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MODELLING THE VOLATILITY OF COMMODITIES PRICES USING A STOCHASTIC VOLATILITY MODEL WITH RANDOM LEVEL SHIFTS

Tesis para optar el grado de Magíster en Economía que presentan

DENNIS LEONARDO ALVARO POLACK

Y

ÁNGEL RÓMEL GUILLÉN LONGA

Dirigido por

GABRIEL RODRÍGUEZ BRIONES

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1 Introduction

The volatility of commodities prices such as oil or minerals is an important issue for small and open economies that depends on raw materials. For example, in many countries of Latin America, the volatility of commodities can affect operational cost or investment schedules of business related to the primary sector. At the macroeconomic level, a high volatility can provocate changes in the current account and in capital inflows, or, on the side of importers, increase uncertainty about production costs and inflation. Therefore, modeling volatility of commodities prices would be useful for private agents and policy makers. For the first ones, it gives valuable information for better options contracts that allow hedge under big uncertainty, and for the second ones, it could help to a better understanding of business cycles given the correlation between mineral prices fluctuations, capital inflows and investment expectations.

We are focused on modeling the volatility of the whole commodities market and some sectors that themselves have a huge repercussion in the global economy (i.e. gold, oil). To get this goal, we appeal to study market commodities indexes, in particular the Standar & Poors Goldman Sachs Commodity Index (hereafter S&P GSCI). As documented in S&P Dow Jones Indices (2014), the S&P GSCI is a benchmark for investment in the commodity markets and a measure of commodity market performance over time. It is also a tradable index that is readily accessible to market participants so we take this index as the best aproximation of commodity market performance. The composition of this index is favorable to energy commodities, where oil accounts for 66% of the total index. Other commodities like industrial metal or precious metals, accounts only 7% and 3% of the index, respectivetely. For this reason, as it is shown in Section 2, we analize volatility of the commodity index as a whole, and of some indexes that compose it such as gold, oil, industrial metals, agriculture and livestock index.

The evolution of commodity prices are studied like any other financial series in the literature. What is more, it exists commodity stocks markets and commodity future markets where a high grade of speculations mixes with fundamentals. The pionner work of Brennan and Schwartz (1985) analized the stochastic nature of natural resources prices and applied stochastic optimal control to the valuation of investments projects. Fama and French (1987) evaluates the commodity futures prices with the theory of storage[1] and as a forecast of a future spot price with a risk premium. Fama and French (1988) focuses on metals futures prices analyzed by theory of storage and the relationship with the stage of the business cycles. Gibson and Schwartz (1990) proposes a two factor model to analyze the pricing of oil contingent claims based on the convenience yield and Schwartz (1997) analized the behavior of commodity prices under several factor models of stochastic basis and found typical features of financial series such as media reversion. Theoretical foundation about stochastic nature of the uncertainty in investment could be found in Ingersoll and Ross (1992). Other works analyzed the rol of some commodities in portflio invesment like Jaffe (1989) who highlighted the rol of gold or precious metals in diversified portfolios and Ankrim and Hensel (1993) who focus on the similarities between commodity and real estate investment as inflation hedges. Moreover, studies like Gorton and Rouwenhorst (2006) describes financial properties of commodities based financial instruments like futures and finds similar behaviour to equities in risk premium and negative correlation with other instruments. All this financial works have to evaluate the volatility dynamics as crucial for their results. Also, evaluating only the behaviour of volatility is found in

[1] Theory of storage explains the spread between spot and futures prices based on the convenience yield on inventory

different investigations like Askari and Krichene (2008) who find that oil is very volatile and sensitive to small shocks even though assumptions about fundamentals of the markets holds. Brunetti and Gilbert (1995) studies volatility of industrial metals since 1972 to 1995 an find that volatility does not increase during that period what was contrary to common opinion. These findings suggest that commodity prices evolve quiet similar to another financial series. Therefore, it is relevant to talk about the returns of commodity prices and its associated volatility.

For this reason, we apply stock return volatility models to mineral prices series. In this field the literature is vast, and the different models proposed can be grouped in two categories: GARCH models and stochastic volatility (SV) models. For a complete survey of theses approaches, see Engle (1995) and Shephard (1995) respectively. The principal characteristic of GARCH models is that they explicitly model the conditional variance of returns given past returns, namely, volatility is predicted one-step-ahead. While, in the SV model, the predictive distribution of returns is specified indirectly via the structure of the model, rather than explicitly. The main advantage of SV models is that they have a strong theoretical support as could be found in Taylor (1986) and Taylor (1994). Also, there are many possible filtering techniques to estimate the volatility as a latent variable.

The SV models are difficult to estimate in the sense that volatility is an unobserved variable. SV models have error terms in the mean and also in the variance equation making the likelihood function difficult to evaluate. Method of Moments was suggested as a possible option to estimate SV models and was developed at Taylor (1986) among others but has efficiency problems. Also, quasi-maximum likelihood estimators coud be found by using Kalman filter in works like Harvey, Ruiz, and Shephard (1994). Finally, Bayesian procedures are the most popular method to evaluate SV models since Jacquier, Polson, and Rossi (1994) who found that this estimation procedure outperforms the other ones. In this field, Kim, Shephard, and Chib (1998) give quite an extensive discussion of various alternative methods for actually implementing Markov Chain Monte Carlo (MCMC) algorithms in order to simulate posterior distributions. The issue of sampling to simulate posterior distributions is relevant in most of Bayesian analysis and makes a very large difference in the computational efficiency of the methods. Kim, Shephard, and Chib (1998) proposes an improved MCMC algorithm based on an offset mixture of normal distributions for the error term. Finally, filtering follows an special case of Pitt and Shephard (1999) and results are better compared to GARCH models.

Volatility in financial series has some important features like clustering, leverage effects and long memory. This work is going to deal with the phenomen of long memory in commodities returns series. Long memory could be described as a slow rate of decay in the autocorrelation function (hereafter ACF) of a particular time series. These processes has been described in the literature as a fractional integrated process $I(d)$ since Adenstedt (1974) and Granger (1980). This phenomen could be found at any time series but is a particular feature that has been observed at financial returns so it has been a lot of work trying to model this characteristic. For example, Baillie (1996) is a complete survey of econometric developments in long memory and applications to economic and financial series. Also, Baillie, Bollerslev, and Mikkelsen (1996) developed a fractionally integrated model for the volatility in the family of ARCH models (FIGARCH).

The other topic of interest in this work is the structural change that is an important feature in macroeconomic time series. The seminal paper of Perron (1989) shows that the presence of a unit root could be confused with structural breaks in the series. This idea has been generalized to the context of volatility where the structural breaks could lead to identify falsely long memory. Confusing long memory processes with the presence of level shifts has been studied since Diebold and Inoue (2001) but that work present evidence against long memory process based on level shifts tests that are biased. Perron and Qu (2007) find theoretical results about the behavior of a short memory process affected by level shifts focusing on the time and spectral domain. They observe that the periodogram of the process described before follows a similar pattern of a long memory process so it is posible to confuse both processes in empirical applications. Also, Perron and Qu (2010) analize the properties of the ACF, the periodogram and the log periodogram estimate of the memory parameter of a short memory process with level shift explained by a mixture model and get a behaviour similar to long memory processes. By confronting data from various indices of stock markets, they identify similarities of the estimated statistics with the theoretical results they get before.

Qu and Perron (2013) and Lu and Perron (2010) estimate two different kinds of random level shift models of volatility (RLS, hereafter). Lu and Perron (2010) estimates their model by an extension of the Kalman filter, the model proposed could be transformed directly in state-space form by assuming a linear combination of a short memory process and a random level shift component to explain the log of the absolute returns as a proxy of volatility. They get that the remaining component if accounting for level shifts is a short memory process. Also, they estimate a GARCH model including the shift component and the GARCH effects disappear. Qu and Perron (2013) proposed a stochastic volatility model affected by random level shifts. Thence, Bayesian estimation follows procedures based on Kim, Shephard, and Chib (1998) for the sampling of the posterior distributions taking in account the random level shift term. The distribution of the probability of shifts follows a binomial distribution so the probability is varying in time. They applied this model to the Nasdaq and S&P 500 from 1980 to 2012 and for different priors getting a sensitivity analysis. Also, they get better results about the interaction of the volatility with indicators of the business cycle in United States.

Our work is based on the last model of Qu and Perron (2013) and tries to distinguish if exists long memory in commodities returns volatility series or it follows a short memory process with structural breaks. Commodities prices and volatilities affect portfolio decisions and business cycle but there is little work for modelling financial series of interest in Peru using econometric techniques. A first work by Humala and Rodríguez (2013) studies returns of exchange rate and Lima Stock Exchange of Peru and they conclude that this series has same statistical properties like any other financial series in developed markets. Recently, Alanya and Rodríguez (2014) use a SV model following Kim, Shephard, and Chib (1998) to track Peruvian stock market and exchange rate volatilities. Our job tries to fill a gap in this line of investigation through analyzing commodities volatilities.

The remainder of this document is structured as follows. Section 2 contains some features of the commodities volatility, Section 3 describes the applied methodology, Section 4 contains the results for a whole and for each kind of commodity, as well as a brief analysis of bussiness cycle comovements. Finally, Section 5 presents the conclusions.

2 Features of Commodity Volatility

In this paper we focus on commodity prices volatility because this variable is relevant to private and public agents in Latin American countries. However, before estimating this volatility is convenient to analize some features of the series and justify the method that would fit better the volatility component of these series. First of all, we use the S&P GSCI as the approximation of commodity market performance. This index include all eligibility contracts that represent transaction of a physical commodity[2] and is built from the weighted-sum of contracts of different commodities. Table 1 shows the component of the S&P GSCI. Clearly, the oil is the commodity with major contribution to the index (67.2%), seconded by Agriculture subindexes (15.3%). We consider to analize the whole commodity market and their components given possible differences between markets that influences volatility. Thus, we study the commodity index, industrial metals, oil, gold, the agriculture index, and the livestock index[3].

In Figure 1 we can see the evolution of daily returns of commodities from January 1983 to December 2013. A first feature of all series is volatile, which grows in certain periods. These periods of high volatility may be common to all series as happened between 2008-2009 that was associated with the international financial crisis, or a particular commodity as the period of late 1990 and early 1991 which was marked by a high volatility in oil prices associated with the Gulf War. In general, we observe that the series behave similarly to any high frequency financial asset such as stock returns. Therefore, it is valid to use financial modeling techniques to analyze the volatility of commodity markets.

A second feature, also linked to the volatility of the series, is the difference in behavior between markets. For example, variations in returns are larger in oil and industrial metals than in agricultural goods or livestock. In addition, these goods have different paths of volatility. For example, gold provided a period of volatility during late 2000 and early 2001, possibly associated with the crisis of the dot-com in the United States; industrial metals showed a period of high volatility between 2005 and 2008, which was probably causes by high demand of developing countries such as China; while agricultural goods witnessed a high volatility period in the late 90s due to the fall of the Soviet block, which was a important crop producer in the world market. Each of these periods of high volatility in some commodity, have not been replicated by other markets. So, while a set of commodities analysis is useful at the aggregate level, it is important to analyze each market separately given the intrinsic characteristics that influence their level of volatility.

Since it is plausible to analyze the returns of commodity prices as if they were financial series, it is worth noting two of the most important features of this type of time series. First, as has been already seen, the presence of clusters of volatility; and second, the volatility persistence or long-memory. This last characteristic has taken important relevance in the volatility literature. A simple way to detect whether the volatility of a serie has long-memory is estimating the ACF of the logarithm of its squared returns. If long-memory exists, then the ACF will decay slowly to zero. As shown in Figure 2, commodities decay slowly to zero after 1500 days, on average. Moreover, after reaching zero, the ACF oscillates around zero until the maximum number of lags. A similar behavior of the autocorrelation function is reported by Perron and Qu (2010) to analyze the S&P 500 index of the New York Stock Exchange, who argue that this behavior is a stylized fact of financial series which is suspected have long-memory[4].

