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ESTIMATION OF THE SOVEREIGN YIELD CURVE OF PERU: THE ROLE OF MACROECONOMIC AND LATENT FACTORS

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RESUMEN EJECUTIVO

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Abstract

The study of the yield curve has been a topic that interested economists for a long time since the term structure of interest rates is an important transmission channel of monetary policy to inflation and real activity. In this paper, following Ang and Piazzesi (2003), we study the relevance of macroeconomic factors on Peruvian sovereign yield curve through an Affine Term Structure model for the period from November 2005 to December 2015. We estimate a Gaussian model to understand the joint dynamics of macro variables -inflation and real activity factors- and Peruvian bond yields in a multifactor model of the term structure. Risk premia are modeled as time varying and depend on both observable and unobservable factors. A Vector Autoregressive (VAR) model is estimated considering no-arbitrage assumptions, which let us to derive Impulse Response Functions and Variance Decompositions. We find evidence that macro factors help to improve the fit of the model and explained a substantial amount of variation in bond yields. Variance decompositions show that macro factors explain a significant amount of the movements in the short and middle segments of the yield curve (up to 50%) while unobservable factors are the main drivers for most of the movements at the long end of the yield curve (up to 80%). Furthermore, we find that setting no-arbitrage restrictions improve the forecasting performance of a VAR and that models that include macro factors forecast better than models with only unobservable components.

JEL Classification Number: C13, C32, E43, E44, E52, G12

Keywords: Affine Term Structure Models, Macroeconomic Factors, Risk Premia, Yield Curve, Financial Markets, Monetary Policy.

1 Introduction

The last international financial crisis has shown that open economies with underdeveloped domestic debt markets are highly vulnerable to external capital flows. Then, the development of the capital markets in Peru is an essential task since it allows local investors to have an alternative source of funding that improves domestic savings-investment ratio and protect the country against scenarios of tight liquidity. Consequently, since the beginning of the Market Maker Program[1], the Peruvian government has shown a great effort to promote the development of local financial markets and to achieve the development of a sovereign yield curve.

With this in mind, the study of the joint behavior of the yield curve and macroeconomic variables becomes relevant for various reasons. One of these reasons is forecasting, which is based on the theory of rational expectations. According to this theory, the yield curve provides information on the future behavior of the economy since yields of long-term bonds represent the expected value of average future short yields. This means that the study of the yield curve is relevant because it provides a support to the consumption and investment decisions of economic agents, from its ability to predict the behavior of short term interest rates, real activity and inflation; see Campbell and Shiller (1991); Ang, Piazzesi and Wei (2006), Fama (1990). Furthermore, their study is important for the measurement of financial instruments, which is necessary for the development of capital markets. In particular, the yield curve is useful for derivative pricing and hedging, see Duffie et al (2000).

Another important reason for studying the yield curve is its influence on Monetary Policy. The study of the yield curve has been a topic that interested economists for a long time since the term structure of interest rates is an important transmission channel of monetary policy to inflation and real activity. In fact, the impact of the reference interest rate in the short end of the yield curve has an impact on other yields since long term yield dynamics are determined from short term rate expectations and agents' aversion risk, see Evans and Marshall (1998, 2001). Thus, the yield curve is a useful tool for monetary policy because it provides relevant information about the expectations of macroeconomic variables. In addition, this study is relevant for the development of Fiscal Policy since the yield curve influences the decisions of Debt Policy. In particular, the knowledge of its behavior gives the government the ability to decide on its debt structure and on its financing costs, through the implementation of debt management operations according to the economic situation.

Furthermore, the ineffectiveness of Fed monetary policy to change the long term yields in the

[1] The Market Maker Program in Peru was born as part of the "Strategy of Auctions and Public Debt Management Operations", Working Paper published by the Ministry of Economy and Finance of Peru in 2003. (Resolución Ministerial 106-2003-EF/75)

US during the crisis reopened an old discussion about whether macro factors that determine short term interest rates also influences the dynamics of long term interest rates to whether the factors that determine interest rates on the short term are the same as those factors that determine the long term yields. This is a very old debate, from models that assert the existence of a relationship between short-term and long-term interest rates through macro factors to statistical models, that deny any relationship and affirm that only statistical factors help to determine the yield curve. In the middle of this discussion, various methodologies that try to explain the peculiarities of the yield curve have been developed. One of these methodologies was developed by Ang and Piazzesi (2003) and is known as Affine Term Structure Models. We apply this methodology to study the relevance of macroeconomic factors on Peruvian sovereign yield curve for the period from November 2005 to December 2015. In particular, we focus on the analysis of compliance of the hypothesis of rational expectations in the framework of a macroeconomic model. Therefore, we estimate the variables that influence the risk aversion of investors to different terms and we assess whether it is possible that in addition to statistical factors, macro variables are relevant to determine the Peruvian sovereign yield curve if the dynamics are properly modeled. In that sense, this research tries to answer the following questions: What variables govern the whole term structure of interest rates and what is their relationship with the state of the real economy and monetary policy that control interest rates anytime soon? How the fundamental projections, incorporated in macroeconomic variables, can be taken into account in the estimation of the bond yields, when they are well described by unobservable variables or latent factors that determine the level, the slope and the curvature of the yield curve?, among other questions of interest to the Peruvian macroeconomic and financial literature.

In particular, we estimate a Gaussian model for the Peruvian Yield curve considering observable macroeconomic variables and unobservable latent factors. In fact, we estimate and compare two models. The first model presents only latent factors and is called the "Yields-Only Model". The second model considers latent factors in interaction with a Taylor Rule, that includes macro factors, and it is called the "Macro Model". Through Principal Component Analysis, we condensed our macro variables from a set of time series in two variables: an inflation factor and a real activity factor. Risk premia are modeled as time varying and depend on both observable and unobservable factors. The Vector Autorregressive (VAR) model is estimated considering no-arbitrage assumptions, so that Impulse Response Functions (IRFs) and Variance Decompositions can be derived.

We find evidence that macro factors help to improve the forecast errors of a VAR model and explain a significant amount of the variance presented in bond yields. In fact, positive shocks to macro factors increases the yields. The response of yields to inflation shocks is greater than the response to real activity shocks across all maturities. Variance decompositions show that macro factors explain a significant amount of the movements at the short and middle segments of the yield curve (up to 50%) while unobservable factors are the main drivers for most of the movements at the long end of the yield curve (up to 80%). Compared to the Yields-Only model, the “level” factor effect prevails when macro factors are incorporated. Finally, we find that no-arbitrage restrictions with the incorporation of macro factors improve forecasts.

The paper is organized as follows. Section 2 presents a brief literature review of studies about the dynamics of bond yields. Section 3 describes the methodology of Affine Term Structure models. Section 4 present the results and a brief discussion of the implied Impulse Response Functions and Variance Decompositions. Finally, in Section 5 the conclusions are presented.

2 Literature Review

This document studies the effects of macroeconomic variables on the Peruvian yield curve, and let us to understand its movements based on the expectations of macroeconomic variables. Thus, this section provides a review of studies about the dynamics of the bond yields, that includes models that take into account macroeconomic variables. It should be noted that this literature has been mainly applied to developed countries, especially in the USA.

In general, the literature that studies the movements in the yield curve can be classified in different ways. One classification is by the number of factors, establishing that there are one-factor models, see Vasicek (1997), and multi-factor models such as in Litterman and Sheinkman (1991). Another classification is by the nature of factors. According to this classification, there are models with observable factors, see Campbell and Shiller (1991) and Evans and Marshall (2001), and latent factors, see Duffee (2002). Finally, in the last years there have been numerous studies that describe the yield curve movements based on the behavior of certain macroeconomic variables and unobservable factors.

2.1 Literature in Developed Countries

Vasicek (1977) and Cox, Ingersoll and Ross (1985) are the first works that impose the no-arbitrage condition. In these studies, the short term interest rate is the only factor that determines the movements of the term structure of interest rates. Since these type of models have a bad performance explaining the movements in the yield curve, multi-factor models appeared. Litterman and Scheinkman (1991) and Diebold and Li (2006) propose that three factors help to explain the term structure of the interest rates: level, slope and curvature. These models became popular for the reduction of the number of parameters, the ease of estimation and the accuracy of the obtained factors.

On the other hand, there is an approach that pursues to explain the behavior of the yield curve and its relationship with macroeconomic variables. Campbell and Shiller (1991) assess the fulfillment of the expectations theory in determining the U.S. term structure postwar. They find that for the majority of combinations of maturities between one month and ten years, higher spread between long and short term interest rates forecasts an increase in the short end and long end of the yield curve. Thus, the pattern found is inconsistent with the expectations theory of the term structure of interest rates, but it is consistent with a model in which the spread between short term and long term interest rates is proportional to the implied value of the theory of expectations. They explain that this deviation could be generated because of time-varying risk premia, that are correlated with expected increases in short term interest rates.

Evans and Marshall (2001) study the effects of different types of macroeconomic impulses on the nominal yield curve. They use different approaches to identify the economic shocks in the form of a VAR model. The first approach applies a structural VAR following the identification proposed by Galí (1992), which includes the variables of industrial production, the benchmark of the United States, the real interest rate and the real monetary levels. Under a second approach, the authors identify fundamental Impulse Response Functions from different empirical measures of economic shocks proposed in the literature. They find that macroeconomic impulses determine most of the variability of long term interest rates for all maturities.

In recent years, a new methodology that tries to explain the joint behavior of bond yields and macroeconomic variables has been developed. These models provide relevant information about the way these macro variables affect the shape of the term structure and vice versa. Most of these models consider economic variables such as inflation and real activity or employment and the policy interest rate. They are based upon the reaction function of the monetary policy to shocks of these two variables and the transmission of changes in the short end to the long end of the yield curve. In fact, the use of the policy interest rate tries to cover any monetary shock unrelated to these variables.

