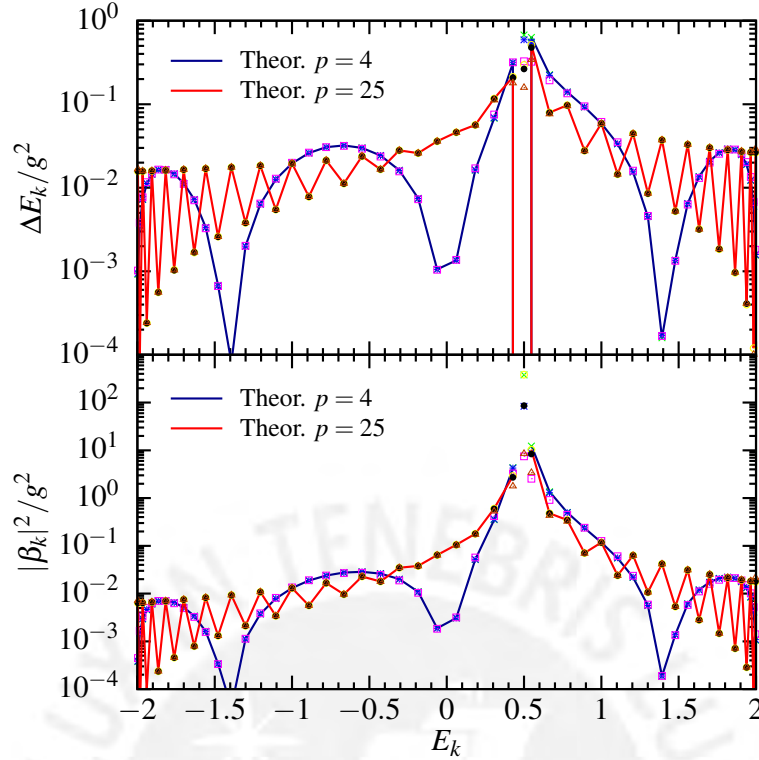
Figure 6: Checking the validity of the two-level variational approach to single qubit detector physics. A hopping model with $L = 50$ sites and open boundaries is attached to a thermometer at site $p = 4$ (blue line) and $p = 25$ (red line). Top: theoretical estimate for the shift in the mode energy $\Delta E_k = \tilde{E}_k - E_k$ due to the presence of the thermometer, as a function of E_k and divided by g^2, Eq. (29). The points are the exact values for $g = 0.05$, 0.1 and 0.25, which are seen to collapse very accurately away from the region $E \sim \mu$. Bottom: theoretical estimate for the occupation of the thermometer site $|\beta_k|^2$ as a function of the unperturbed energy E_k, divided also by g^2, Eq. (30). The points are again the exact values for $g = 0.05$, 0.1 and 0.25.

Returning to expression (19) we can state that $|D_{0l}|^2 = |\beta_l|^2$ and, approximately, $U_{kl} \approx \delta_{lk}$, thus obtaining

$$n_0 \approx \sum_{k \in K} \frac{g^2 |\psi_p^k|^2}{(E_k - \mu)^2}. \tag{31}$$

Notice that the local occupation (and, therefore, the local temperature) is related to the form of the orbitals and the *energy content* at the site to which the detector is attached. The long-term average occupation always depends quadratically with the coupling constant, $n_0 \sim g^2$, for low enough g. Let us remark again that in order to define a proper local temperature one should always take the limit $g \to 0$. As we mentioned before, strictly speaking one should distinguish between the *observed* local temperature, at finite g, and the actual limit value, which is zero everywhere for a ground state.

6   Conclusions and Further Work

In this work, we have presented an operational definition of the local temperature of a quantum system, via the interaction with a single qubit Unruh-DeWitt detector characterized as a two-level system with a (large enough) energy gap μ and a (small enough) coupling constant g. The main observable is the long-term average occupation of the detector, which is shown to have a mild dependence on μ if it is sufficiently above the Fermi energy[3].