As mentioned above, the assumption of long memory must be carefully analyzed. The empirical evidence (see, for example, Perron and Qu (2010)) suggests that the long-memory phenomenon can be confused with a process having discrete level changes, unusual but alter the levels of volatility in the long run. A first approach to assess whether a process

[2] For more details of S&P GSCI methodology see S&P Dow Jones Indices (2014).

[3] We separate oil and gold of their respective subindex due to the individual importance of these commotities in the global economy.

[4] According to Qu and Perron (2013) a process has long-memory if $\gamma_z(\tau) = g(\tau)\tau^{2d-1}$ as $\tau \to \infty$, where z_t is a stationary time series, $\gamma_z(\tau)$ its autocorrelation function, $d > 0$ and $g(\tau)$ is a slowly varying function as $\tau \to \infty$. The ACF decreases to zero at a hyperbolic rate, in contrast to the fast geometric rate observed for short-memory processes with $d \in (0, 1/2)$.

has long-memory is by estimating the parameter d using the log-periodogram, proposed by Geweke and Porter-Hudak (1983). The results of this estimation are shown in Figure 3. Each frame shows the estimation of the parameter memory, d, for each commodity, which is in the y axis, while the frequency of the data is on the x axis. If the process is long-memory then the parameter d should be the same for all sizes of frequency. However, the parameter memory tends to decay as the frequency is higher. The vertical lines crossing each of the figures represent the $T^{1/3}$ frequencies, $T^{1/2}$ and $T^{2/3}$ for a sample of $T = 7818$. Thus it is for low frequencies (between $T^{1/3}$ and $T^{1/2}$) the parameter d greater than 0.5, on average, while for higher frequencies tend to decline, this decline continued even for frequencies greater than $T^{2/3}$.

The results found in the log-periodogram are similar to those found by Perron and Qu (2010) who analyze the volatility of S&P 500. According to these authors, the fall in the long memory parameter with increasing frequency is due to the existence of two components in volatility, a first component, short-run, present throughout the entire series, and another component, level shifts, that causes jumps in volatility levels resembling long memory processes[5]. The latter component is dominant at low frequencies, but as the number of frequency increases, the short-term component is dominant and hence the parameter d tends to decline.

A second approach to assess long-memory processes is to rule whether or not these are spurious. For this, we used the test of Qu (2011), whereby, under the null hypothesis, the process has long-memory, while under the alternative hypothesis, the process is a short memory with level changes. The results of the test applied to the volatility of commodities are presented in Table 2. The first column shows the estimated d for $T = 0.7$; that is, to a frequency which is slightly right of $T^{2/3}$. It has none of the estimated d exceeds 0.5, which is consistent with the literature. On the other hand, the next two columns show the test statistics for two types of trimming. All volatilities of commodity returns reject the null hypothesis of long memory with a significance level of 1%. This would indicate that commodity volatilities would present discrete steps that can be interpreted as structural changes or strong shocks that permanently altered the level of volatility, simulating an apparent long memory.

In summary, after analyzing the series of commodity prices, we observed: i) high volatility of the series, accompanied by volatility cluster and high persistence, similar to that found in financial series; ii) certain differences between the commodities markets, suggesting a separate analysis for each series; and iii) the apparent long-memory of the series is actually caused by discrete jumps in volatility whose occurrence is relatively low. In view of this evidence, it is reasonable to model the volatility of commodity returns using an econometric model of volatility including the posibility of level shifts.

In the econometric literature, the SV models have been improved to include level changes; for example, the work done by Qu and Perron (2013). One advantage of this model is that volatility can be easily represented as the aggregation of two latent variables, one short term and one long term, the latter with level jumps, and both components can be estimated. GARCH type models also include level jumps, as Stărică and Granger (2005). Another type GARCH model, but applied to the volatility of oil prices is developed by Charles and Darné (2014). Both models agree that the level jumps are relevant in explaining the series with high persistence, but the jumps are exogenous to the model. In the present study we choose to follow the proposal of Qu and Perron (2013) and apply a SV model with random level shifts to model the volatility of commodity prices. The

[5] As noted by Perron (1989) a serie with the presence of breaks or level shifts resembles to the behavior of a non stationary serie, which is equivalent to a very persistent process.

methodology used is described below.

3 Methodology

The SV model with random level shifts follows the method for estimate and inference with Bayesian analysis due to Qu and Perron (2013). The objective of the paper is to model volatilities of the returns of principal commodities exported by Peru with a short memory component and random level shifts instead of assuming the presence of long memory discarded in the last section. First, the process of the return is mean corrected and is expressed by

$$x_t = \exp(h_t/2 + \mu_t/2)\varepsilon_t, \tag{1}$$

where the error term ε_t is an i.i.d. standard normal random variable. The term h_t gives us the stochastic volatility while the second term μ_t expresses the random level shifts components.

The volatility h_t is explained by an stationary $AR(1)$ process with v_t as a Normal standarized error term:

$$h_t = \phi h_{t-1} + \sigma_v v_{t-1}. \tag{2}$$

On the other hand, the level shifts component is given by the random Bernoulli variable δ_t that takes value 1 with probability p, also the size of the shift is stochastic and is given by the Normal standarized random variable η_t:

$$\mu_t = \mu_{t-1} + \delta_{t-1}\sigma_\eta \eta_{t-1}. \tag{3}$$

The random variables $\varepsilon_i, v_j, \delta_k, \eta_l$ are mutually independent for all $1 \leqslant i, j, k, l \leqslant n$. The level shifts component allows us to have different sized random shifts. Allowing this characteristic of the process, we can determine the component h_t as a short memory process for the variables analyzed.

Our proxy for volatility will be the log squared mean corrected returns $\log x_t^2$, then our model can be expresed by the following form:

$$\begin{aligned} \log x_t^2 &= h_t + \mu_t + \log \varepsilon_t^2, && (4)\\ h_{t+1} &= \phi h_t + \sigma_v v_t, && (5)\\ \mu_{t+1} &= \mu_t + \delta_t \sigma_\eta \eta_t. && (6) \end{aligned}$$

Because of ε_t is Normally distributed, the model is a partial non-Gaussian state space model. The form to address this problem is by filtering as in Kim, Shephard, and Chib (1998) with approximation of the term $\log \varepsilon_t^2$ by a mixture of Normals. A new error process is defined by ε_t^* as:

$$\varepsilon_t^* = \log \varepsilon_t^2 - E(\log \varepsilon_t^2). \tag{7}$$

Following Kim, Shephard, and Chib (1998), we aproximate the distribution of this new process by the mixture of normals:

$$\varepsilon_t^* \sim \sum_{i=1}^{K} q_i N(m_i, \sigma_i^2), \tag{8}$$

where, the parameters K, q_i, m_i, σ_i^2 that describe the distribution could be found in the work mentioned. We identify $w_t = j$, where w_t is asigned that value if ε_t^* is a realization of the j^{th} component of the mixture of normals. This way of threat the nonlinearity of $\log \varepsilon_t^2$ allow us to put all the model in a Gaussian state-space model conditioned on the mixture.

Finally, to complete the specification of the model we adress the problem of return values close to zero that distorts results of the estimations. We define another variable y_t by:

$$y_t = \log(x_t^2 + c) - E(\log \varepsilon_t^2), \tag{9}$$

where c is a small number that make the number inside the logarithm far away from the value of zero. This specification was first used by Fuller (1996) on the literature of sthocastic volatility. The value of the "offset" c is 0.001 just as Qu and Perron (2013). At last, we have the model expressed by:

$$y_t = h_t + \mu_t + \varepsilon_t^*, \tag{10}$$

$$h_{t+1} = \phi h_t + \sigma_v v_t, \tag{11}$$

$$\mu_{t+1} = \mu_t + \delta_t \sigma_\eta \eta_t, \tag{12}$$

with initial conditions $(h_0, \mu_o) = 0$ and $(h_1, \mu_1)' \sim N(0, P)$.

3.1 Sampling Procedure

We express variables and paramteres in vector notations following Qu and Perron (2013). Let $\alpha_1 = (h_1, \mu_1)$, $R = \left\{(v_1, \eta_1)', ..., (v_T, \eta_T)'\right\}$, $\delta = (\delta_1, ..., \delta_T)$, $\omega = (\omega_1, ..., \omega_T)$, $\theta = (\phi, \sigma_v, \sigma_\eta, p)$ and $y = (y_1, ..., y_T)$. The location of shifts is relate to the variable δ, whereas δ, R and α_1 jointly give the stochastic volatility process. Sampling from the joint posterior distribution $f(\theta, \alpha_1, R, \delta, \omega|y)$ is equivalent to sample from the following four block: (1) $f\left(\theta_{(-p)}, \alpha_1, R|p, \delta, \omega, y\right)$, where $\theta_{(-p)}$ denote the vector of parameters excluding p; (2) $f(\delta|\theta, \alpha_1, R, \omega, y)$; (3) $f\left(p|\theta_{(-p)}, \alpha_1, R, \delta, \omega, y\right)$; and (4) $f(\omega|\theta, \alpha_1, R, \delta, y)$. Each of this blocks generate draws using Gibbs sampling procedure.

3.2 Specification of Priors

We use the priors distribution of Kim, Shephard, and Chib (1998). For ϕ:

$$\pi(\phi) \propto \left\{\frac{1+\phi}{2}\right\}^{\phi^{(1)}-1} \left\{\frac{1-\phi}{2}\right\}^{\phi^{(2)}-1}$$

with $\phi^{(1)}$, $\phi^{(2)} > \frac{1}{2}$. We set $\phi^{(1)} = 20$ and $\phi^{(2)} = 1.5$, implying a prior mean of 0.86. For σ_v, we use Inverse-Gamma distribution so $\sigma_v^2 \sim \mathcal{IG}(\sigma_r/2, S_\sigma/2)$ with $\sigma_r = 5$ and $S_\sigma = 0.01 \times S_\sigma$. In the case of p and σ_η, we use the priors ditribution of Qu and Perron (2013) which are Beta and Inverse-Gamma. For $p \sim beta(\gamma_1, \gamma_2)$ with $\gamma_1 = 1$ and $\gamma_2 = 40$, which impplies a prior mean 1/41 or a shift each 41 days. For $\sigma_\eta \sim \mathcal{IG}(\sigma_r^*/2, S_\sigma^*/2)$ wih $\sigma_r^* = 20$ and $S_\sigma^* = 60$ which impplies a prior mean of 3.33 and a variance of 1.39. For the initial conditional state we use diffuse priors with $(h_1, \mu_1) \sim N(0, P)$ with $P = Diag\left(1 \times 10^6, 1 \times 10^6\right)$.