An example of this approach is Ang and Piazzesi (2003). They describe the joint behavior of the yield curve and macroeconomic variables through the use of an Affine Term Structure model. In particular, they use a Taylor rule for the short term interest rate and an affine model for the rest of the yield curve. Thus, the model is estimated in two steps. First, they estimate the macro dynamics and the short rate equation treating latent variables as monetary policy shocks. In the second step, the previous parameters calculated are hold fixed, and the other parameters of the term structure model are estimated through the maximum likelihood. The authors find that the performance of a VAR improves when they introduce no-arbitrage restrictions and they show that models with macro factors forecast better than models with only unobservable factors. Furthermore, Impulse Response Functions and Variance Decomposition analysis show that macro factors (inflation and real activity) explain up to 85% of the variation in bond yields at the short end and middle of the yield curve, but only 40% at the long end of the curve. Thus, they conclude that macro factors mainly determine movements at the short end and middle end of the yield curve while unobservable factors explain most of the movements at the long end of the yield curve.

Pericoli and Taboga (2008) develop a canonical representation for the no-arbitrage discrete-time term structure models, that consider observable and unobservable state variables. They analyze how different parameterization can affect estimated risk premia, Impulse Response Functions and Variance Decompositions. Thus, their specification provides a better comprehension on the advantages and disadvantages of alternative modeling approaches. They identify a trade-off between the need to achieve parsimonious parameterizations and the effectiveness of the models to match observed patterns of variation in risk premia. Furthermore, they notice that an ample set of parameterizations are required to capture the empirical properties of bond returns and the autocorrelation structure of the state variables that drive bond yields.

Halberstadt and Stapf (2012), analyze the dynamics of the German yield curve and the risk premium for the period between 1999 and 2010. The authors estimate two model specifications. While the first presents only latent factors, the other model consider latent factors and a Taylor rule that includes a price factor and a real activity factor. These factors are derived from an ample macroeconomic data set. Halberstadt and Stapf (2012) find that macroeconomic factors, especially the real activity factor, help to improve the accuracy of the model. Furthermore, they analyze the effect of the macroeconomic factors on the risk aversion of market participants. The authors also notice that, in the recent financial crisis, the market prices of risk for the macro factors changed significantly. In times of crisis, the increase in yield risk premia is fewer at the short end as compared to longer maturities since offsetting safe haven flows affects shorter maturities. Finally, they include a liquidity stress factor in the macro model to show the influence of the slope during times of crises, which is associated with the effect of the safe haven flows.

2.2 Literature in Emerging Countries

The financial markets of emerging countries are incipient and their debt instruments are illiquid. However, there is little literature developed about the interaction of macroeconomic variables and the term structure of interest rates.

In Mexico, Cortés and Ramos (2008) investigate the way that different macroeconomic shocks affect the term structure of interest rates. In particular, they elaborate a model that includes a no-arbitrage specification for the term structure in a context of a small open economy. They find that the level of the yield curve is affected by persistent shocks on inflation Furthermore, the notice that increases in expected future short rates and the expansion of risk premia affect the medium and long term bond yields. Finally, the authors demonstrate that a positive demand shock cause an upward flattening shift in the yield curve. This result is explained by both the monetary policy response and the time varying term premia.

In Chile, Morales (2008) estimates a dynamic model for the yield curve incorporating latent and macro factors. He uses the yield curve following Nelson and Siegel (1987), but considering a dynamic interpretation based on Diebold and Li (2006). After assuming that the data generating process for the latent and macro factors can be represented by a VAR process, he uses a state-space representation and he estimates the yields-macro model by a Kalman Filter approach and by using a simplified two-step procedure proposed by Diebold and Li (2006). He finds that the results are not significantly affected by the use of the simplified approach. Furthermore, he concludes that the level and the slope seemed to be responsive to real activity and monetary policy shocks.

In Colombia, Melo and Castro (2010) apply the Diebold, Rudebusch and Arouba (2004) methodology to represent the term structure of interest rates. They model the yield curve with three latent factors following Nelson and Siegel (1987) and macroeconomic variables. In particular, they use a state-space representation and estimate a VAR with these factors. They conclude that there is a bidirectional relationship between macro variables and latent factors. However, they find that Granger causality is stronger from macro variables to latent factors.

2.3 Literature in Peru

The literature on yield curves in Peru is limited especially when this literature is linked to macroeconomic variables. However, there are some studies that should be mentioned. Rodriguez and Villavicencio (2005) discuss the formation of the Peruvian yield curve and the evolution of its different sections as responses to different domestic policies and external events. They estimate the structure of the zero coupon spot yields applying the method proposed by Nelson and Siegel (1987). They find that the yield curve is very sensitive to internal events, such as the issuance of a new long term bond, and to external events, such as changes in international interest rates.

Pereda (2009) estimates two models for the Peruvian yield curve using the methodologies developed by Nelson and Siegel (1987) and Svensson (1994), respectively. In particular, he compares the performance of both models in terms of accuracy, flexibility and stability of parameters. Although he finds that the model of Svensson (1994) has a better performance, it is less stable while there is not enough data for the different term maturities estimated. This result is explained by the absence of issues and the illiquidity of secondary market. Thus, he concludes that the use of the model of Nelson and Siegel (1987) is recommended.

Jauregui and Valdivia (2012) study the behavior of Peruvian sovereign curve to predict levels and movements in the term structure of interest rates through models based on condition "arbitrage" and statistical models. They find that the Diebold-Li model presents a greater predictive power than the CIR model since it forecasts with a lower minor deviation out of sample. They also develop the estimation of the expanded model with macroeconomic variables and they find evidence in favor of the interaction between the yield curve and macroeconomic variables.

Carrillo and Montes (2014) study also the relationship between macro variables and the Peruvian yield curve. They use a dynamic version of Nelson and Siegel (1987) model that let them to obtain the three dynamic factors of the yield curve: level, slope and curvature. They represent the interaction of these factors with macroeconomic variables through a VAR. The state space-representation measures the effects of the state variables (latent factors and macro variables) on the yields of different terms. Thus, they find evidence in favor of the dynamic interaction between the yield curve and macroeconomic variables, such us inflation, product and the interbank rates.

In this document, the Peruvian sovereign yield curve will be represented by a Gaussian Arbitrage-free affine model. Therefore, we test the hypothesis of rational expectations based on a macroeconomic model. In particular, macroeconomic variables are comprised in a price factor and a real activity factor, which are incorporated into the yield curve model based on a representation of pricing Kernel, that determines the prices for every existing bond. In that sense, this paper aims to provide a contribution to the study of the joint behavior of macroeconomic factors and Peruvian sovereign bonds.

3 Methodology

This section describes the methodology of Affine Term Structure models that is used to estimate bond yields. In particular, following the approach of Ang and Piazzesi (2003), we describe a Gaussian affine model with two state vectors, one containing the latent factors and the second containing macroeconomic variables. The model incorporates observable macroeconomic variables with Unobservable Components or latent factors. Risk premia are modeled as time varying, because they are consider to be affine in potentially all of the underlying factors. Our notation closely follows that adopted in Ang and Piazzesi (2003).

3.1 General Setup

3.1.1 State Dynamics

The model assumes the existence of k_1 observable macro variables (f_t^o) and k_2 latent factors (f_t^u) In particular, the vector $F_t = (f_t^o, f_t^u)$ follows a Gaussian VAR (p) process:

$$F_t = \Phi_0 + \Phi_1 F_{t-1} + \Phi_2 F_{t-2} + ... + \Phi_p F_{t-p} + \theta u_t, \tag{1}$$

where $u_t \sim IIDN(O,\ I)$. The state of the economy is then described by a k-dimensional vector of state variables X_t where $k = k_1 p + k_2$ This vector is partitioned into $k_1 p$ observable variables and k_2 unobservable variables X_t^u. The observable vector consists of current and past levels of macroeconomic variables $X_t^o = (f_t^{o'}, f_{t-1}^{o'}, ..., f_{t-p-1}^{o'})'$ where $X_t^u = f_t^u$ only incorporates contemporaneous latent factors (Unobservable Components). Thus, latent factors f_t^u follow an AR(1) process in which the coefficients $\Phi_0 ....\Phi_p$ of the equation corresponding to $X_t^u = f_t^u$ are equal to 0.

There are two groups of macroeconomic variables: the group of inflation measures and the group that captures real activity. Following Ang and Piazzesi (2003) the dynamics of $X_t = (X_t^{o'}, X_t^{u'})$ is set as a first order Gaussian VAR:

$$X_t = \mu_t + \Phi X_t + \Sigma \epsilon_t, \tag{2}$$

with $\epsilon_t = \left(u_t^{0'}, 0, ...0, u_t^{u'}\right)$ where u_t^0 and u_t^u are the shocks to the observable and unobservable factors, respectively. In the first order companion form, the $k \times k$ matrix Σ contains blocks of zeros to accommodate higher order lags in F_t.

3.1.2 The Short Rate Equation

The short rate r_t is assumed to be an affine function of all state variables:

$$r_t = \gamma_0 + \gamma_1' X_t, \tag{3}$$

where the three-month yield y_t^3 is used as an observable short rate r_t. In particular, we estimate a model based on the policy rule recommended by Taylor (1993) and it is called the "Macro Model". According to this rule, the evolution of the short rate r_t is circumscribed to movements in contemporaneous macro variables f_t^o and a component that is not explained by these variables, an orthogonal shock υ_t.

$$r_t = a_0 + a_1' f_t^o + \upsilon_t, \tag{4}$$

where υ_t can be interpreted as a monetary policy shock according to the assumptions considered by Christiano et al. (1996). The original specification for the Taylor Rule uses two macro variables as factors in f_t^o: the annual inflation rate, similar to our inflation factor, and the output gap, analogous to our real activity factor. Thus, the coefficient γ_1 is constrained to depend only on contemporaneous factor values $X_t^o = f_t^o$.