We have studied the behavior of the detector occupation and the associated local temperature on the ground state of free fermionic systems in 1D with inhomogeneous hopping parameters $t(x)$, which can be understood as the time-lapse, $t(x) \sim |g_{00}(x)|^{1/2}$, of a background static geometry (or local speed of light). Since we operate at zero temperature, the energy to excite the detector must come only from the coupling between the thermometer and the system, measured by the coupling constant g. Indeed, the local thermometer occupation is always proportional to g^2 and tends to zero as $g \to 0$. Thus, properly speaking, the measured temperature is always zero. Yet, for small but finite values of g we find an approximate inverse proportionality between the long-term average occupation of the detector and the time-lapse, $n_0^{-1} \sim t(x)/g^2$. Thus, for finite g, the *observed* local temperatures are larger where the local speed of light is smaller, bearing similarity to the Tolman-Ehrenfest theorem from thermodynamics on curved space-time, which states that for a system in thermal equilibrium on a static metric, $T(x) \cdot t(x)$ is a constant. Yet, in opposition to it, we find that, for finite g, $T_g(x) \cdot \log t(x) \sim$ const. Our result does not contradict the Tolman-Ehrenfest theorem because the limit of $T_g(x)$ for vanishing coupling g is zero.

Our main claim, $n_0(x)^{-1} \sim t(x)/g^2$, seems to remain approximately valid for a wide variety of inhomogeneities: linear (Rindler), exponential (rainbow) or sinuosidal hoppings. Nonetheless, a theoretical explanation and a discussion of its validity are left for further work. It is relevant to ask whether it remains valid in higher dimensions, in different topologies, or in the presence of interactions.

The most relevant question is how our technique will work in the case of equilibrium states at a finite temperature or non-equilibrium systems. In that case, there are some relevant approaches in the literature to define local and effective temperatures. A well

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[3] We would like to stress that, despite of our use of Unruh-DeWitt detectors, our measurement is not of an Unruh temperature.

tested procedure is to attach a local thermal reservoir at temperature T_0 locally to the system, and find the value of T_0 for which the heat flow between the bath and the system vanishes [35–40]. This approach is operational, like ours, and will also yield zero temperature for the ground state. The main advantage of our approach is that it explores the possibility of *measuring the temperature without a thermal bath*, and thus it is better suited for pure quantum environments, such as ultracold atomic gases. Another tested approach is based on the fluctuation-dissipation theorem [33]. The temperature is defined from the relation between the response to an impulse perturbation and the correlation function. Thus, as opposed to the previous case, it is not an operational definition, and it presents several technical issues in the quantum regime [34]. Nonetheless, one of the most relevant insights from the technique is that the temperature can be frequency-dependent, a feature that can be obtained from our full frequency occupation $\hat{n}_0(\omega)$, Eq. (17). A good extension of the definition of temperature should respect the principles of thermodynamics. Both approaches mentioned above are known to respect the second principle, but for our technique this is still to be proved.

Although our procedure is inspired by the Unruh-DeWitt detector, it is also important to stress the difference between the local temperature measured and the Unruh temperature. In order to observe the Unruh effect, an observer will move with constant acceleration through the Minkowski vacuum. From her point of view, this motion will translate into a change of her metric, which will become Rindler. Thus, as opposed to our case, she will observe the Minkowski vacuum through the lens of a Rindler Hamiltonian, as shown in [5].

As a last remark, we would like to stress that our proposal for the definition of the local temperature is operational, and therefore it can lead to experimental observation. An interesting setting would be using ultracold atoms on an optical lattice.

References

[1] M. Lewenstein, A. Sanpera and V. Ahufinger, *Ultracold Atoms in Optical Lattices: Simulating Quantum Many-body Systems*. EBSCO ebook academic collection. OUP Oxford, 2012. (Cited on page 1.)

[2] O. Boada, A. Celi, J. I. Latorre and M. Lewenstein, *Dirac equation for cold atoms in artificial curved spacetimes*, *New Journal of Physics* **13** (2011) 035002, [1010.1716]. (Cited on pages 1 and 6.)

[3] O. Boada, A. Celi, J. I. Latorre and M. Lewenstein, *Quantum simulation of an extra dimension*, *Phys. Rev. Lett.* **108** (2012) 133001, [1112.1019]. (Cited on page 1.)

[4] O. Boada, A. Celi, J. Rodríguez-Laguna, J. I. Latorre and M. Lewenstein, *Quantum simulation of non-trivial topology*, *New Journal of Physics* **17** (2015) 045007, [1409.4770]. (Cited on page 1.)