3.3 Filtering

In the filtering process we want to obtain recursively a sample of draws from $(\alpha_t|X_t,\theta)$ for $t=1,...,T$. We use particle filter as Kim, Shepard and Chib (1998), where, for a given sample M draws $\alpha_t^{(j)}$ $(j=1,...,M)$ from the distribution of $(\alpha_t|X_t,\theta)$, a sample from $f(\alpha_{t+1}|X_{t+1},\theta)$ is obtained by drawing from $f\left(\alpha_{t+1}|\alpha_t^{(j)},X_{t+1},\theta\right)$ and reweighting them using $f\left(\alpha_{t+1}|\alpha_{t+1}^{(j)},X_{t+1},\theta\right)$. The distribution $f(\alpha_{t+1}|\alpha_t^{(j)},X_t,\theta)/f(x_{t+1}|X_t,\theta)$ depends on wheter a shift occurs at time t and is given by:

$$\alpha_{t+1}|\left(\alpha_t^{(j)},X_t,\theta\right)\sim\delta_t W_{1t}^{(j)}+(1-\delta_t)\,W_{2t}^{(j)}$$

with

$$W_{1t}^{(j)}\sim N\left(\begin{bmatrix}\phi & 0\\ 0 & 1\end{bmatrix}\alpha_t^{(j)},\begin{bmatrix}\sigma_v^2 & 0\\ 0 & \sigma_\eta^2\end{bmatrix}\right)\text{ and }W_{2t}^{(j)}\sim N\left(\begin{bmatrix}\phi & 0\\ 0 & 1\end{bmatrix}\alpha_t^{(j)},\begin{bmatrix}\sigma_v^2 & 0\\ 0 & 0\end{bmatrix}\right).$$

The associated weights are given by $\omega_{t+1}^{(j)}=f\left(x_{t+1}|\alpha_{t+1}^{(j)},X_t,\theta\right)/\sum_{j=1}^{M}f\left(x_{t+1}|\alpha_{t+1}^{(j)},X_t,\theta\right)$, where $f\left(x_{t+1}|\alpha_{t+1}^{(j)},X_t,\theta\right)\sim N\left(0,\exp\left(h_{t+1}^{(j)}+\mu_{t+1}^{(j)}\right)\right)$.

4 Results

We use a SV model with random level shifts to estimate the commodity volatility. In particular, we applied this methodology to six indexes of SPGS: agriculture, livestock, gold, oil, industrial metals and a general commodity index. The data to be used has daily frequency over the period January of 1983 to December of 2013. The products analized are the most representative of the Peruvian trade balance and, in many cases, their behavior have a big impact on business cycles of the real economy. The analysis of commodity volatility is useful for both private and public agents, for the first ones, commodity volatility gives insights about risk management and for the second ones, a better understanding of business cycle. We intent to describe the results in a comprehensive way, thus we start with posterior distributions results description of each commodity, then we analyze the contributions of level shifts component over all volatility, and finally we analized the possible comovements between volatility and several indicators relate to Peruvian business cycle.

4.1 Posterior Distributions Results

The estimates of volatility parameters are shown at Table 3. A first interesting result is the size of the probability of level shifts. Usually this probability is small, without taking in account the gold, a break occurs between 300 to 1000 days. Another interesting result is the big differences between the variance of jumps σ_η^2 and the variance of short-memory component σ_v^2. These findings are consistente with the theorical proposal of Qu and Perron (2013) that jumps are uncommon events causes by structural breaks or big shocks that change the level of volatility abruptally and explain the most part of it. Respect to the size of the persistence of volatility, measured by the ϕ parameter, for most commodities the value of ϕ is between 0.93 and 0.98, which indicates that volatility shocks on average have a half-life from 9 days to 30 days, depending on the analyzed market. This findings are consistent with Qu and Perron (2013) that get similar results for stock index when level shifts are counted, and it goes in opposition of studies that hold long-memory assumptions, for example Vivian and Wohar (2012) estimate half-life of commodity volatility shocks between 90 and 300 days.

4.1.1 Commodity Index

As proposed above, we estimate stochastic volatility of commodity price as a whole. The model captures major shifts associated with huge shocks in the commodity markets. The panel b) of Figure 4 shows the level shift component, the line with discrete changes, and the log volatiliy, overall measure of volatility, that fluctuated around the level shifts. Some major jumps are associated to important events of commodities markets. For example, the jump ocurred at the begining of 1986 is relate to a crash in oil markets. This crisis was consequence of the "oil glut" in the first half of 1980s. After a big expansion of oil production and oversurplus, oil prices falls by over 50% in 1986. The next important jump occurs during Gulf War in response to fear of drastic oil production-cut in 1990 to 1991. The sequence of events and the evolution of volatility can see in the Table below. First, the posterior mean of the level shift variable μ_t holded low even in tension periods between Irak and Kuwait from July 15 to August 1 of 1990, but when Irak attacks Kuwait the log volatility hit a big jump from -0.45 to 1.87[6]. The volatility keeps high until the coalition force, leading by United States, attacks at January 17 of 1991, then it decreased progressevely to reach 1.42 at the end of January, when Irak forces retired from Kuwait, after that the level shift component falls to its previous levels of -0.44. After the war, the level of μ_t decreases and stays at these magnitudes for about three years. This result is consistent with the findings of Jacks, O'Rourke, and Williamson (2011) that pointing out high volatility periods in commodities during wars.

Date Event	07/15-08/01/90 Tensions	**08/02/90-01/16/91** **Irak's attack**	01/17-01/31/91 Coalition's attack	02/01-02/28/91 End of Gulf War
μ_t	-0.45	**1.87** to **1.89**	1.89 to 1.42	-0.44

In the other hand, two important increases of volatility are reported previous financial crisis of 2008. One at the begininig of 1996 and the other in 2000, both are linked to American economy performance, but in a oppositive sense, the first is associated to a fall in gold price due to a strong dollar, while the second one to dotcom crash and the subsequent United States recession. Finally, the international financial crisis of 2008 caused a jump of volatility of several markets (oil, gold, industrial metals). However the jump in volatility occurs two months before the crash in September 2008 and the level keep high for many time that in previous crisis. As we can see in the table below the log volatility increase progressevely from 1.33 in April 2008 to 1.65 in July 2008, and then volatility pick just a small jump to 1.72 and keep that level by nine months. For this phenomenom we ensay some explanations. First, the increase of volatility was progressive and it anticipated the crash due to a sequence of bad news[7]. So, the crash did not represent a great jump in volatility. Some studies like Cashin and McDermott (2002) and Vivian and Wohar (2012)

[6] These numbers represent theorically the level shift component of the volatility μ_t of commodity price returns. For example, according to (1), a value of $\mu_t = 0$ (as h_t component have mean zero and short variance) implies that commodity returns tends to a standard Normal distribution at any $t \geq 0$. For positive μ_t we have a commodity returns distribution with fat tails (more probability of extreme values), while for negative μ_t we have a distribution with a mass concentration around zero (less probability of extreme values). So we are very interesting how μ_t parameter evolves.

[7] For example the close of IndyMac Bank, the rescue of Fannie Mae and Freddie Mac, and several negatives announcements about housing markets and financial indicators.

highlight that the commodity markets are always volatile and the last financial crisis does not necessarily impregn a plus of volatility over historical registers. This fact is sopported by our estimations, for example the level of volatility was higher in the Gulf War. However, our study provides new evidence of the duration of periods of high volatility, thus volatility during 2008 crisis remains high for a long time (nine months), more than in previuos crisis. Thereby, the magnitud of a crisis could be a important source of both the magnitude and the duration of volatility.

Date Event	01/02/08-04/24 Pre-crash	04/25-07/01/08 Bad news	**07/02/08-03/25/09** **Crash**	03/26-11/09/09 Post-crash	11/10-12/31/09 Recovery
μ_t	0.65	1.33 to 1.65	**1.72**	1.69 to 1.22	0.71 to 0.51

The method applied reproduces level shifts that are coherent with commodity markets evolution and have permanent effects in the level of volatility[8]. An interesting result of estimation is that shifts are uncommon. According to posterior distributions reported in Figure 5. The probability of level shifts, p, has a posterior mean of 0.00149, which implies that a jump occurs each 671 days, roughly each 2.8 years. This make many sense if we believe that jumps have to be provoked by rare and unexpected events with big impact in commodity markets such as wars, market crashes, recessions or financial turmoils. Following with the parameters show in Figure 5 we have the posterior density of short memory parameter ϕ with a mean of 0.948 and a 95% confidence interval of $(0.913, 0.971)$. This value indicated a persistency of the log volatiliy that is consistent with the theory, but it is less than in long-memory process that report autorregresive coefficients very close to 1. Respect to variances of volatility components, we have that level shift variance has a posterior mean of 1.649, while short memory component has a posterior mean of 0.145. Namely, perturbations to the permanent component, despite being rares, causes a major impact in the volatility of the serie. As we will analyse in the next section, this component is key at the moment to explain change in the volatility of commodities. The rest of panels of the Figure 5 show the correlograms of the parameters, in general, these graphics indicated that the Bayesian estimation have no problems of autocorrelations and therefore the estimation is correct.

The estimation of parameters is robust to different priors. In Table 4 we report the posterior means of commodity volatility under different priors. For example, we choose a range of prior of p from 0.0167 to 0.001 , which impplies level shifts between 60 and 960 days. The results are not sensitive to this especification, the posterior mean of p are between 0.0013 and 0.0021 or a time occurency of level shifts between 462 and 763 days that is consistent with our estimation of 671 days. The rest of the parameters remain unchange, for example short-memory component is around 0.95 , while the variance of the level shits component is at least ten times higher than the variance of short-memory component. We also change the prior of the variance of the level shift component and the result are almost the same. We repeat this exercise for the rest of commodities, we find that posterior means and volatility components are not sensible to prior especification. However, prior distributions do affect the level of autocorrelation of posterior distribution.

4.1.2 Industrial Metals

Now, we focus on the industrial metal index which includes copper, aluminium, lead, nickel and zinc. Copper, lead and zinc are the main exports of Peru, specially copper which

[8] We say permanent in the sense that the level shift hold until another structural change or big shock causes another jump.

accounts for 23 percent of total exports in the country at 2013. The filtered volatility series and the shift levels are found at Figure 6. We could analyze if the model identifies shifts that coincide with special events for this index. Specifically, the model identifies relevant positive shifts at 1987, 2006 and 2008. In the table below we have the evolution of level shift component during 1987 and three years later. In the first four months of 1987, the level shift component was in average -0.79, which was relate to a slightly increase of index price of 0.63% per month. Then, in April 20, the model detects a level shift that increased volatility and hold it for six months. This period of high volatility coincided with a sharply increased of prices at a rate of 5% per month. The major shift occurs at October 20, a day after "Black Monday"[9], volatility jumps from 0.55 to 2.28. Prices remain very volatile for the next six months, increased 70% in the first three months to fall again previous levels just two months later. After the crash, the volatility downs progressevely and by the end of January 1991 a new level shift drops out volatility to -0.72. This period coincide with the end of Gulf War.

Date Event	01/02-04/16/87 Price estability	04/20-10/19/87 Price rising	**10/20/87-04/14/88** **Black Monday**	04/15/88-01/23/91 After crash	01/24-03/23/91 Price estability
μ_t	-0.79	0.55	**2.28**	1.78 to 0.90	-0.72

Is important highlighted that volatility of industrial metals in 1987-1991 would be explained mainly by supply and demand fundamentals. Even during the stock crash, demand side would have been the channel of volatility impact over, i.e agents' expectations or uncertainty about American economy. This argument is in the line with Brunetti and Gilbert (1995) when the high volatility in 1987-90 is associted with tight demand. According to these authors not was until 1994 when industrial metals attracted hedges funds and investment institution. They argue also that the participation of financial institutions in metals market did not increase volatility relative to historically levels. This last argument is examined in the next table, when we report the level shift volatility component from 2006 to 2009, a period of huge financial especulation in commodity markets and with a financial crisis through. The level shifts stay low for more than ten years, from 1991 to mid-2006. However, on February 2006 a major shift occurs (μ_t component jumps from 0.36 to 1.32). In this case a mix of fundamentals and especulation explain the high volatility period. A commodity boom was cause mainly by the high demand in developing countries, specially China, but market speculation contributes to raise prices in 50% in just six months. After this period, a new plateu was reached with the volatility fluctuated around one. Then, in the financial crash, volatility jump to 1.51 and increase progressively for three months, coinciding with a collapse of 50% in the price level. Both periods, even though they were highly volatile do not reach the levels reported in 1988. This behaviour is also highlighted by Vivian and Wohar (2012) but in the case of copper, they do not found a significance difference beetween high volatility in recent years versus volatility in 1980s.