In the case of Affine term structure models, as Duffie and Kan (1996) mentioned, the short rate is based on a equation similar to (4) with an assumption on risk premia. Thus, the short rate is established to be an affine (constant and a linear term) function of underlying latent (unobservable) factors:

$$r_t = c_0 + c_1' X_t^u + \upsilon_t, \tag{5}$$

The unobserved components themselves follow affine processes so the VAR constitutes the special Gaussian case. If pricing is risk neutral, prices of bonds of longer maturities depend upon parameters of X_t^u. Nevertheless, in the general case, that is considered in this paper, the risk adjustment must be specified carefully to obtain closed-form solutions for bond yields.

We can combine (4) and (5) since the short rate is specified in both equations as affine function of factors. Considering that $X_t = (X_t^o, X_t^u)$, we can rewrite (3) as:

$$r_t = \gamma_0 + \gamma_{11}' X_t^o + \gamma_{12}' X_t^u, \tag{6}$$

Following Ang and Piazzesi (2003), the latent factors X_t^u are specified to be orthogonal to the macro factors X_t^o. Thus, the short rate dynamics of the model can be explained as a Taylor rule with the errors $\upsilon_t = \gamma_{12}' X_t^u$ being unobserved factors.

3.1.3 The Pricing Kernel

The model uses the assumption of no-arbitrage (Harrison and Kreps, 1979) that guarantees the existence of an equivalent martingale measure Q, which is a risk-neutral measure. Therefore, the price of any asset V_t that does not pay any dividends at time $t+1$ satisfies $V_t = E_t^Q\left[\exp(-r_t)V_{t+1})\right]$, in which the expectation is taken under the measure Q. This assumption is equivalent to the assumption of the existence of a stochastic discount factor ξ_{t+1}, which allows us to price any asset in the economy:

$$\xi_{t+1} = \xi_t \exp\left(\frac{1}{2}\lambda_t' \lambda_t - \lambda_t' \epsilon_{t+1}\right), \tag{7}$$

where λ_t parameters are the time-varying market prices of risk related to the sources of uncertainty ϵ_t. Thus, λ_t is parameterized as an affine process:

$$\lambda_t = \lambda_0 + \lambda_1 X_t, \tag{8}$$

for a k-dimensional vector λ_0 and a $k \times k$ matrix λ_1. Shocks in the underlying state variables (macro and latent factors) are related to ξ_{t+1} through equations (7) and (8) and consequently determine how factor shocks affect all yields. To ensure that both the macro and unobservable factors are priced, the parameters in λ_0 and λ_1 corresponding to lagged macro variables are established to be zero. Again, following Ang and Piazzesi (2003), the pricing kernel m_{t+1} is define as:

$$m_{t+1} = \exp(-r_t)\frac{\xi_{t+1}}{\xi_t}, \tag{9}$$

$$m_{t+1} = \left(-\frac{1}{2}\lambda_t'\lambda_t - \gamma_0 - \delta_1' X_t - \lambda_t'\epsilon_{t+1}\right). \tag{10}$$

3.1.4 The Bond Pricing

Once defined the nominal pricing kernel, which prices all nominal assets in the economy, we state that the total gross return process R_{t+1} for any asset satisfies:

$$E_t(m_{t+1}R_{t+1}) = 1. \tag{11}$$

If p_t^n represents the price of an n period zero coupon bond, we can estimate bond prices recursively through (11):

$$p_t^n = E_t(m_{t+1}p_{t+1}^n). \tag{12}$$

With this in mind, the discrete-time Gaussian k factor model with k_1p observable variables and k_2 unobservable factors is formed by: i) the state dynamics of X_t given by (2), ii) the dynamics of the short rate equation r_t, given by (3) and iii) the Radon-Nikodym derivative, given by (7), where p is the number of lags in the autoregressive representation of the observable factors. Therefore, we can define an n-period bond price as:

$$p_t^{n+1} = \exp\left(\overline{A}_n + \overline{B}_n' X_t\right), \tag{13}$$

where the coefficients $\overline{A}_n$ and $\overline{B}_n$ are described by the following equations:

$$\overline{A}_{n+1} = \overline{A}_n + \overline{B}_n'\left(\mu - \Sigma\lambda_0\right) + \frac{1}{2}\overline{B}_n\Sigma\Sigma'\overline{B}_n - \gamma_0, \tag{14}$$

$$\overline{B}_{n+1}' = \overline{B}_n'(q - \Sigma\lambda_1) - \gamma_1', \tag{15}$$

with $\overline{A}_1 = -\gamma_0$ and $\overline{B}_1 = -\gamma_1$. Thus, the continuously compounded yield y_t^n of an n-period zero coupon bond is given by:

$$y_t^n = \frac{\log(p_t^n)}{n}, \tag{16}$$

$$y_t^n = \overline{A}_n + \overline{B}_n' X_t, \tag{17}$$

where $A_n = -\overline{A}_n/n$ and $B_n = -\bar{B}_n/n$ From (17), we see that the expected excess return comprises three terms: (i) a Jensen's inequality term $-\frac{1}{2}\overline{B}'_{n-1}\Sigma\Sigma\overline{B}_{n-1}$, (ii) a constant risk premium $\overline{B}'_{n-1}\Sigma\lambda_0$, and (iii) a time-varying risk premium $\overline{B}'_{n-1}\Sigma\lambda_1$. The parameters in the matrix λ_1 govern the time variation while the term premium is determined by the vector λ. Considering a positive shock at ϵ_{t+1} in a state variable. This shock affects all bond prices and alters bond returns according to (13), (14) and (15) . When λ is negative, the shock also drives up the log value of the pricing kernel (10), which involves a negative correlation between bond returns and the pricing kernel. Since this correlation has a hedge value, the risk premia on bonds are positive. Therefore, when λ is negative a positive shock determines a positive bond risk premium.

The variance decompositions can be computed following standard methods because both bond yields and the expected holding period returns of bonds are affine functions of X_t. In this model, the dynamics of the term structure rely on the risk premia parameters λ_0 and λ_1. This means that a non-zero vector λ_0 impacts the long-run mean of yields since this parameter impacts the constant term in (16). On the other hand, a non-zero matrix λ_1 impacts the time-variation of risk-premia, because it impacts the slope coefficients in (16). Thus, a model that presents a non-zero λ_0 and zero matrix λ_1, lets the average yield curve to be upward sloping. However, the risk premia in this model cannot be time-varying. In fact, if investors are risk neutral, $\lambda_0 = 0$ and $\lambda_1 = 0$ we are in the well-known case called Expectations Hypothesis. In general, the main drivers of the zero coupon bond yields are: i) the expected future path of short term interest rates and ii) the extra returns that investors require as compensation for the risk of holding longer-term instruments; see Cortés and Ramos (2008).

3.1.5 Assumptions on Model Parameters

Following Ang and Piazzesi (2003), the model consists of three latent factor and two macro factors In fact, we assume the existence of three latent factors that follow AR(1) processes. In particular, we estimate:

$$f_t^u = \rho f_{t-1}^u + u_t^u, \tag{18}$$

with 3-dimensional shock vector $u_t^u \sim IIDN(0, I)$ and a lower-triangular 3×3 companion matrix ρ. Furthermore, we assume that latent factors are independent of macro factors. This implies that the upper-right 2×3 corner and the lower-left corner 3×2 of Φ and Σ in the compact form in (2) include only zeros. According to this approach, the pricing kernel that includes observed macro factors specifies all uncertainties setting in the latent factors as orthogonal to the macro variables. This means that there is not a bi-directional relationship between latent factors and macro factors. Besides there is empirical evidence that there is a feedback effect between unobservable and observable factors for developed countries, such us Rudebusch and Wu (2008) and Diebold and Li (2006) for the USA and Hordahl et al (2006) for Germany, García and Montes (2014), shows that there is no a significant effect of latent factors on macro variables in the case of Peru.

As we mentioned before, risk premia are represented through parameters λ_0 and λ_1. When risk neutral measure and the data-generating coincide, $\lambda_t = 0$ for all t, and this case is well known as "Local Expectations Hypothesis". In our model, while a non-zero vector λ_0 impacts the long-run mean of yields since it impacts the constant term in (8), a non-zero matrix λ_1 influences the time-variation of risk premia, because it influences the slope coefficients in (17).

The number of parameters to estimate in λ is very large since λ_0 has $K_1 + K_2 = 5$ parameters and λ_1 has $(K_1 + K_2)^2 = 25$ parameters in the Macro model. Thus, following Ang and Piazzesi (2003), λ_1 matrix is specified to be block-diagonal, setting zero restrictions on the upper-right $2 \rtimes 3$ and lower-left $3 \rtimes 2$ corner blocks. This parameterization assumption implies the orthogonalization of macro and latent variables. Consequently, time variation in the compensation for shocks to unobservable components is only driven by the unobservable components themselves. We can apply the same argument for the payment for shocks in observable macro factors.

In summary, there are 5 parameters estimated in λ_0 and $4 + 9$ parameters in λ_1. On one hand, the parameters in λ_0 are related to the current observable macro variables and latent variables. On the other hand, the parameters in λ_1 are comprised in two non-zero matrices on the diagonal: a lower-right matrix $3 \rtimes 3$ for the unobservable components and the upper-left $2 \rtimes 2$ matrix for current macro variables.