[5] J. Rodríguez-Laguna, L. Tarruell, M. Lewenstein and A. Celi, *Synthetic unruh effect in cold atoms*, *Phys. Rev. A* **95** (2017) 013627, [1606.09505]. (Cited on pages 1 and 18.)

[6] G. Vitagliano, A. Riera and J. I. Latorre, *Volume-law scaling for the entanglement entropy in spin-1/2 chains*, *New Journal of Physics* **12** (2010) 113049, [1003.1292]. (Cited on pages 1 and 10.)

[7] G. Ramírez, J. Rodríguez-Laguna and G. Sierra, *From conformal to volume law for the entanglement entropy in exponentially deformed critical spin 1/2 chains*, *Journal of Statistical Mechanics: Theory and Experiment* **2014** (2014) P10004, [1407.3456]. (Cited on pages 1 and 10.)

[8] M. Srednicki, *Entropy and area*, *Phys. Rev. Lett.* **71** (1993) 666–669, [hep-th/9303048]. (Cited on page 1.)

[9] J. Eisert, M. Cramer and M. B. Plenio, *Colloquium: Area laws for the entanglement entropy*, *Rev. Mod. Phys.* **82** (2010) 277–306, [0808.3773]. (Cited on page 1.)

[10] G. Ramírez, J. Rodríguez-Laguna and G. Sierra, *Entanglement over the rainbow*, *Journal of Statistical Mechanics: Theory and Experiment* **2015** (2015) P06002, [1503.02695]. (Cited on pages 1 and 10.)

[11] R. C. Tolman and P. Ehrenfest, *Temperature equilibrium in a static gravitational field*, *Phys. Rev.* **36** (1930) 1791–1798. (Cited on page 1.)

[12] S. Takagi, *Vacuum noise and stress induced by uniform accelerationhawking-unruh effect in rindler manifold of arbitrary dimension*, *Progress of Theoretical Physics Supplement* **88** (1986) 1–142. (Cited on pages 1 and 2.)

[13] C. Rovelli and M. Smerlak, *Thermal time and tolman–ehrenfest effect: 'temperature as the speed of time'*, *Classical and Quantum Gravity* **28** (2011) 075007, [1005.2985]. (Cited on page 1.)

[14] W. G. Unruh, *Notes on black-hole evaporation*, *Phys. Rev. D* **14** (1976) 870–892. (Cited on page 1.)

[15] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, 1982. (Cited on pages 1, 2, and 5.)

[16] D. Buchholz and R. Verch, *Macroscopic aspects of the unruh effect*, *Classical and Quantum Gravity* **32** (2015) 245004, [`1412.5892`]. (Cited on page 2.)

[17] D. Buchholz and R. Verch, *Unruh versus Tolman: on the heat of acceleration*, *General Relativity and Gravitation* **48** (2016) 32, [`1505.01686`]. (Cited on page 2.)

[18] L. C. B. Crispino, A. Higuchi and G. E. A. Matsas, *The Unruh effect and its applications*, *Rev. Mod. Phys.* **80** (2008) 787–838, [`0710.5373`]. (Cited on page 2.)

[19] E. Martín-Martínez, A. Dragan, R. B. Mann and I. Fuentes, *Berry phase quantum thermometer*, *New Journal of Physics* **15** (2013) 053036, [`1112.3530`]. (Cited on page 2.)

[20] J. Bell and J. Leinaas, *Electrons as accelerated thermometers*, *Nuclear Physics B* **212** (1983) 131 – 150. (Cited on page 2.)

[21] E. T. Akhmedov and D. Singleton, *On the relation between unruh and sokolov–ternov effectss*, *International Journal of Modern Physics A* **22** (2007) 4797–4823, [`hep-ph/0610391`]. (Cited on page 2.)

[22] M. Kliesch, C. Gogolin, M. J. Kastoryano, A. Riera and J. Eisert, *Locality of temperature*, *Phys. Rev. X* **4** (2014) 031019, [`1309.0816`]. (Cited on page 2.)

[23] S. Hernández-Santana, A. Riera, K. V. Hovhannisyan, M. Perarnau-Llobet, L. Tagliacozzo and A. Acín, *Locality of temperature in spin chains*, *New Journal of Physics* **17** (2015) 085007, [`1506.04060`]. (Cited on page 2.)