Date Event	02/08/06 Pre-boom	**02/09-08/11/06** **Market Boom**	08/14/06-08/15/08 Plateau	**08/18-11/03/08** **Crash**	11/04/08-09/02/09 Post-crash
μ_t	0.36	**1.32 to 1.63**	0.97	**1.51 to 1.91**	1.90 to 1.30

Posteriors distributions and correlograms of the draws are found at Figure 7. The probability p has a posterior mean of 0.00292 which is higher than the value of p for commodity index. This value of p implies that we have a shift occurring every 342 days and this is still higher than our initial prior of every 41 days. The parameter ϕ is has

[9] The S&P 500 falls about 20 percent in a day.

a mean value of 0.932, which impplies a half-life cycle of 10 days, a very short memory proccess. Respect to variances of volatily components, as such as in the previous case, level shift component has a variance ten times that short-memory component. Jumps in volatility are caused by unsual big shocks, whereas small and usual shock determine the estationary dynamic of volatility in the short term. In panel f), g) h) and i) we report the ACF for the posterior draws. The ACF decay around zero beetween the period 100 and 200. Although, the ACF slightly out of the confidence bands for ϕ and σ_v parameters.

4.1.3 Gold

Gold volatility has some different characteristics from the other commodities. First, it has averaged more hops than other commodities, which can be clearly seen in Figure 8. Second, many of the periods identified level changes are not necessarily common to all commodities, such as breaks in the mid-90s, early 2000s or late 2011. Third, if we look at the posterior distributions in Figure 9, have the autoregressive component is about 0.1; i.e. very quickly converges to the average. Fourth, the difference between the size of the variance component of long-term and short-term is less than in other commodities. This would indicate that the volatility in gold has a very short memory, the past has little nothing in this volatility. The long-term impacts are not very large and its frequency is relatively higher. This finding is consistent with studies of Hammoudeh and Yuan (2008) and Batten, Ciner, and Lucey (2010) that show that gold is susceptible to various shocks such as economic crises, wars, changes in interest rates or supply shocks and is generally more volatile than other metals. Another feature of gold is its dual role as a financial instrument and as a hedge against inflationary periods. In terms of their volatility, this means that during periods of uncertainty, gold volatility can be increased sharply, as in systems with high inflation expectations. The table below shows this behavior through the presence of jumps from level to mid 90s. First, from April 1993 until September of that year, an increase occurs in the component level jumps in volatility due to inflation expectations for American economy. Later, after increasing interest rates throughout 1994, volatility declined instead of rising, that because the officers had already made the adjustment of interest rates. Similarly, prior to the crisis dotcom in 2000. Uncertainty about a possible bubble led to greater demand for gold by investors seeking a safe asset, this involved a rapid increase in the volatility months before the crisis, when the crisis erupted, the volatility of gold declined instead of increasing, as most of the agents already had positions in this asset.

Date	04/93-09/93	02/94-11/94	**09/99-10/09**	03/00-12/00
Event	Inflation expectations	Interest rates up	**Uncertainty**	Dot-com crash
μ_t	-1.27 to 0.00	-0.89 to -1.92	$\mathbf{-1.35}$ **to 0.5**	0.05 to -0.92

From the above, it appears that the gold level jumps seem to anticipate periods of crisis, in contrast to the volatility of other commodities which react primarily during periods of crisis. This idea is reinforced in the following table, where the periods prior to the 2008 financial crisis and the European debt crisis that intensified in 2012. More volatility is observed during the pre-crisis periods of these events. This would indicate that the mainly private operators, while not anticipate the crisis, if they perceive a scenario of greater risk to their financial positions, and therefore choose to use gold as a haven, its price suddenly increased and thus the level of volatility . This pattern is repeated in the three crisis periods analyzed, the component level shift which anticipates periods of crisis. The study of this component as a predictor of the business cycle is beyond the scope of this investigation, but lets not be interesting advantage of the method used to enable better analysis of the changes experienced volatility in relation to periods of crisis.

Date Event	**10/07-08/08** **Uncertainty**	09/08-02/09 Financial crisis	**08/11-10/11** **Uncertainty**	01/12-10/12 European debt-crisis
μ_t	$\mathbf{-0.20}$ **to** $\mathbf{1.22}$	1.24 to 1.11	$\mathbf{-0.47}$ **to** $\mathbf{1.10}$	0.17 to -0.71

In Figure 9, we find posterior distributions and the correlograms for the draws. This index has a particular result in parameter ϕ because its posterior mean is 0.078. This is the lowest value for the parameter ϕ and is near to zero, so the short memory component has not persistance at all. Also, the volatility of the gold index has the biggest probability of shifts for our six indexes. Posterior mean of p is 0.00684 or in terms of duration of the shift, it occurs every 146 days, this is the reason why we found so many shifts in this series. Another important result is the one of the parameter σ_ν that has the posterior mean value of 0.822 which is very high compared to the rest that have a maximum of 0.15. This parameter gives us the variance of the shock to the short memory component so it implies that this component is very volatile for the gold. In Figure 8, we found that gold has many shifts during our period of analysis. Also, we report ACF of posterior draws, almost all parameters do not have autocorrelations problems and ACF with the exeption of p, which fall to zero very slowly. We found that the ACF of p is sensible to prior specification, for example we explore a sensibility analysis for gold index, similar to the reported in Table 4 for commodity index, and for some values of priors the ACF converges rapidly to zero, while for others not.

4.1.4 Oil

Oil price volatility is also analyzed. In Figure 10 we show the serie of returns of oil price, for panel a), and the level shift component and the log volalitity are represented at panel b). The results are close to the ones obtained by commodity index because oil is the main component of the general index. It exists three major shifts in the evolution of oil volatility. First, the jump in volatility for the "oil glut" of 1985 to 1986. Second, the jump related to the Gulf War of 1990 to 1991 , and finally, the high volatility period of the internacional financial crisis at 2008. As we revised the Gulf War period in the analysis of commodity index, now we are going to focus on the "oil glut" in the mids of 1980s, and the last financial turmoil. Regard the first one, we have in the table below the behavior of level shift component μ_t from 1985 to 1986. During almost 1985, level of volatility remains low (around 0.16). In parallel, many negotiations beetween OPEC members was carried out in order to regulate overproduction. But, this negotiations failed and in December of 1985 and a price war begins, which cause a falling of prices by over 50 percent in the next three months. High volatility was exhacerbated by Irak-Iran war, and it holded until August of 1986 when OPEC gets an agreement.

Date Event	04/26-12/03/85 Negotiations	12/04/85-01/22/86 **Price war**	01/23-08/13/86 **Price war**	08/14-10/01/86 OPEC Agreement
μ_t	0.16	**1.88**	**2.13** to **2.70**	0.84

When we examines commodity index during last financial crisis, the probabilities of jumps was under 0.5 and in that case we argued that a possible explanation was the mix of commodities with different volatility path. This is the case of oil for example, as oppositive to gold, level shifts manifiested in differences dates, and as we can see in panel c) of Figure 10 they have a significance probability of ocurrency at the begining and the end of the crisis. The level of volatility previous crisis is estimated in 1.23 and was relative stable since the begining of 2000s. But it had a big jum few weeks previos Lemahn Borthers banckruptcy and keep high six months post the crash (see table below). This "long" period of high volatility was consistent with other commodities and with the estimations of Qu and Perron (2013) for the S&P 500 Index, and reflect the magnitude of the last financial crisis in comparison to previous crisis.

Date Event	06/04-08/20/08 Pre-crash	**08/21/08-04/20/09** **Crash**	04/21-09/25/09 Post-crash
μ_t	1.23	**2.84**	1.32 to 1.24

In Figure 11 we show the posterior distributions of parameters. Respect to the posterior mean of probability p, it has a value of 0.00178 which impplies a shift even 562 days. Namely, a level shifts is a rare event, but when it happens, its variance σ_η^2 is ten times higher than the variance of short-memory volatility component σ_v^2. Another important feature is the autorregresive estimator, which is 0.942 imppliying a half-life cycle of 12 days, very close to the cycle of industrial metals. As other commodities, persistence of volatility is manifested through high values of ϕ, but less than 1. This findings are in opposition of Vivian and Wohar (2012) that assume a long-memory process, but in concordance with Charles and Darné (2014) that include structural changes in the behavior of volatility. The ACF reported in panels f) to i) present some autocorrelation problems. Similarly to gold case, the ACF in sensible to prior especification, but it does not affect the estimation of volatility.

4.1.5 Agriculture

The agriculture index is constructed with the information of the following commodities: wheat, corn, soybeans, coffee, sugar, cocoa and cotton. The majority of these commodities are import products for Peru with the remarkable exception of coffee which is an important export of this country.

In Figure 12, we can observe that shifts are rare and the model identifies three major shifts that increases volatility which coincides with specific context of the agriculture commodities. In 1988, the volatility of the index increases dramatically between May and August of that year. This volatile period was related to the drought conditions in the United States. This increases the prices of wheat, corn and soybeans that were produced in that country. Those increments in volatility are identified by the shift component of the model which increases from -0.52 to 1.18 in May of 1988 and stayed there for three months and then drop to -0.46 in the end of August of that year as observed in the Table above.

Date Event	03/09-05/12/88 Low volatility	**05/13-08/30/88** **Drought**	08/31 Low volatility
μ_t	-0.52	**1.18**	-0.46

At 2007, the model identifies two major shifts coincident with the world food price crisis which recorded increases in prices of these commodities for different causes like financial speculation and the use of food for fuel. At March 30th in 2007, level shift component raises from 0.03 to 0.46 ans stays at that level until May 18th when other hight shifts increases that component to 0.92. After that period, the model identifies a regime where the level shift component stays at high levels between 0.93 to 0.73 from May of 2007 to

October of 2012. However, our model shows that the long period of high volatility in food prices has ended since october 2012 which see two major shifts downwards that moves level shift component to 0.4 and -0.02.

Date Event	07/28/06-03/29/07 Low volatility	**03/30-05/17** **Speculation**	05/18/07-10/01/12	10/02/12	10/22/12 Low volatility
μ_t	0.03	**0.46**	0.92 to 0.73	0.40	-0.02

In Figure 13, we can find the posteriors distributions and correlograms of the draws for the 4 parameters. The probability p has a posterior mean of 0.00099 which is very different from the prior of 1/41 and gives us that the probability of shifts is very low. This implies in average a shift happens every 1010 days. Also, we find that parameter ϕ is 0.973 and is closer to 1. The implicity half-life cycle of short-memory component is 25 days, which doubling the size of industrial or oil index. Respect to the variances of volatility components, we have a posterior mean of σ_η equal to 1.65 and for σ_ν the posterior mean is 0.12. Similarly to other indexes, the variance of level shift component is higher ten times variance of short-memory component. In general, estimators behave accordingly to expected and draws of posterior distributions and do not present problems of serial correlation. As it is shown in panels f) to i), the ACF decays maximum in 50 periods to zero for all parametes.