3.2 Estimation Method

The assumption that yields are analytical functions of the state variables implies the transformation of the system of yields and observable variables $(Y_t', X_t^{o\prime})$ into a system of observables and unobservables $X_t = (X_t^{o\prime}, X_t^{u\prime})$. Thus, the unobservable factors can be inferred from yields. The estimation method is based on the maximum likelihood derived by Ang and Piazzesi (2003). In this case, the likelihood for the VAR is a function of $(Y_t', X_t^{o\prime})$ that let us to to draw inferences about yield curve movements and macro shocks from the parameters in Φ coefficients and covariance terms. Thus, we estimate a VAR of $(Y_t', X_t^{o\prime})$ with assumptions that guarantee no arbitrage and identify unobservable components orthogonal to macro shocks.

The estimation consists of a two-steps procedure, which let us avoid the issues related to the estimation of a model with many factors using maximum likelihood with highly persistent yields. In the first step, the macro dynamics and the coefficients of the macro factors in the short rate equation are estimated. Then, in the second step, holding all pre-estimated parameters fixed, the rest of the parameters of the term structure model are estimated. This procedure also eludes the problem of calculating an extensive number of lag coefficients in the bivariate VAR for the macro variables in the term structure model.

In the first step, the short rate equation coefficients of the macro variables in (6) and the macro dynamics are estimated treating latent variables as monetary policy shocks. In particular, they are computed by ordinary least squares, as informed in Table 4. Because the macro factors constructed have zero mean, the constant γ_0 in the short rate equation portrays the unconditional mean of the 3 month yield, which reaches 3.75% on an annualized basis. To obtain an estimate γ_0 at a monthly frequency, this number obtained must be divided by 12. The coefficients γ_{11} of the short rate equation represents the greatest magnitude of short rate movements explained by the macro factors, with all remaining orthogonal factors being unobservables. Then, we use the no- arbitrage assumptions to distinguish the unexplained proportion.

In the second step, we hold the previous estimates fixed and derive a likelihood function of observables $(Y_t', X_t^{o\prime})$ from that of $X_t = (X_t^{o\prime}, X_t^{u\prime})$. In order to achieve convergence, we need to find good starting values since the model is a highly non-linear system. Given the difficulty to estimate unconditional means of persistent series, we estimate the model in several iterative rounds. In fact, the likelihood surface, which determines the mean of long yields, is very flat in λ_0, see Ang and Piazzesi (2003).

The estimation begin with starting values for ρ in (18) obtained from the estimation of the model under the Expectations Hypothesis, with λ_0 and λ_1 equal to zero. Then, starting values for λ_1 are computed holding λ_0 at zero. Next, λ_0 is estimated setting any insignificant parameters in λ_1 at 10% level equal to zero. After that, the insignificant λ_0 parameters are set to zero and re-estimate. This procedure generates the zero of Φ and λ_1 matrices reported in Table 6 and 7. The majority of the non-zero parameters in Φ and λ_1 are significant, and it is expected that their effects remain in other iterative estimation structures. Since this procedure could be considered path dependent, future research should be developed to get feasible alternatives that calculates unconditional means for long yields close to those in the data.

Finally, following Chenn and Scott (1993) the likelihood construction proposed by Ang and Piazzesi (2003) is solved for the unobservable factors from the joint dynamics of the zero coupon bond yields and the macro factors. This implies to assume that there are as many yields measured without error as unobservable factors, and there are yields that are measured with error. In particular, we assume the 1 year and 9 year yields are measured with error.

4 Estimation Results

4.1 The Data

To study the joint behavior of the yield curve and macroeconomic factors, we use monthly information on yields and variables associated with the performance of the output and the price level, which are obtained from the Superintendency of Banking and Insurance (SBS) and the Central Reserve Bank (BCRP), respectively.

4.1.1 The Yield Data

We use monthly data on zero coupon bond yields of maturities of 3 months, 1, 2, 9 and 10 years from November 2005 to December 2015. The bond yields are obtained from SBS Price Vector. Regarding the choice of maturity to explain the behavior of the long end of the yield curve, in line with the proposal of Halberstadt and Stapf (2012) we take as reference the yield of the 10-year bond since bonds with longer maturities have not significantly different trading frequencies. Therefore, the estimation with longer yields almost does not affect results. On the other hand, at the short term of the curve, the 3 month yield is chosen as the risk-free rate since it is the smallest term available.

Figure 1 plots the yields that are considered to be measured without error while Table 1 presents the main sample statistics and some stylized facts. The average yield curve is upward sloping; the standard deviation of yields mostly decrease with maturity; and yields are remarkably autocorrelated, with declining autocorrelation at longer maturities. The yield levels exhibit mild excess kurtosis at short maturities which increases with maturity. Overall, the distribution observed in the Figure 1 and the evidence from the statistics of the series of monthly yields seems to not reject a Normal distribution. In fact, the Jarque-Bera normality test does not reject Gaussianity for yields. Thus, since we are interested in modelling the joint behavior of yields and macroeconomic variables, the Gaussian assumption that we made in later sections is a sufficient approximation to the dynamics of the yield curve.

An important stylized fact is that yields at near maturity are highly correlated. We can observe that the correlation between 9 and 10 years yield is 99%. In the estimation developed, the five yields are used to estimate the model. We set that the 3-month yield, 2-year and 10-year yields are measured without errors and they represent the short, medium and long ends of the yield curve in the model with 3 unknown factors. The 1 year yield has a 96% correlation with the 2-year yield and the 9-year yield has 99% correlation with the 10-year yield.

4.1.2 Macro Variables

Following Ang and Piazzesi (2003), macro variables are sorted in two groups. The first group comprises various inflation measures which are based on Consumer Price Index (CPI), the CPI of Food and Energy (CPI-FE) and Imported Inflation (CPI-M). The second group consists of variables that capture real activity: the index of Primary Gross Domestic Product (PRIM-GDP), the Non Primary Gross Domestic Product index (NO PRIM-GDP) and the Urban Employment Index for firms with 10 and more employees (EMP). This list of variables includes most variables that have been used in monthly VARs in the macro literature. All growth rates are measured as the difference in logs of the index at time t and $t-12$.

The main sample statistics of macro variables are presented in Table 1. In the first group, we observe that the average of inflation is around the target limit (3%) with a low standard deviation. This average is greater in the case of the CPI-FE and lower in the case of CPI-M, with higher volatility in both cases. An important stylized fact is that all inflation measures are highly autocorrelated (up to 90%). With respect to the variables associated with real activity, as we expect, we observe that the growth rate of NO PRIM-GDP has a higher mean than the PRIM-GDP and a lower standard deviation, while the growth rate of EMP presents the lowest standard deviation and the highest autocorrelation. The autocorrelation is lower for the growth of PRIM-GDP, which can be explained by the volatility of this sector.

The dimensionality of the system is reduced through by extracting the first principal component of each group of variables independently. In particular, we extract the first principal component from the inflation measures group, and we extract the first principal component from the real activity measures group. Thus, we keep with two variables which, in line with Ang and Piazzesi (2003), are called "inflation" and "real activity". In particular, we first normalize each series independently to have zero mean and unit variance. We then assemble the three variables associated with inflation (real activity) into a vector $Z_t^1(Z_t^2)$. For each group i, the vector Z_t^i can be represented as:

$$Z_t^i = Cf_t^{o,i} + \epsilon_t^i, \tag{19}$$

where $Z_t^1 = (CPI_t, CPI - FE_t, CPI - M_t)$ for the group "inflation" and $Z_t^2 = (PRIM - GDP_t, NOPRIM - GDP, EMP_t)$ for the group "real activity". The error term ϵ_t^i satisfies $E(\epsilon_t^i) = 0$ and $Var(\epsilon_t^i) = \Gamma$, where Γ is diagonal. The matrices C and Γ are 3×1 and 3×3, respectively, for each group. The extracted macro factor $f_t^{o,i}$ inherits the zero mean from Z_t^i ($E(f_t^{o,i}) = 0$) and has unit variance ($Var(f_t^{o,i}) = 1$).

Table 2 presents the loadings of the first three principal components for the two groups. The factor loadings shows that the first principal component can explain the variation in each group up to 50%. More precisely, over 75% (60%) of the variance of nominal variables (real variables) is explained by just the first principal component of the group. The first principal component of the inflation and real activity measures loads positively on CPI, CPI-FE, and CPI-M and PRIM-GDP, NO PRIM-GDP and EMP, respectively. We plot these macro factors in Figure 2. The figure indicates that some conditional correlations between the inflation factor and real activity factor might be important.

Table 3 shows the correlation between the original macro series in each group and the extracted principal components. These correlations demonstrate that the inflation factor is most closely correlated with CPI and CPI-FE (92% and 98% respectively) and less correlated with imported inflation (70%). The real activity factor is most closely correlated with NO PRIM-GDP growth (91%) and EMP (91%).

Furthermore, we can infer from correlation matrix in Table 3 some pioneer information about the relationship between the macro factors and the yield curve. The correlation between yields at longer maturities and real activity factor is higher than the correlation between these yields and inflation. In fact, the correlation of inflation is highest for short yields (56% correlation between inflation and the 3-month yield), and somewhat smaller for long yields (26% correlation between inflation and the 10-year yield). Real activity correlation increases with maturity (53% correlation between real activity and 2-year yield) and then declines at the longest maturity (28%).