[24] A. de Pasquale, D. Rossini, R. Fazio and V. Giovannetti, *Local quantum thermal susceptibility*, *Nature Communications* **7** (2016) 12782, [`1504.07787`]. (Cited on page 2.)

[25] S. Jevtic, D. Newman, T. Rudolph and T. M. Stace, *Single-qubit thermometry*, *Phys. Rev. A* **91** (2015) 012331, [`1408.6967`]. (Cited on page 2.)

[26] L. A. Correa, M. Mehboudi, G. Adesso and A. Sanpera, *Individual quantum probes for optimal thermometry*, *Phys. Rev. Lett.* **114** (2015) 220405, [`1411.2437`]. (Cited on page 2.)

[27] C. Raitz, A. M. Souza, R. Auccaise, R. S. Sarthour and I. S. Oliveira, *Experimental implementation of a nonthermalizing quantum thermometer*, *Quantum Information Processing* **14** (2015) 37–46. (Cited on page 2.)

[28] M. Brunelli, S. Olivares and M. G. A. Paris, *Qubit thermometry for micromechanical resonators*, *Phys. Rev. A* **84** (2011) 032105, [`1103.2875`]. (Cited on page 2.)

[29] J. Uffink and J. van Lith, *Thermodynamic uncertainty relations*, *Foundations of Physics* **29** (1999) 655–692, [`cond-mat/9806102`]. (Cited on page 2.)

[30] M. Mehboudi, M. Moreno-Cardoner, G. D. Chiara and A. Sanpera, *Thermometry precision in strongly correlated ultracold lattice gases*, *New Journal of Physics* **17** (2015) 055020, [`1501.03095`]. (Cited on page 2.)

[31] M. G. A. Paris, *Achieving the landau bound to precision of quantum thermometry in systems with vanishing gap*, *Journal of Physics A: Mathematical and Theoretical* **49** (2016) 03LT02, [`1510.08111`]. (Cited on page 2.)

[32] L. F. Cugliandolo, J. Kurchan and L. Peliti, *Energy flow, partial equilibration, and effective temperatures in systems with slow dynamics*, *Phys. Rev. E* **55** (1997) 3898–3914, [`cond-mat/9611044`]. (Cited on page 3.)

[33] L. F. Cugliandolo, *The effective temperature*, *Journal of Physics A: Mathematical and Theoretical* **44** (2011) 483001, [`1104.4901`]. (Cited on pages 3 and 18.)

[34] L. Foini, L. F. Cugliandolo and A. Gambassi, *Fluctuation-dissipation relations and critical quenches in the transverse field ising chain*, *Phys. Rev. B* **84** (2011) 212404, [`1107.5956`]. (Cited on pages 3 and 18.)

[35] H.-L. Engquist and P. W. Anderson, *Definition and measurement of the electrical and thermal resistances*, *Phys. Rev. B* **24** (1981) 1151–1154. (Cited on pages 3 and 18.)

[36] Y. Dubi and M. Di Ventra, *Thermoelectric effects in nanoscale junctions*, *Nano Letters* **9** (2009) 97–101. (Cited on pages 3 and 18.)

[37] A. Caso, L. Arrachea and G. S. Lozano, *Local and effective temperatures of quantum driven systems*, *Phys. Rev. B* **81** (2010) 041301, [`0907.1515`]. (Cited on pages 3 and 18.)

[38] A. Caso, L. Arrachea and G. S. Lozano, *Local temperatures and heat flow in quantum driven systems*, *Phys. Rev. B* **83** (2011) 165419, [`1102.4305`]. (Cited on pages 3 and 18.)

[39] F. G. Eich, M. Di Ventra and G. Vignale, *Temperature-driven transient charge and heat currents in nanoscale conductors*, *Phys. Rev. B* **93** (2016) 134309, [`1601.07807`]. (Cited on pages 3 and 18.)

[40] A. Shastry and C. A. Stafford, *Cold spots in quantum systems far from equilibrium: Local entropies and temperatures near absolute zero*, *Phys. Rev. B* **92** (2015) 245417, [`1508.03385`]. (Cited on pages 3 and 18.)