4.1.6 Livestock

Finally, the analysis of the livestock volatility will not be so exhaustive because it is not of main importance for external trade of Peru. The results could be found in Figure 14 where is observed that livestock volatility stays constant in perfectly identified regimes of volatility. Livestock volatility presents the less ammount of shifts in volatility. The posterior parameters could be found at Figure 15 and at Table 3 and reinforce the results observed in the evolution of the series. We found that the posterior mean of p is 0.00081 which is the lowest value for all of the probabilities of shifts in our series. What is more, the lowest value of the confidence interval of probabilities p is 0.00016 that is to close to zero. In average, it is expected a shift every 1234 days, so shifts are very rare. Also, the parameter ϕ has a value of 0.977 thus we have more persistance for the short-memory component of the volatility than in others commodities, which a implicity half-life cycle of 30 days. In this case, the short-memory component has the lowest variance ($\sigma_v = 0.076$) in comparison with other indexes, while the variance of level shift component is twenty times higher than it. Although level shifts are very uncommon events, they impregnate a high variation in volatility. Respect to serial correlation of draws, only the ACF of σ_η holds in bandwidths.

4.2 Contributions to the Overall Variation in Volatility

The model has the particular feature to split the global volatility in two components: level shifts and a short memory component. If we think that this model could replicate empirical features of the data, we have to analyze if this decomposition is signifcant. To see this, we could divide the contributions to the overall volatility following Qu and Perron (2013): $s_t = \mu_t + h_t$ with s_t beint the overall volatility, μ_t and h_t are the level shift and short memory components, respectively. If, we denote by s, μ and h the sample means of the correspondent processes then we have that $(s_t - s) = (\mu_t - \mu) + (h_t - h)$ so the following ratios:

$$\frac{\sum_{i=1}^{n}(\mu_t - \mu)^2}{\sum_{i=1}^{n}(s_t - s)^2} \text{ and } \frac{\sum_{i=1}^{n}(h_t - h)^2}{\sum_{i=1}^{n}(s_t - s)^2},$$

give us the contributions of μ_t and h_t to the global variation in volatility of our indicators. Qu and Perron (2013) found that level shift component is more important that the short-memory component to explain the variations in volatility of the S&P 500 and Nasdaq daily returns. Table 5 resumes our results for the six indexes and find similar results to Qu and Perron (2013) for all of them.

Level shift component has an important part in the explanation of the volatility variation. The maximum contribution of the level shift component to the volatility is 0.84 and corresponds to the industrial metals. Gold index and commodities index level shift components follow closely Industrial metals in the ammount of contribution to the overall variation in volatility. Those volatilities has different evolutions as observed in the Section before but they have in common that accounting for level shift components is relevant for volatility modelling. Agriculture level shift component accounts for 54 percent of the variation in volatility what is an important part but less than the other ones. This ammount is similar to what is observed for livestock volatility and those are the cases where level shift component explains the less of the variation in volatility. However, it explains more than 50 percent of volatility with not many shifts as seen before. Finally, Oil index has same results as commodities index because it is the main component of it and has many shifts but no so much as Gold.

With this measurement, we can conclude that variation in volatility could be better predicted with the level shift component that is less volatile than short memory component which is a noisy process. Therefore, commodities volatilities could be better predicted and analyzed with a level shift framework instead of a long memory analysis.

4.3 Business Cycle Comovements

An important aspect of commodities index volatility is the presence of comovements with business cycle indicators in small and commodities exporting economies like Peru. We estimate the correlation between components of commodities returns volatility and some indicators of Peruvian economy by common regressions. The indicators used are consumption of cement, production of electricity, expectations of economy[10] and money supply because they are observed constantly by private and government analysts in Peru. Also, we are going to measure the correlations obtained between volatility of commodities with some indicators of production: total and sectorial gross domestic product, the sector analyzed will be agriculture, mining, construction and manufacture.

The data is obtained from the Central Bank of Peru in monthly frecuency. So, we transform our results of level shift component, short memory component and the overall volatility of the series to monthy data by montly averages. After the transformation of frecuency, we get the correlations with the interannual variation of the business cycle indicators. Results are presented in Table 6. First, all commodities prices volatilities are correlated with indicators of business cycle, but not in the same direction, industrial minerals and oil volatility present a positive correlation, while gold a negative one. This may be explained by the correlation between financial markets and gold volatility, while some periods of high volatility in industrial minerals or oil have been linked to the "boom"

[10] Expectations indicator building by the Central Bank of Peru.

of commodities. Second, only gold is a significant variable to explain expectations, which suggest the relevance of gold volatility as indicator of financial stability and therefore outcome performance in the future.

It is expected that industrial metals and oil being highly correlated to business cycle indicators and this is true with the indicators of consumption of cement and production of electricity. Also, we get some spurious correlations of the agriculture and livestock indexes volatilities with those indicators because they are not expected to affect or being affected by peruvian business cycle.

Also, we get some correlations with gross domestic product (GDP) indicators. Agriculture volatility is correlated positively with total GDP and Agriculture GDP. Industrial Metals and oil volatilities are highly correlated with total, manufacturing and construction GDP but the correlations with mining GDP are not high but still significative. Gold volatility does not present correlation with total and mining GDP. Finally, the volatiltity of the index of all commodities shows correlations with total and all sector GDPs because it is mainly composed by oil and industrial metas indexes.

When we analyze the correlations for all the components we get that the short memory component h_t has no correlation at all with the indicators of the business cycle. The level shift component accounts for all the correlation that the volatility of commodities index has with economic activity indicators. These could be interpreted as that level shift component captures macroeconomic drivers behing volatility while the short memory component accounts for the noise of daily activity in stock markets.

4.4 Analysis of Residuals

One way to observe if the model fit well for our analysis of the data is to observe the behaviour of residuals. From equation (1) we have that $x_t = \exp(h_t/2 + \mu_t/2)\varepsilon_t$ and the series x_t, h_t and μ_t are outputs from the estimation and filtering. Thence, $\widehat{\varepsilon_t}$ could be extracted directly from our results as an estimation of ε_t. The assumptions are that ε_t is i.i.d and with Normal distribution. Therefore, we could observe if the the standardized estimated residuals $\widehat{\varepsilon_t}$ behaves as Gaussian and are independent by applying some well known graphical analysis.

QQ plots are used to ensure that our residuals approximates to a random variable with Normal distribution. To analyse independence in estimated residuals we could analyse the ACF of residuals and squared residuals. However, as the returns does not exhibit autocorrelations then we only need to analyse if our meausurements of volatility of the residuals presents autocorrelations. The results presented includes the Figures of the ACFs obtained from the log squared residuals ε_t

In Figure 16 and Figure 17 are presented the results of residual analysis of Commodity, Industrial metals, Gold, Oil, Agriculture and Livestock indexes, respectively. All the series have the characteristic that their estimated residuals $\widehat{\varepsilon_t}$ does not exhibit significant autocorrelation in their log squared and absolute values[11]. The values of the autocorrelation are in general minor than 0.05 and they are inside the Bartlett windows.

On the other hand, all the series does not have the same QQ plot results. Agriculture and Livestock estimated standardized residuals presents the best QQ-plots results in the sense that their estimated distribution aproximates more to the standard Normal distribution. On the other hand, gold index residuals does not have the same behaviour. It exhibits large fat tails that indicates the presence of large shocks even though we include

[11] The ACF of absolute mean error is done too but no reported, the results indicate no problems of serial correlation.

the level shift component. This is not surprising after all because of the gold index is the most volatile of the all six indeces analyzed. Finally, industrial metals, oil and all commodities indices exhibit reasonable QQ-plots results.

By the analysis of residuals, we can conclude that the model fit quite well for our series. However, the only exception would be the volatile series of gold because does not match the Normal distribution assumption but it accomplishes the independent assumption.

5 Conclusions

This study modeled the volatility of commodities indexes of S&P GSCI following the methodology of Qu and Perron (2013) that included random level shifts in the SV model of Kim, Shephard, and Chib (1998).

The main results seem to confirm the relevance of shifts in the volatility of the studied series. After considering these breaks, the alleged long memory disappears and volatility converges to its mean in a short period of time.

The persistence of the short memory component is lower than one so the average life of a shock reduces compared to standard SV models. However, we have the exception of Livestock index that present extremely rare shifts and those shifts does not explain variations in volatility. Also, the persistance of its noise component is near to one. Despite of these results, Livestock index is not so important for peruvian trade. Also, gold index has different results because presents so many shifts and parameter ϕ is close to zero.

Shifts are rare in volatilities but they account for most of the variation in them for all commodities indexes except for Livestock. It was no important that gold has more shift than industrial metals or oil more than agriculture; all of them presents that level shift component was significant in volatility modelling.

The analysis of residuals presents that autocorrelation in log squared and absolute value of standardized residuals dissapears. This means that the model captures all of the second moment autocorrelations of the series. The QQ plot gives us similar results with the standardized residuals being close to the Normal distribution as supposed in the model with the exception of gold Index which has fat tails.

Finally, we find that the components of level shifts in the volatility of commodities prices are strongly correlated with indicators of Peruvian economic cycle like capital goods imports, economy expectations, production of electricity and internal consumption of cement. However, Livestock index and Agriculture index are the exception, they do not account for much of the international trade of Peru. Not only that, if we include indicators of sectoral gross domestic product, the volatility is still highly correlated with interanual variations of these indicators.

With the new estimated parameters, we could construct better measurements of risk for the commodities prices to help private companies or to create special government funds in order to avoid being affected for high volatile prices of the traded commodities.

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6 Annexes: Tables and Figures

Table 1. Composition of SP GSCI

	Weight	Included Commodities
Energy	69.8%	-
- Oil	67.2%	-
- Natural Gas	2.6%	-
Industrial Metals	6.7%	-
- Copper	3.2%	-
- Others	3.5%	Aluminum, Lead, Nickel and Zinc
Precious Metals	3.3%	-
- Gold	2.8%	-
- Silver	0.5%	-
Agriculture	15.3%	Wheat, Corn, Soybeans, Coffee, Sugar, Cocoa, Cotton
Livestock	4.9%	Lean Hogs, Live Cattle, Feeder Cattle

Source: S&P GSCI Methodology, 2014.

Table 2. Test Against Spurious Long Memory

	$\widetilde{d}$ (local Whittle)	W ($\varepsilon = 0.02$)	W ($\varepsilon = 0.05$)
Commodity Index	0.37	2.14**	2.14**
Copper	0.41	2.03**	1.63**
Gold	0.37	1.56**	1.37*
Oil	0.34	2.03**	2.03**
Agriculture	0.34	1.84**	1.84**
Livestock	0.24	2.12**	1.79**

H0: serie is a stationary long-memory process, H1: serie is affected by regime change or a smoothly varying trend.