The unconditional correlation between the two macro factors is small (0.35), as reported in Table 3. Although the unconditional correlation is weak, based on the Figure 2, we estimate a VAR for the macro factors and we find that the conditional correlation is significant. Specifically, we estimate a bivariate process with 1 lag for the macro factors: $f_t^{o,i} = (f_t^{o,1}, f_t^{o,2})'$:

$$f_t^o = \rho_1 f_{t-1}^o + \Omega u_t^o, \tag{20}$$

where ρ_1 and Ω are 2×2 matrices with $u_t^o \sim IIDN(0,1)$. Figure 3[2] presents the Impulse Response Functions (IRFs) from a VAR(1) fitted to the macro factors. The response of inflation to shocks in real activity is positive and hump-shaped, while the response of real activity to inflation shocks is also positive, and then turns negative before dying out.

[2]The IRFs are computed using a Cholesky orthogonalization. There is no significant difference reversing the order of the variables.

4.2 Short Rate Dynamics

Based on the independence assumption on X_t^o and X_t^u, we estimate the coefficients on inflation and real activity in the short rate equation by ordinary least squares. Table 4 reports the estimation results from two regressions: the original Taylor rule and the forward-looking version of the Taylor rule, that includes lags of the macro variables[3]. The R^2 of the estimated Taylor rule is 36%, while the forward looking version is 44%. These results indicate that macro factors should have explanatory power for the movements of the yield curve.

The performance of the residuals provides some knowledge about what to expect from a model with unobservable components. In particular, we can infer some preliminary information from Figure 4, which plots the residuals and the demeaned short rate The residuals from both versions of the Taylor rule are highly autocorrelated. While the autocorrelation of residuals from the short rate equation with only contemporaneous macro factors is 0.914, the autocorrelation from the equation that incorporates lagged macro factors is slightly lower, 0.847. The short rate itself has an autocorrelation of 0.943, which indicates that macro variables could explain some of the persistent shocks to the short rate. In addition, unless a variable that replicates the short rate itself is set on the right hand side of the Taylor Rule Equation, the residuals will follow the same broad pattern as that of the short rate. Thus, we can infer that the "level" factor found in first term structure research, see Vasicek (1977), prevails when macro variables are included in a linear version of the short rate in a term structure model.

Finally, the coefficients of inflation and real activity in the simple Taylor rule are positive and significant, which is consistent with previous estimates found in the literature. In contrast to the simple Taylor rule estimation, Table 4- Panel B reports that most parameter estimates for the forward-looking version of the Taylor rule are not significant, except for the 11th lag on real activity. This suggests that using many lags in the Taylor rule may lead to an over-parameterized and potentially poorly behaved system. Moreover, the optimal Schwartz (BIC) choice rejects the forward-looking Taylor rule (-1.21) in favor of the original Taylor rule (-1.83).

4.3 A Term Structure Model without and with Macro Factors

We compare the yields estimates resulting from the Yields-only model and the Macro model to asses the relevance of macroeconomic information for the yield estimation. We find that the root mean

[3]Since one of the specifications establishes that latent variables are orthogonal to macro variables, following Ang and Piazzesi (2003), we modiffy the implementation of the forward-looking Taylor rule proposed by Clarida et al. (2000). In particular, we add lagged macro variables as arguments in (6) instead of redefine v_t to include forecast errors $f_{t+1}^o - E(f_{t+1}^o)$. We do this procedure because following Clarida et al (2000) implies to include forecast errors into some latent variables and, consequently, to drop the assumption of independence between macro variables and latent factors.

squared error (RMSE) of the yield estimates is indeed smaller for the macro model ($RMSE^{mf} = 0.06$) compared to the Yields-only model ($RMSE^{lat} = 0.17$). The maturity-specific $RMSEs$ are provided in Table 5. We can see that $RMSEs$ are lower for the shortest and the longest maturities in both models. Thus, the macroeconomic factors have some relevance for the yield estimation. These results are in line with many papers that include macro factors as sources of risk, such as Ang and Piazzesi (2003), Pericoli and Taboga (2006), Rudebusch and Wu (2008) and Halberstadt and Stapf (2012). Furthermore, we are avoiding the detection of structural breaks -that lead to changes in the influence of macro factors- since we have estimated over the whole sample period, this constitutes a first approach to check for the explanatory power of our macro factors over yield curve estimation.

4.3.1 The Yields-Only Model

Table 6 shows the estimation results for the Yields-Only Model. The estimation results are presented by ordering the latent factors by decreasing autocorrelation. The model has one very persistent factor, one less persistent but still strongly, and one last factor that is strongly mean-reverting. This is consistent with previous multi-factor estimates for other countries such as the USA or Germany.

These unobservable factors are known in the literature such as “level”, “slope” and “curvature”, respectively because of the effects of these factors on the yield curve (Litterman and Cheinkman, 1991). The level factor is related to the long end of the yield curve and therefore may be associated with y_t^{120} as empirical proxy. In turn, the slope factor is associated with the behavior of the short end of the curve and his empirical proxy can be defined as $y_t^{120} - y_t^3$. Finally, the empirical proxy for the curvature factor, related to the mid end, can be represented as $y_t^{120} - 2y_t^{24} + y_t^3$ since is related to the middle end of the yield curve.

Then, we assess how closely the Unobservable Components (UC) obtained in the model are related to the empirical factors through the analysis of the correlation between them. We find that the first latent variable, UC 1, has an 83% correlation with the “level” transformation of the yield curve while the correlation between UC 2 and the “slope” transformation is 85%. Finally, the UC 3 has a 94% correlation with the “curvature” transformation. Thus, we can infer that the model represents the different sections of the Peruvian yield curve very well.

Furthermore, the estimated vector λ_0 has one significant negative parameter that corresponds to the most highly autocorrelated factor. Negative parameters in λ_0 imply that long yields have to be on average higher than short yields, because bond prices are estimated under the risk neutral measure. Therefore, the unconditional mean of the short rate under the risk-neutral measure results higher than under the data-generating measure. According to this result, the average Peruvian yield curve is upward sloping. Furthermore, the model also shows that time-variation in risk premia, associated with the elements of λ_1 primarily depend on the the "level" and the "curvature" of the yield curve, the first and third unobservable factor, respectively.

4.3.2 The Model with Yields and Macro Variables

We present the estimation results of the Macro Model in Table 7. We can observe that the auto-correlation of the UC 1 is almost the same as the one found in the Yields-Only model. Thus, the first latent factor has a similar persistent effect across the models considered. However, the same does not occur with UC 2 and UC 3 variables since they vary more across the models. On the other hand, the risk premia estimate in Table 7 λ_0 is significant, which means that long term yields are on average higher than short yields.

Furthermore, we find that the observable macro factors also affects time-variation in risk premia since market price of risk coefficients of these observable variables are highly significant. In particular, the elements corresponding to inflation, $\lambda_{1,11}$ and real activity $\lambda_{1,22}$ are both negative in the Macro model (where $\lambda_{1,ij}$ refers to the λ_1 element of the ith row and the jth column). This means that a positive shock at "$t+1$" on state variables leads to a positive risk premium, and consequently higher returns for long end yields, which is consistent with the economy theory. While the inflation expectations increase, investors demand more compensation for the risk of holding longer-term instruments. Similarly, expectations of high economic growth lead to higher yields because it implies an increase in inflation expectations. Finally, the inflation-real activity cross terms, $\lambda_{1,12}$ and $\lambda_{1,12}$, are also significant but positive. This can be explained for the undetermined effect of positive shocks of economic growth expectations and inflation expectations on investor's decisions

4.3.3 Impulse Response Functions

The effect of each factor on the yield curve is determined by the weights of B_n from equation (17). These effects represent the initial response of yields to movements in the various factors. Thus, the weights of B_n are plotted as a function of yield maturity for the Yields Only model in Figure 4 and for the Macro Model in Figure 5. The B_n coefficients are related to movements of one standard deviation of the factors and are presented in an annualized way.

In the Yields-Only model, the "level" factor can be linked to the weight on the most persistent factor (UC 1) since it is almost horizontal and affects yields of all maturities the same way. The "slope" factor could be related to the coefficient of the second factor (UC 2) because it is upward sloping and it mainly affects the short end of the yield curve relative to the long end. Finally, the coefficient on the least persistent factor (UC 3) can correspond to the "curvature" factor since it is hump-shaped and thus has a twisting effect on the yield curve.

In the Macro model, the coefficients look very similar. The coefficients from UC 1 through UC 3 represent "level", "slope" and "curvature" factors. On the other hand, the B_n coefficients corresponding to inflation and real activity, are represented as stars and circles, respectively. We find that the effects of inflation and real activity mostly affect short and middle yields and less so long yields. In particular, we observe that the magnitude of the inflation and real activity weights are higher than the level factor weights at short and middle maturities. Thus, macro factors would have an explanatory power for yield curve dynamics. These results are in harmony with the estimates obtained of the Taylor rule in Table 4. The inflation and real activity have a significant effect on the short rate, so we get a strong initial effect on yields. In particular, the real activity factor seems to have a stronger initial effect than the inflation factor at short yields while this difference disappear at the middle and long end of the yield curve.

With respect to the time variation of prices of risk, it should be noted that these prices control the way that yields at the long end respond relative to the short rate. In the Macro model, as we mentioned before, the time varying prices of risk of macro factors are both negative. The more negative terms the more positively yields of the long end react to positive factor shocks. Therefore, the initial effect of inflation is larger than real activity across the yield curve.

Impulse Response functions (IRFs), which show how a shock on a macro factor affects the yields, are also derived. Figure 6 shows the IRFs of 3 month, 2 and 10 year yields from the Macro model and from an unrestricted VAR (1), with macro factors and 5 yields. A one-standard deviation shock to the inflation factor seems to have stronger and more persistent effects compared to innovations in the real activity factor across all maturities in both models. This can be explained by the fact that the loading on real activity (0.029) in the Taylor rule is smaller than the inflation (0.054). IRFs of macro shocks for the unrestricted VAR are hump shaped while the IRFs derived from Macro model do not follow the same pattern. Furthermore, the magnitudes of IRFs differ across models.