Table 3. Posterior Estimates for Commodities Indexes volatilities

Index	Parameters			
	p		ϕ	
	Posterior Mean	95% credible set	Posterior Mean	95% credible set
Commodity Index	0.00149	[0.00068, 0.00258]	0.948	[0.913, 0.971]
Agriculture	0.00099	[0.00035, 0.00198]	0.973	[0.960, 0.983]
Livestock	0.00081	[0.00016, 0.00177]	0.977	[0.959, 0.992]
Industrial Metals	0.00292	[0.00166, 0.00451]	0.932	[0.902, 0.960]
Oil	0.00178	[0.00079, 0.00319]	0.942	[0.914, 0.964]
Gold	0.00684	[0.00461, 0.00949]	0.078	[0.012, 0.177]
Index	Parameters			
	σ_ν		σ_η	
	Posterior Mean	95% credible set	Posterior Mean	95% credible set
Commodity Index	0.145	[0.107, 0.195]	1.649	[1.273, 2.157]
Agriculture	0.123	[0.101, 0.147]	1.650	[1.260, 2.187]
Livestock	0.076	[0.056, 0.104]	1.645	[1.245, 2.213]
Industrial Metals	0.152	[0.118, 0.189]	1.479	[1.177, 1.891]
Oil	0.168	[0.133, 0.206]	1.652	[1.271, 2.147]
Gold	0.822	[0.773, 0.878]	1.267	[1.064, 1.531]

Table 4. Posterior Means for Commodity Index Volatility Under Different Priors

Index	(a) Vary γ_1			
	$\gamma_1 = 0.25$		$\gamma_1 = 4$	
	Posterior Mean	95% credible set	Posterior Mean	95% credible set
p	0.00146	[0.00068, 0.00265]	0.00216	[0.00108, 0.00352]
ϕ	0.945	[0.915, 0.967]	0.948	[0.916, 0.970]
σ_ν	0.147	[0.113, 0.186]	0.140	[0.108, 0.181]
σ_η	1.626	[1.264, 2.143]	1.610	[1.242, 2.112]
Index	(b) Vary γ_2			
	$\gamma_2 = 60$		$\gamma_2 = 960$	
	Posterior Mean	95% credible set	Posterior Mean	95% credible set
p	0.00180	[0.00089, 0.00304]	0.00129	[0.00060, 0.00222]
ϕ	0.933	[0.887, 0.963]	0.952	[0.929, 0.970]
σ_ν	0.158	[0.117, 0.212]	0.144	[0.114, 0.179]
σ_η	1.566	[1.226, 2.018]	1.628	[1.266, 2.134]
Index	(c) Vary σ_r^*			
	$\sigma_r^* = 10$		$\sigma_r^* = 40$	
	Posterior Mean	95% credible set	Posterior Mean	95% credible set
p	0.00138	[0.00063, 0.00236]	0.00170	[0.00076, 0.00302]
ϕ	0.936	[0.895, 0.965]	0.942	[0.911, 0.968]
σ_ν	0.159	[0.115, 0.210]	0.149	[0.111, 0.197]
σ_η	2.026	[1.487, 2.794]	1.217	[1.000, 1.491]
Index	(d) Vary S_σ^*			
	$S_\sigma^* = 30$		$S_\sigma^* = 120$	
	Posterior Mean	95% credible set	Posterior Mean	95% credible set
p	0.00162	[0.00072, 0.00287]	0.00131	[0.00060, 0.00224]
ϕ	0.951	[0.917, 0.975]	0.949	[0.917, 0.970]
σ_ν	0.140	[0.105, 0.191]	0.145	[0.113, 0.190]
σ_η	1.271	[0.969, 1.664]	2.195	[1.702, 2.870]

Table 5. Contributions to overall volatility

Index	Component	
	Level Shift	Stationary
Commodities	0.81	0.14
Agriculture	0.54	0.35
Livestock	0.52	0.36
Industrial Metals	0.84	0.10
Oil	0.70	0.23
Gold	0.80	0.17

Note: The contributions are obtained from the decomposition $s_t = \mu_t + h_t$ where μ_t corresponds to the level shift component while h_t is the stationary component. The contributions to the overall volatilities are obtained from the following: $\frac{\sum_{i=1}^{n}(\mu_t-\bar{\mu})^2}{\sum_{i=1}^{n}(s_t-\bar{s})^2}$ and $\frac{\sum_{i=1}^{n}(h_t-\bar{h})^2}{\sum_{i=1}^{n}(s_t-\bar{s})^2}$

Table 6. Comovement between volatilty components and business cycle indicators

Panel (a). Agriculture Index						
	μ_t		h_t		$\mu_t + h_t$	
Variables	Coefficient (t-stat)	R^2	Coefficient (t-stat)	R^2	Coefficient (t-stat)	R^2
Consumption of Cement	10.64	0.16	-0.67	0.00	5.89	0.09
	(6.70)		(-0.29)		(4.68)	
Production of Electricity	3.82	0.06	0.21	0.00	2.26	0.04
	(3.72)		(0.15)		(2.88)	
Expectations of the economy	4.18	0.01	-22.16	0.09	-2.51	0.00
	(1.04)		(-3.47)		(-0.77)	
Money Supply	-2.17	0.00	0.28	0.00	-1.13	0.00
	(-0.63)		(0.06)		(-0.44)	
GDP	7.37	0.45	2.09	0.01	5.34	0.36
	(9.95)		(1.24)		(8.21)	
Agriculture production	4.59	0.16	3.56	0.03	3.84	0.17
	(4.71)		(2.02)		(4.89)	

Panel (b). Industrial Metals Index						
	μ_t		h_t		$\mu_t + h_t$	
Variables	Coefficient (t-stat)	R^2	Coefficient (t-stat)	R^2	Coefficient (t-stat)	R^2
Consumption of Cement	9.35	0.25	4.26	0.01	8.33	0.23
	(8.63)		(1.16)		(8.27)	
Production of Electricity	4.42	0.15	2.01	0.00	3.94	0.14
	(6.32)		(0.90)		(6.09)	
Expectations of the economy	0.63	0.00	-14.11	0.01	-0.27	0.00
	(0.23)		(-1.33)		(-0.10)	
Money Supply	5.97	0.02	6.52	0.00	5.73	0.02
	(2.39)		(0.90)		(2.49)	
GDP	5.62	0.56	-1.13	0.00	4.91	0.48
	(12.29)		(-0.42)		(10.52)	
Mining GDP	1.69	0.05	4.25	0.02	1.75	0.06
	(2.50)		(1.59)		(2.75)	
Manufacturing GDP	6.30	0.31	-2.88	0.00	5.40	0.26
	(7.37)		(-0.72)		(6.48)	
Construction GDP	10.91	0.44	1.31	0.00	9.73	0.39
	(9.60)		(0.22)		(8.76)	

Table 6. (continued)

Panel (c).Gold Index

Variables	μ_t Coefficient (t-stat)	R^2	h_t Coefficient (t-stat)	R^2	$\mu_t + h_t$ Coefficient (t-stat)	R^2
Consumption of Cement	-1.82	0.02	7.33	0.00	-1.71	0.02
	(-2.00)		(0.76)		(-1.90)	
Production of Electricity	-2.42	0.08	7.99	0.01	-2.29	0.08
	(-4.54)		(1.37)		(-4.34)	
Expectations of the economy	-20.75	0.17	15.56	0.00	-19.16	0.15
	(-5.12)		(0.46)		(-4.84)	
Money Supply	-14.37	0.28	1.01	0.00	-14.02	0.27
	(-9.77)		(0.05)		(-9.60)	
GDP	0.67	0.00	-0.43	0.00	0.62	0.00
	(0.60)		(-0.05)		(0.58)	
Mining production	2.45	0.04	7.16	0.01	2.39	0.04
	(2.23)		(0.78)		(2.25)	

Panel (d). Oil Index

Variables	μ_t Coefficient (t-stat)	R^2	h_t Coefficient (t-stat)	R^2	$\mu_t + h_t$ Coefficient (t-stat)	R^2
Consumption of Cement	4.71	0.19	-7.32	0.02	4.01	0.15
	(7.37)		(-2.34)		(6.36)	
Production of Electricity	4.72	0.53	-1.68	0.00	4.28	0.47
	(15.85)		(-0.88)		(14.13)	
Expectations of the economy	-0.82	0.00	-18.78	0.03	-1.55	0.00
	(-0.41)		(-1.98)		(-0.80)	
Money Supply	12.58	0.30	-4.17	0.00	11.25	0.26
	(10.35)		(-0.70)		(9.45)	
GDP	4.40	0.60	-3.26	0.02	4.04	0.54
	(13.48)		(-1.35)		(11.74)	
Mining production	2.13	0.14	0.21	0.00	2.03	0.13
	(4.39)		(0.09)		(4.28)	
Manufacturing production	5.14	0.37	-7.65	0.04	4.56	0.31
	(8.35)		(-2.14)		(7.24)	
Construction production	7.93	0.41	-10.49	0.03	7.09	0.34
	(9.04)		(-1.99)		(7.87)	

Table 6. (continued)

Panel (e).Commodities Index						
	μ_t		h_t		$\mu_t + h_t$	
Covariables	Coefficient (t-stat)	R^2	Coefficient (t-stat)	R^2	Coefficient (t-stat)	R^2
Consumption of Cement	5.62	0.09	-6.35	0.02	3.62	0.05
	(4.59)		(-1.98)		(3.28)	
Production of Electricity	5.27	0.20	-3.57	0.01	3.69	0.13
	(7.61)		(-1.83)		(5.75)	
Expectations of the economy	-1.19	0.00	-35.84	0.11	-4.10	0.02
	(-0.38)		(-3.96)		(-1.46)	
Money Supply	-1.67	0.00	-0.63	0.00	-1.43	0.00
	(-0.68)		(-0.10)		(-0.65)	
GDP	5.53	0.40	-0.73	0.00	4.38	0.31
	(8.84)		(-0.30)		(7.30)	
Agricultural production	3.56	0.15	0.16	0.00	2.87	0.12
	(4.53)		(0.06)		(4.01)	
Mining production	2.58	0.08	4.13	0.02	2.43	0.09
	(3.32)		(1.70)		(3.51)	
Manufacturing production	6.20	0.22	-5.71	0.02	4.48	0.14
	(5.84)		(-1.59)		(4.49)	
Construction production	10.41	0.29	-5.93	0.01	7.84	0.21
	(6.99)		(-1.12)		(5.55)	

Figure 1. Returns of Commodities

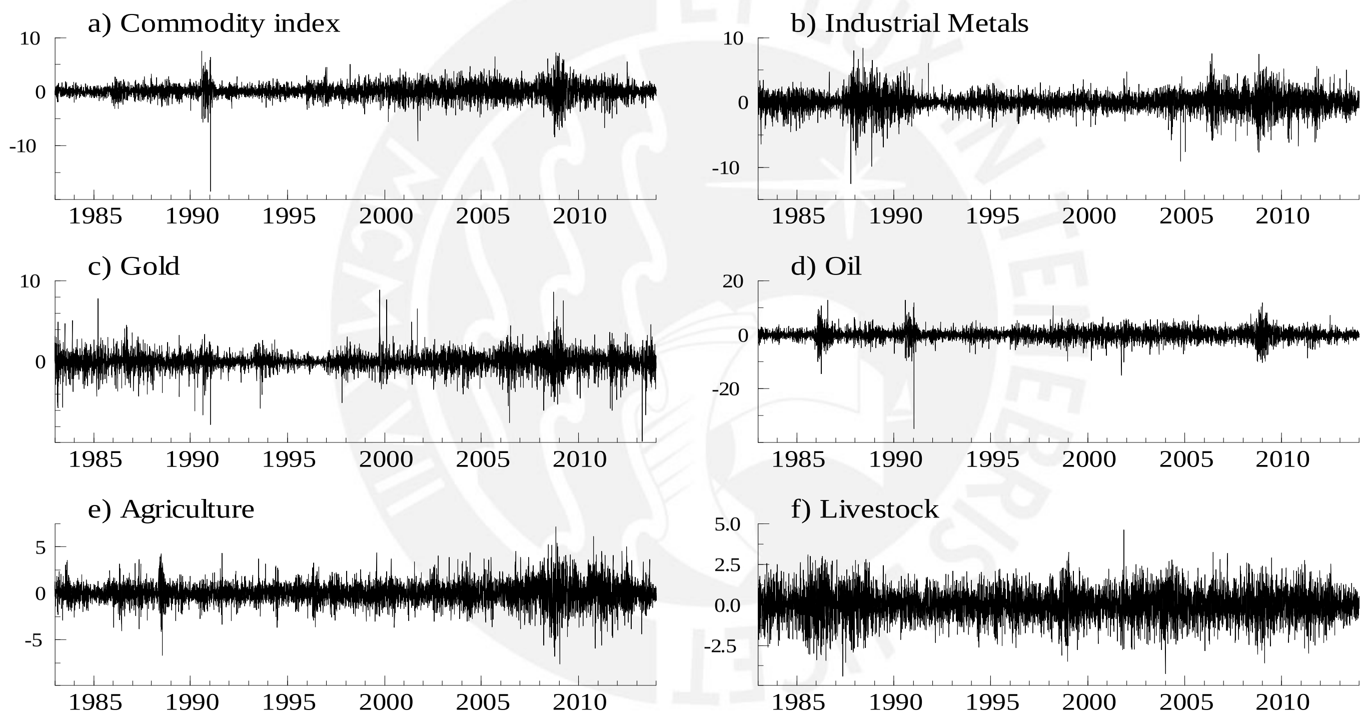

Figure 2. The autocorrelation function of commodities log-squared returns

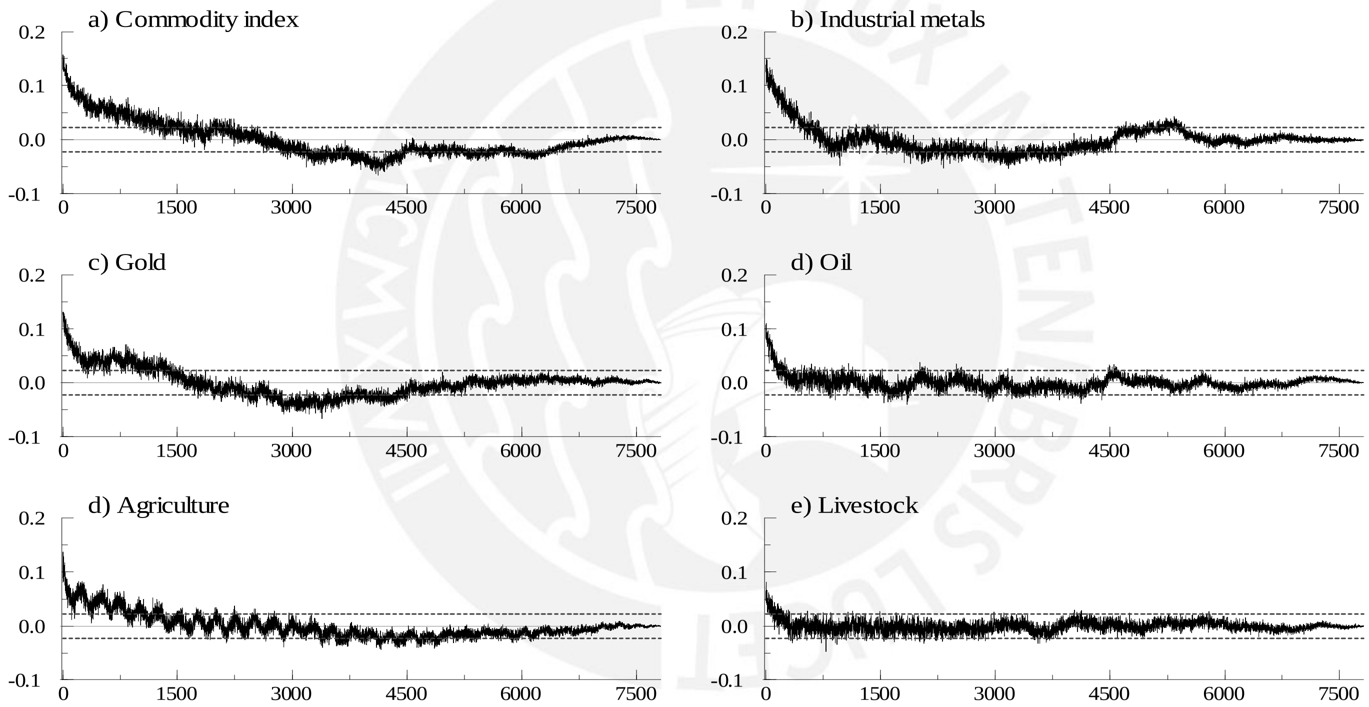

Figure 3. The log periodogram estimate of fractional d as a function of m

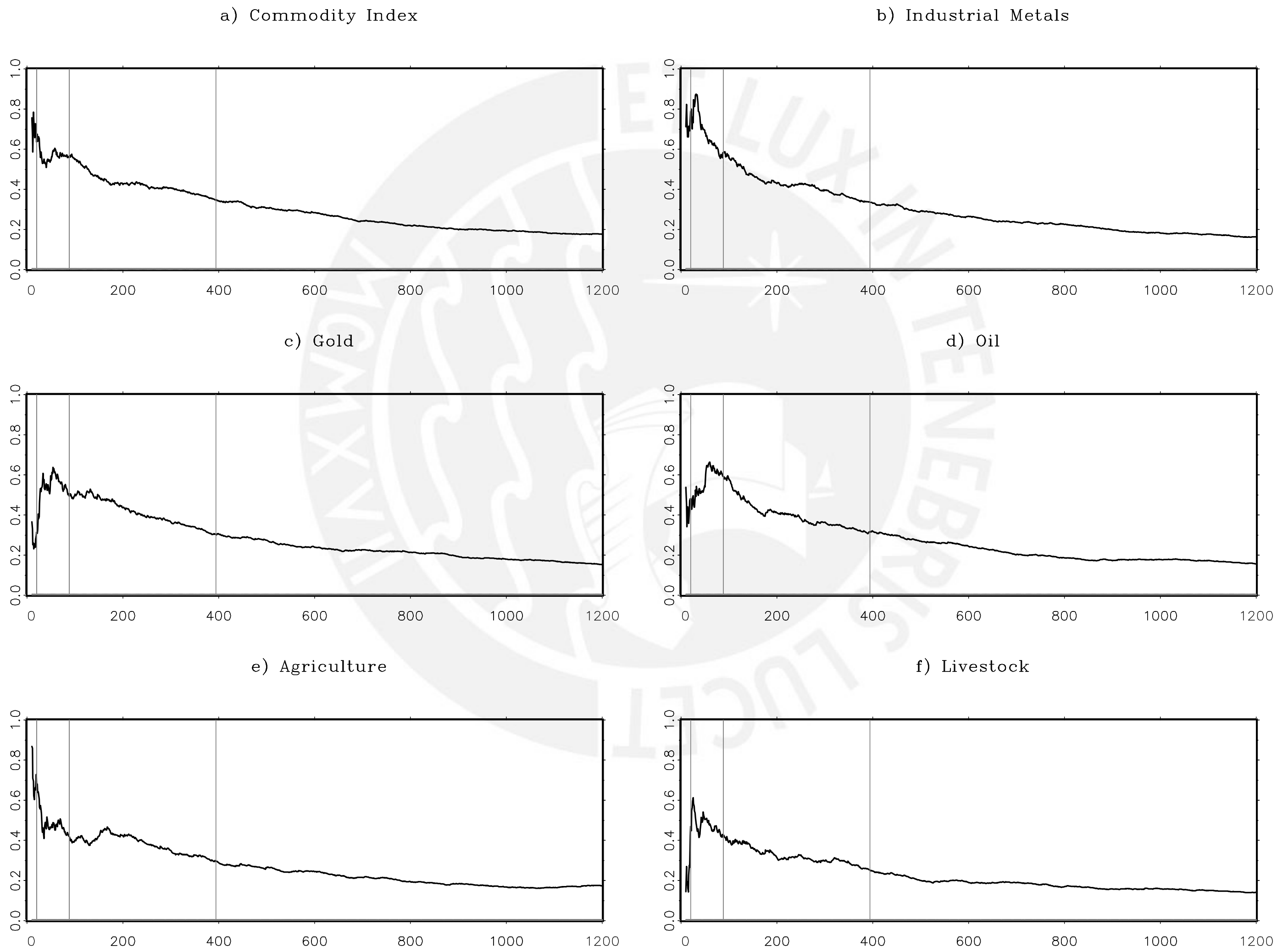

Figure 4. Results for Commodity Index Volatility

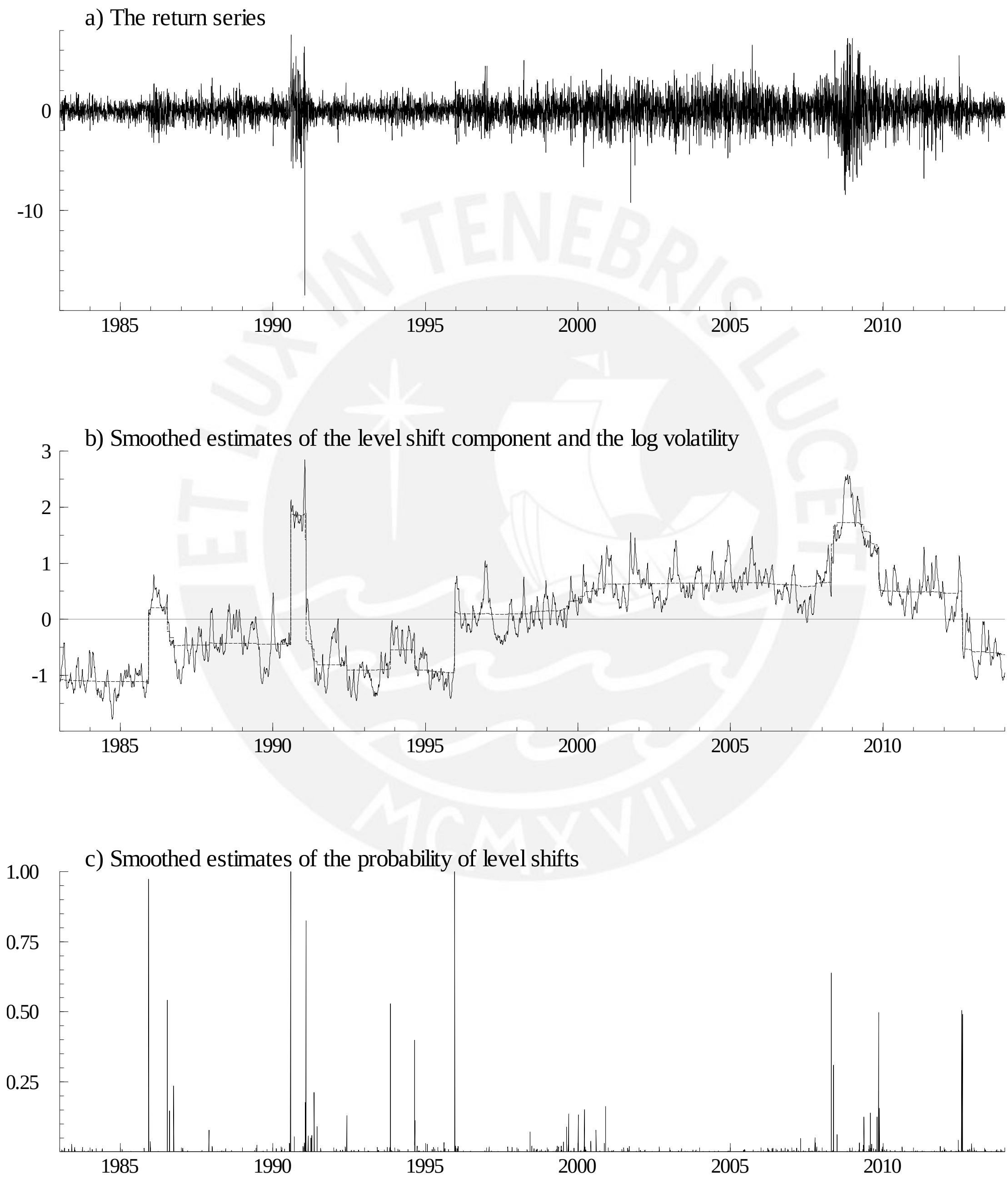