Turning to the first column of Figure 6, we can observe that the hump in the unrestricted responses to inflation takes place after an average of 5 months, while the hump in the responses of real activity shocks is greater and occurs later, after 9 months approximately. In particular, a one-standard deviation shock to inflation increases the 3-month yield about 10 basis points (bps) at the beginning. The response peaks after about 6 months at 15 bps and then dies out slowly. The responses of longer yields are smaller but follow the same pattern. The initial response of the 2-year yield (10-year yield) is only 8 bps (7.5 bps). The response increases to around 12 bps (9 bps) after 5 months (4 months), and then slowly levels off. On the other hand, the response of yields to real activity shocks is smaller at the beginning than the response to inflation shocks but with a bigger hump occurring after 9 months or more.

The IRFs for the Macro Model are plotted in the second column of Figure 6. The IRFs derived do not present a hump-shaped, but are much larger and persistent. For example, one standard-deviation shock to inflation factor (real activity factor) generates an initial response of the 3-month yield of 65 bps (35 bps), which is approximately seven times (ten times) the effect of the IRFs from the unrestricted VAR (1). For the 2 year yield, the initial response to inflation (real activity factor) is about 60 bps (32 bps), compared to a move of 8 bps (7.5 bps) founded before. Finally, for the long-term yield the initial responses to inflation factor and real activity factor are also greater and persistent (42 bps and 23 bps, respectively).

These results are in line with the findings from Halberstadt and Stapf (2012) derived for German data and Ang and Piazzesi (2003) derived for US data. Reactions to innovations in the price factor and the real activity shocks declined very slowly across all maturities. This is due primarily to the estimates of the time-varying price of risk. The diagonal elements of λ_1 in the Macro model are negative. As we mentioned before, negative prices of risk have higher positive impacts from the macro factors to long yields.

4.3.4 Variance Decompositions

We construct variance decompositions to determine the relative contributions of the macro factors and latent factors to forecast variances. These show the proportion of the forecast variance attributable to each observable and unobservable factor. Table 8 lists variance decompositions for the 3-month, 2-year and 10-year yield at different forecast horizons derived from the Yields-Only model and the Macro model while Table 9 provides a summary of the proportion of the forecast variance explained by macro factors.

The results show that the proportion of unconditional variance explained by macro factors decrease with the maturity of the yields. The 3-month yield presents the largest effect since macro factors explained the 69% of the unconditional variance for the Macro model. This effect decreases for the 2-year yield (59%) and results very much smaller for the 10-year yield (17%). Thus, after the removal of the effects of inflation and real activity, the latent factors explain the residuals in the Taylor rule considered for the Macro Model. In general, the proportion of the forecast variance explained by the latent factors increases as the yield maturity increases. As we can deduct from Table 9, the latent factors account for 83% of the unconditional variance for the 10-year yield in the Macro model. This result is supported on the dominance of persistent unobserved factors (the near unit-root factor), which lead to a low variance decomposition of long-term yield attributable to macro factors. Since the highest autocorrelation corresponds to the level factor, its influence is highest for long maturities.

Turning now to the Macro model, we observe that the explanatory power of the inflation is greater than the real activity across the maturities and for all the horizons. The macro factors explain a significant proportion (up to 50%) of the unconditional at long-horizons variances for the short and medium segments of the yield curve. With the exception of the 3-month yield, the proportion of the variance explained by inflation generally decreases with the forecast interval h. In contrast, the explanatory power of real activity generally increases with the forecast interval h: at short horizons, real activity has a little explanatory power for the forecast variance across the yield curve; however as the horizon raises, the proportion due to real activity shocks raises to 31% of the 3-month and 27% of the 2-year yield. Nevertheless, at long end yields, the higher persistence of the latent factor dominates and the effect of macro factors decreases (17% of the unconditional variance for the 10-year yield).

In particular, the most persistent latent factor is the Unobs 1 variable and corresponds to the level effect (see Table 8). For the Yields-Only model, this factor explains a significant part of the variance at the long end of the yield curve at all horizons and at the short and middle ranges of the yield curve at long horizons. However, its influence decreases at the short end of the yield curve, where the slope factor explains much of the variance. With regard to the Macro model, the level factor effect is significantly reduced for the 3-month yield, where the macro factors -especially the inflation factor- play a major role due to the effects they have in the Taylor rule. For the 2-year yield, macro factors still dominate the variance decomposition, but in a smaller way. Finally, for the 10-year yield, Unobs 1 has the greatest influence, explaining the 80% of the unconditional variance for the Macro model. Thus, as maturity increases, the explanatory power of macro factors becomes smaller.

5 Conclusions

We use and estimate an Affine Term Structure model that characterizes the dynamics of Peruvian yield curve using monthly information for the period 2005 to 2015. The model follows the methodology proposed by Ang and Piazzesi (2003).

In particular, we estimate a model to understand the joint dynamics of macro variables and bond prices in a multifactor model of the term structure. Through Principal Component Analysis, we condensed our macro variables from a set of time series in two factors: an inflation factor and a real activity factor. Risk premia are modeled as time varying and depend on both observable and unobservable factors. The VAR model is estimated considering no-arbitrage assumptions.

We find evidence that our macro factors help to improve the fit of the model and explain a significant amount of variation in bond yields. Positive shocks to macro factors raises the yields, while the response to inflation shocks are greater than the real activity across all maturities. Variance decompositions demonstrate that macro factors mainly explain movements in the short and middle segments of the yield curve (up to 50%) while unobservable components are the main drivers of the majority of the movements at the long end of the yield curve (up to 80%). Comparing to the Yields-Only model, the "level" factor effect prevails when macro factors are incorporated. Finally, we find that no-arbitrage restrictions with the incorporation of macro factors improve forecasts.

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Table 1: Summary statistics of data: 2005:11 to 2015:12

	Central Moments					Autocorrelations		
	Mean	Median	Std Dev.	Skew	Kurt	Lag 1	Lag 2	Lag 3
3 mth	3.7546	3.9061	1.3870	-0.2589	2.9928	0.9410	0.8512	0.7521
1 year	3.9700	3.9359	1.4320	0.0427	3.0676	0.9530	0.8720	0.7720
2 years	4.3276	4.0174	1.3820	0.2610	3.2779	0.9454	0.8671	0.7679
9 years	6.2112	6.0402	0.9991	0.4050	3.7072	0.9120	0.8100	0.7130
10 years	6.3630	6.2403	0.9724	0.3367	3.6864	0.9070	0.8010	0.7040
CPI	2.9844	2.9815	1.4435	0.2834	3.0064	0.9532	0.8854	0.8059
CPI-FE	3.7924	3.7686	2.2706	0.1901	2.8609	0.9432	0.8625	0.7767
CPI-M	2.0724	2.5712	4.2486	-0.5107	4.0585	0.9710	0.9030	0.8110
PRIM-GDP	2.9132	3.0452	5.4527	-0.0022	3.3372	0.3613	0.2701	0.3175
NO PRIM-GDP	6.5881	6.7354	3.4917	-0.0651	2.3196	0.8586	0.8473	0.7697
EMP	4.3133	4.1623	2.6659	0.1638	1.8891	0.9723	0.9375	0.8990

The 3 month, 2 and 10 year yields are annual zero coupon bond yields from Price Vector of SBS The inflation measures CPI, CPI - FE and CPI-M correspond to Core Inflation, Non Core Inflation and Price Index of Imports, respectively. The inflation measure at time t is calculated using $\log(P_t/P_{t-12})$ where P_t is the price index. The real activity measures NO PRIM - GDP, PRIM-GDP and EMP refer to the growth rate of the Index of Non Primary GDP, the growth rate of the Index of Primary GDP and the growth rate of Urban Employement Index for firms with 10 or more workers. The growth rate of the Non Primary GDP, the Primary GDP and the Employement Index are calculated using $\log\,(I_t/I_{t-12})$ where I_t is the employment or production index.

Table 2. Principal component Analysis. 2005:11 to 2015:12

	Principal Components: Inflation		
	1st	2nd	3rd
CPI	0.6324	-0.2955	-0.7161
CPI-FE	0.6244	-0.3527	0.6969
CPI-M	0.4585	0.8878	0.0386
% variance explained	0.7674	0.9856	1.0000
	Principal Components: Real activity		
	1st	2nd	3rd
PRIM-GDP	0.6701	-0.2366	0.7036
NO PRIM-GDP	0.6751	-0.1998	-0.7102
EMP	0.3085	0.9509	0.0258
% variance explained	0.6122	0.9163	1.0000

Table 3. Selected correlations. 2005:11 to 2015:12

	CPI	CPI-FE	CPI-M	
Inflation	0.9222	0.9827	0.7007	
	PRIM-GDP	NO PRIM-GDP	EMP	
Real Activity	0.4181	0.9081	0.9149	
	Inflation	Real Activity	3 mth	2 year
Real Activity	0.3479			
3 mth	0.5588	0.4159		
2 year	0.4519	0.5314	0.9230	
10 year	0.2557	0.2808	0.5776	0.7477

Table 4. The dependence of the short rate on macro variables. 2005:11 to 2015:12.

Coeff.	Constant	Inflation	Real Activity	Adj. R^2
Panel A: y_t^3 on constant, inflation and real activity				
t	0.3129^a	0.0462^a	0.0478^a	0.3341
	(0.0086)	(0.0086)	(0.0086)	
Panel B: y_t^3 on constant, 12 lags of inflation and real activity				
t	0.2911^a	0.0324	-0.0389	0.5949
	(0.0086)	(0.0442)	(0.0279)	
$t-1$		0.0205	-0.0131	
		(0.0686)	(0.0296)	
$t-2$		0.0141	0.0181	
		(0.069)	(0.0348)	
$t-3$		-0.0271	0.0356	
		(0.0697)	(0.0343)	
$t-4$		0.0317	-0.0038	
		(0.0694)	(0.0344)	
$t-5$		-0.0334	-0.0002	
		(0.0702)	(0.0343)	
$t-6$		0.0273	0.0003	
		(0.0681)	(0.0346)	
$t-7$		-0.0216	0.0124	
		(0.066)	(0.032)	
$t-8$		-0.0314	-0.0131	
		(0.0647)	(0.0321)	
$t-9$		0.0032	-0.0013	
		(0.0641)	(0.0321)	
$t-10$		-0.0152	0.0192	
		(0.0645)	(0.0288)	
$t-11$		0.003	0.0413	
		(0.0427)	(0.0271)	

Panel A presents the regression results of the 3 month yield y_t^3 on a constant, the inflation factor and the real activity factor. Panel B presents the regress of y_t^3 on a constant, inflation, real activity and 11 lags of inflation and real activity. The OLS standard errors are reported in parenthesis. Standard errors significant at the 10%, 5% and 1% level are denoted by $(^a)$, $(^b)$, $(^c)$, respectively.

Table 5. Forecast Comparisons. RMSE (2005:11 - 2015:12)

	Yields Only	Macro Model
3 months	0.16	0.10
1 year	0.13	0.13
2 years	0.12	0.02
9 years	0.21	0.06
10 years	0.25	0.01

Table 6. Yields-only model estimates. 2005:11 to 2015:12.

Companion from Φ		
0.9997	0.0000	0.0000
(0.0000)		
0.0000	0.9707	0.0000
	(0.0012)	
0.0000	-0.0029	0.8457
	(0.0023)	(0.0082)

Short rate parameters γ_1 (x100)		
UC 1	UC 2	UC 3
0.0258	-0.0298	0.0158
(0.0000)	(0.0000)	(0.0000)

Prices of risk λ_0 and λ_1		λ_1 matrix		
	λ_0	UC 1	UC 2	UC 3
UC 1	-0.2431	-0.0008	0.0000	0.0000
	(0.0274)	(0.0000)		
UC 2	0.0000	-0.0002	0.0000	-0.0191
		(0.0000)		(0.0024)
UC 3	0.0000	0.0002	0.0000	0.0153
		(0.0001)		(0.0023)
Loglike	-2,297.706			

The table reports parameter estimates and standard errors in parenthesis for the 3-factor Yields-Only Model: $X_t = \Phi X_{t-1} + \varepsilon_t$, con $\varepsilon_t \sim N(0, I)$, Φ lower triangular and the short rate equation given by.$r_t = \gamma_0 + \gamma_1 X_t$. All factor $X_t = f_t^u$ are unobservable. The coefficient γ_0 is set to the sample unconditional mean of the short rate. Market prices of risk $\lambda_t = \lambda_0 + \lambda_1 X_t$ are restricted to be block diagonal.

Tabla 7. Macro Model estimates. 2005:11 to 2015:12

Companion from Φ		
0.9937	0.0000	0.0000
(0.0001)		
0.0000	0.9707	0.0000
	(0.0000)	
0.0000	0.0043	0.8689
	(0.0002)	(0.0000)

Short rate parameters γ_1 (x100)

UC 1	UC 2	UC 3
0.0145	-0.05834	0.09965
(0.0000)	(0.0000)	(0.0000)

Prices of risk λ_0 and λ_1		λ_1 matrix				
	λ_0	Inflation	Real activity	UC 1	UC 2	UC 3
Inflation	0.0000	-0.8803	0.2548	0.0000	0.0000	0.0000
		(0.0000)	(0.0000)			
Real activity	0.0000	0.5959	-0.1103	0.0000	0.0000	0.0000
		(0.0000)	(0.0000)			
UC 1	-0.0775	0.0000	0.0000	-0.0023	0.0000	0.0000
	(0.0029)			(0.0003)		
UC 2	0.0000	0.0000	0.0000	0.0069	0.0000	-0.0239
				(0.0001)		(0.0000)
UC 3	0.0000	0.0000	0.0000	-0.0010	0.0000	-0.0016
				(0.0010)		(0.0006)
Loglike	-3,723.31					

The table reports parameter estimates and standard errors in parenthesis for the Macro Model with the short rate equation specified with 1 lags of inflation and current real activity. The short rate equation is given by $r_t = \gamma_0 + \gamma_1 X_t$; where γ_1' only picks up current inflation, current real activity and the latent factors. The dynamics of the inflation and real activity are given by VAR(1). The model is $X_t = \Phi X_{t-1} + \varepsilon_t$, with $\varepsilon_t \sim N(0, I)$, X_t contains 1 lag of inflation and real activity and three latent variables, which are independent at all lags to the macro variables. The coefficient γ_0 is set to the sample unconditional mean of the short rate. Market prices of risk $\lambda_t = \lambda_0 + \lambda_1 X_t$ are restricted to be block diagonal.

Table 8. Variance Decompositions

	h	Macro factors: Inflation	Macro factors: Real activity	Latent factors: UC 1	Latent factors: UC 2	Latent factors: UC 3
3 month yield	3			0.43	0.50	0.06
Yields-Only	12			0.58	0.40	0.02
	60			0.84	0.16	0.01
	∞			0.99	0.01	0.00
Macro	3	0.32	0.09	0.00	0.49	0.10
	12	0.34	0.12	0.00	0.51	0.03
	60	0.36	0.22	0.00	0.40	0.01
	∞	0.38	0.31	0.03	0.27	0.01
2 year yield	3			0.52	0.47	0.01
Yields-Only	12			0.65	0.35	0.00
	60			0.87	0.13	0.00
	∞			0.99	0.01	0.00
Macro	3	0.38	0.11	0.02	0.49	0.01
	12	0.37	0.14	0.02	0.47	0.00
	60	0.38	0.23	0.04	0.35	0.00
	∞	0.32	0.27	0.22	0.19	0.00
10 year yield	3			0.60	0.40	0.00
Yields-Only	12			0.72	0.28	0.00
	60			0.90	0.10	0.00
	∞			0.99	0.01	0.00
Macro	3	0.41	0.12	0.21	0.26	0.00
	12	0.37	0.13	0.26	0.23	0.00
	60	0.29	0.17	0.40	0.14	0.00
	∞	0.10	0.08	0.80	0.03	0.00

Contribution of the factor i to the h-step ahead forecast variance of the 3 month yield (short end), 2 year yield (middle) and 10 year yield (long end) for the Macro Model.

Table 9. Proportion of variance explained by macro factors

	Forecast horizon h			
	3 mth	2 year	10 year	∞
Short end	41%	46%	58%	69%
Middle	49%	51%	61%	59%
Long end	52%	51%	46%	17%

Contribution of the macro factors to the h-step ahead forecast variance of the 3 month yield (short end), 2 year yield (middle) and 10 year yield (long end) for the Macro Model. These are the sum of the variance decompositions from the macro factors in Table 8.

Figure 1. Monthly Zero Coupon Bond Yields

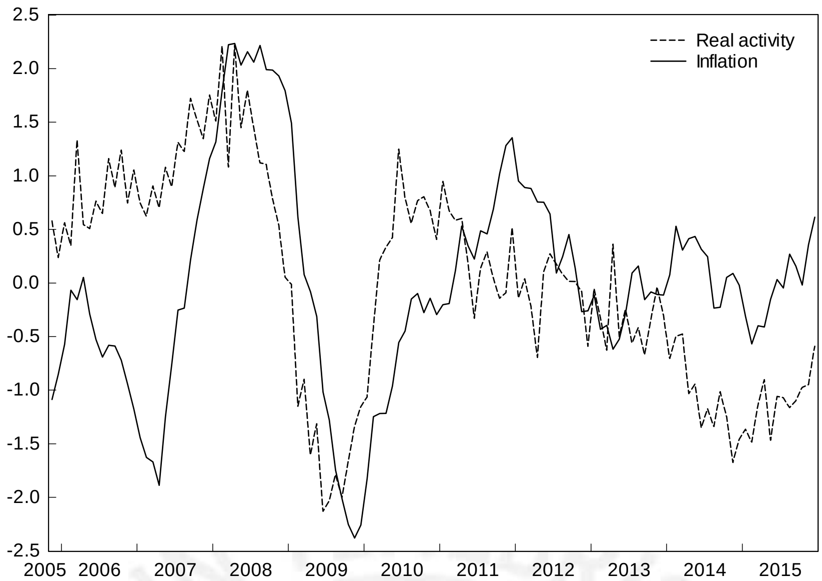


Figure 2. Macro Factors

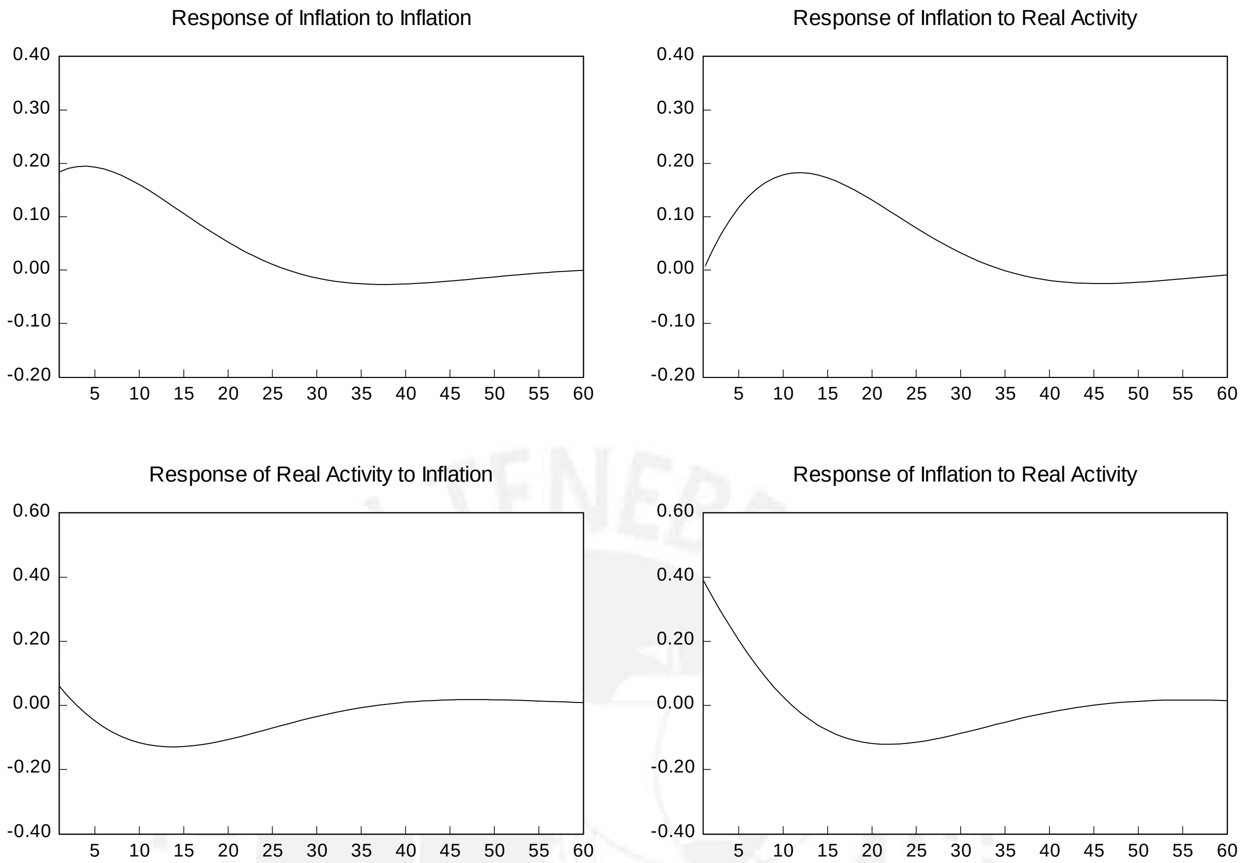


Figure 3. Impulse Response Functions (IRFs) from the VAR (1) on macro factors. The VAR(1) is fitted to the inflation and real activity macro factors, where inflation is ordered first.

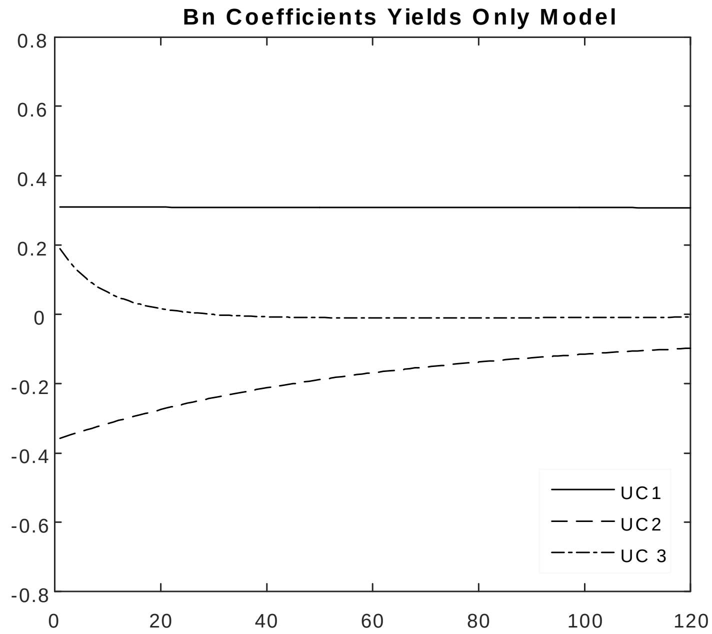


Figure 4. B_n yield weights for the Yields only model. (The plots show only the B_n yield weights corresponding to contemporaneous state variables in each system. The weights are scaled to correspond to one standard deviation movements in the factors and are annualized by multiplying by 1200)

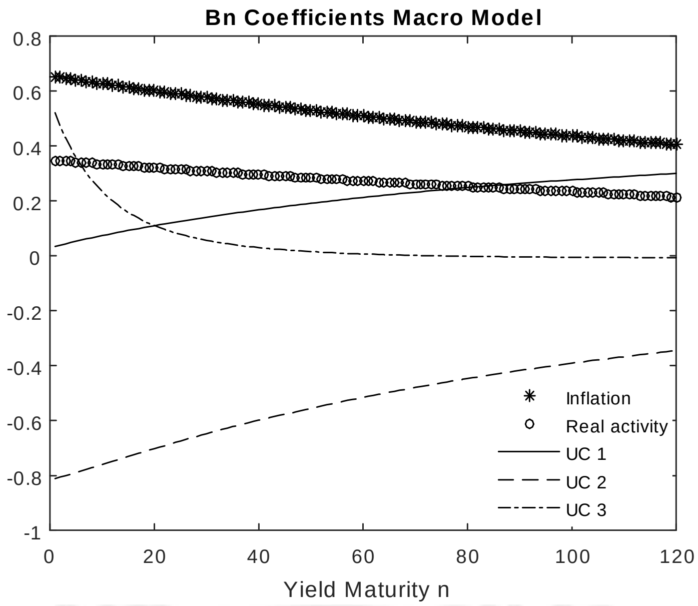
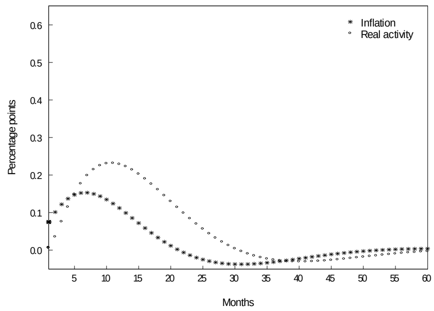
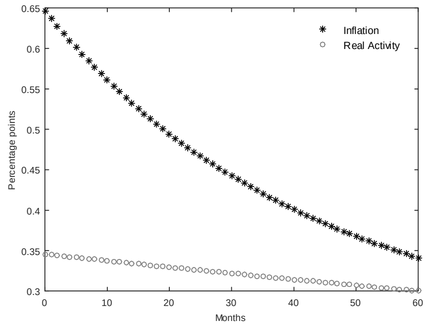


Figure 5. B_n yield weights for the Macro model. (The plots show only the B_n yield weights corresponding to contemporaneous state variables in each system. The weights are scaled to correspond to one standard deviation movements in the factors and are annualized by multiplying by 1200)

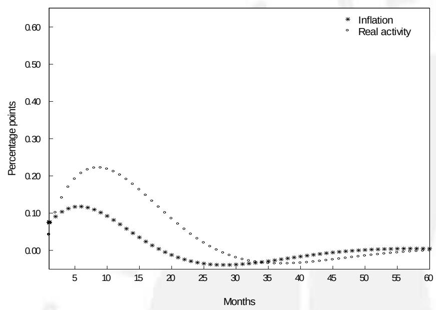
Unrestricted VAR (1) | Macro Model



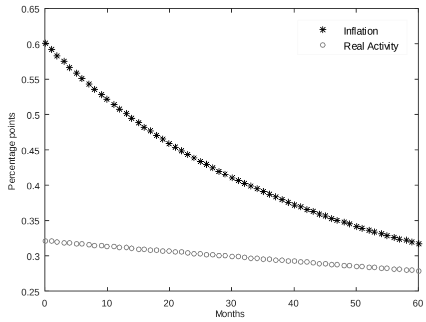
3 month yield



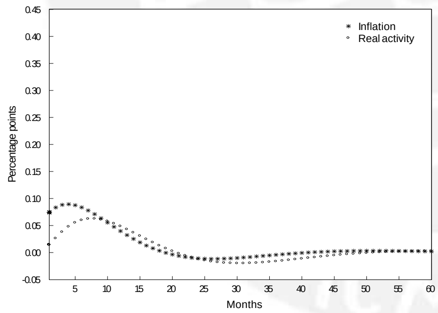
3 month yield



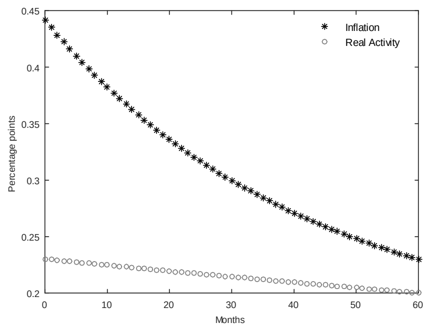
2 year yield



2 year yield



10 year yield



10 year yield

Figure 6. Impulse Response Functions. IRFs for 3 month (top row), 24 month (middle row) and 120 month (bottom row) yields. (The first column presents IRFs from an unrestricted VAR(1) fitted to macro variables and yields; the right column presents IRFs from the Macro model. The IRFs from inflation (real activity) are drawn as stars (circles). All IRFs are from a one standard deviation shock.)