Figure 5. Posterior Estimates for Commodity Index Volatility

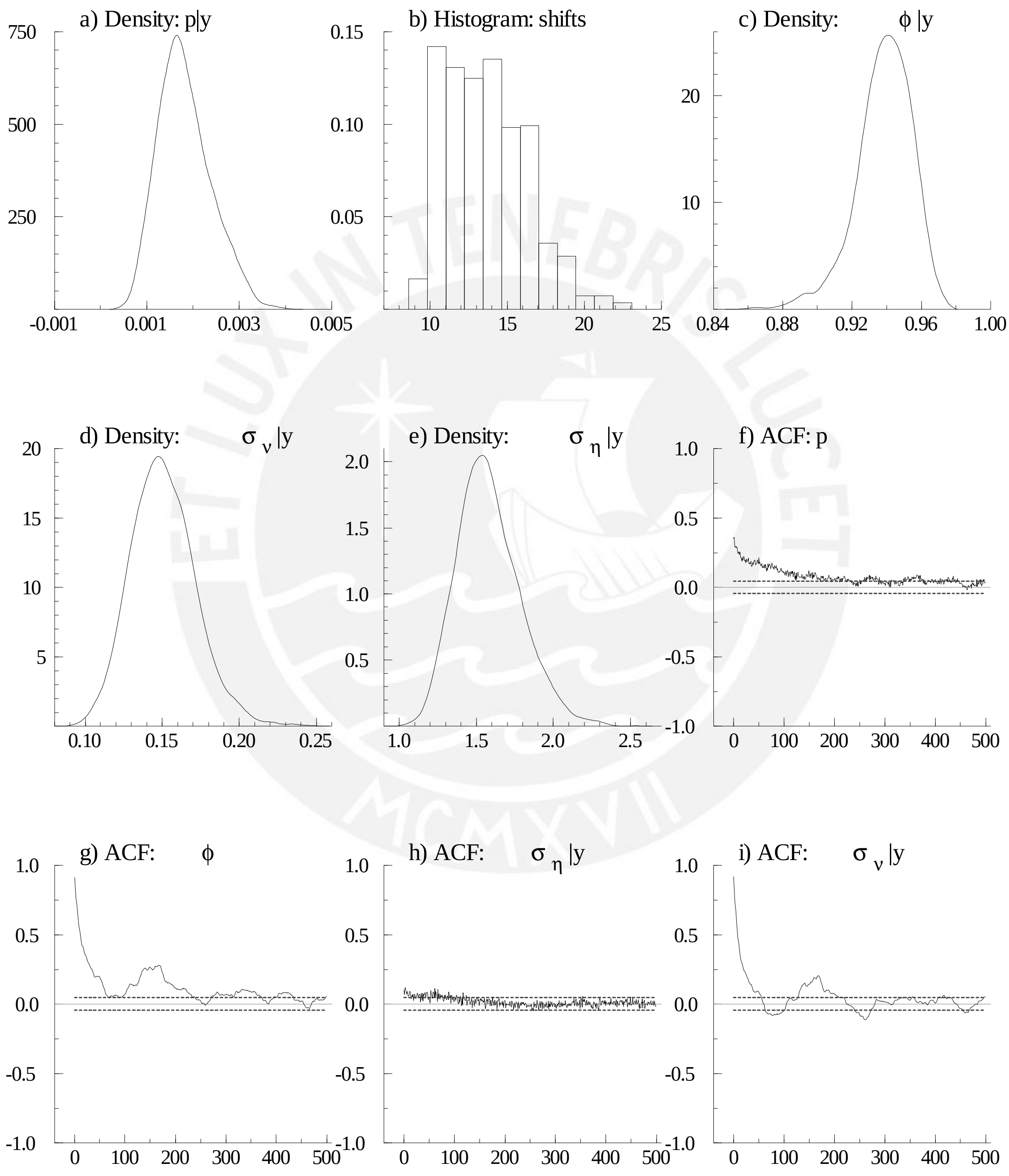

Figure 6. Results for Industrial Metals Index Volatility

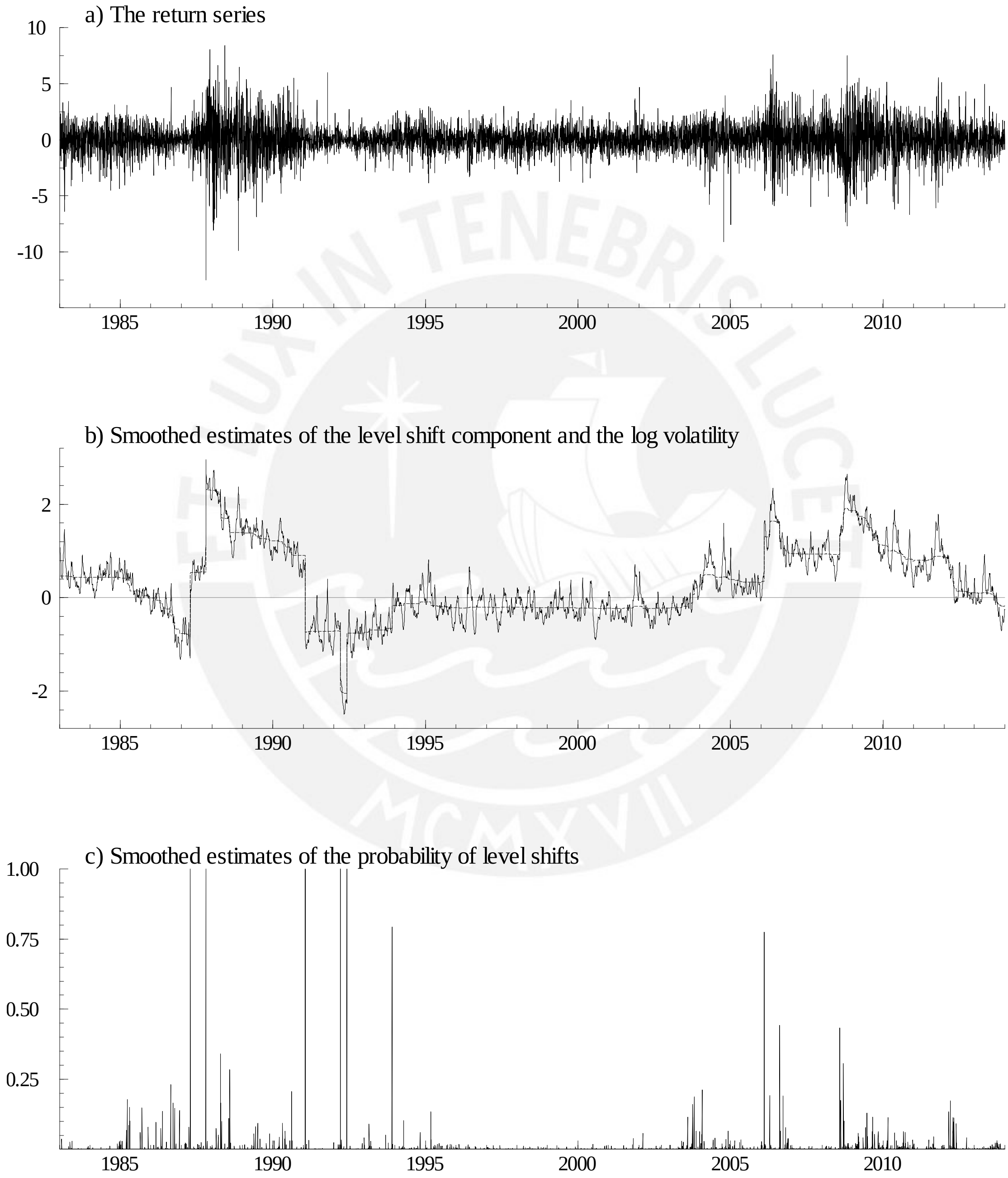

Figure 7. Posterior Estimates for Industrial Metals Index Volatility

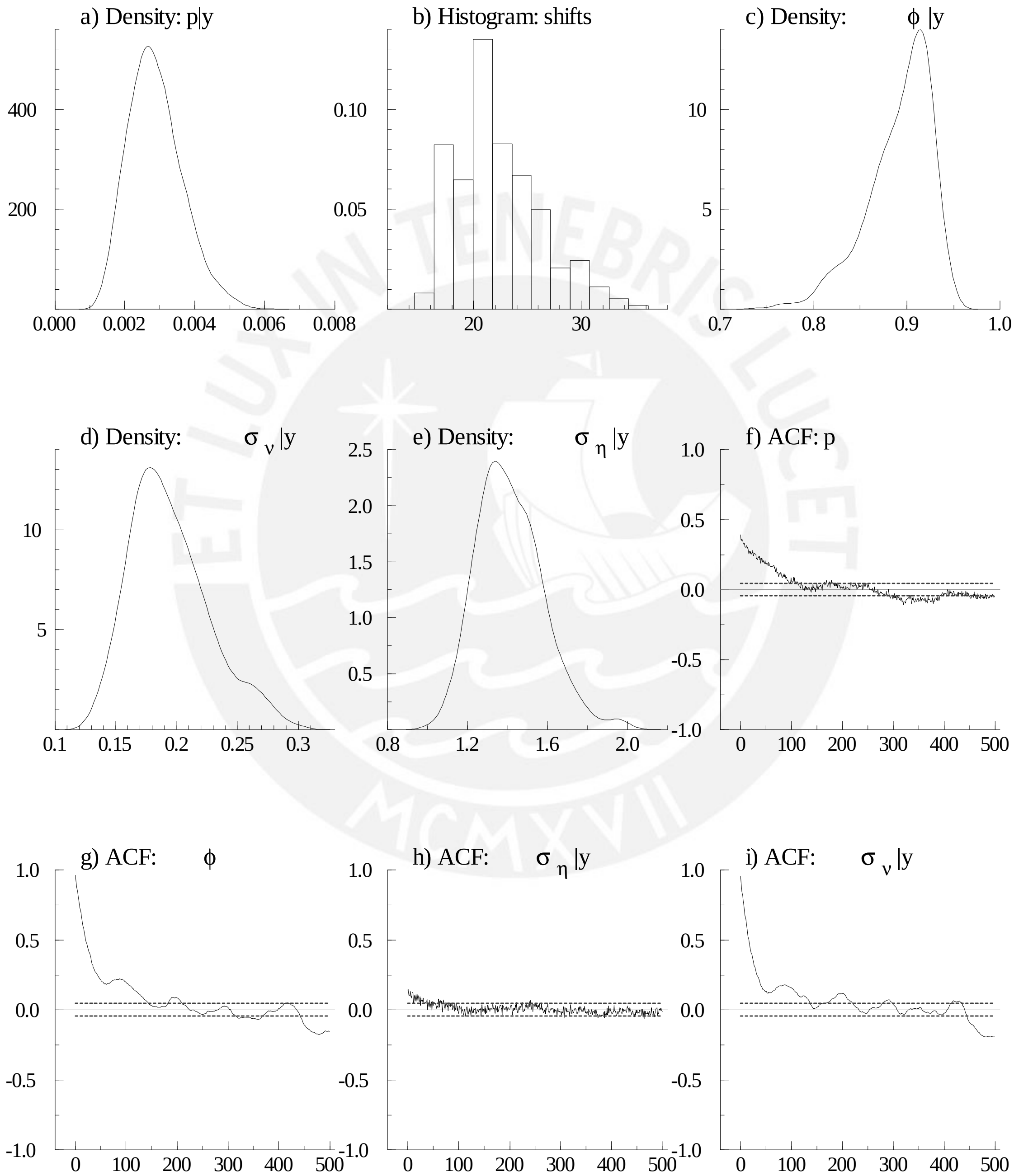

Figure 8. Results for Gold Index Volatility

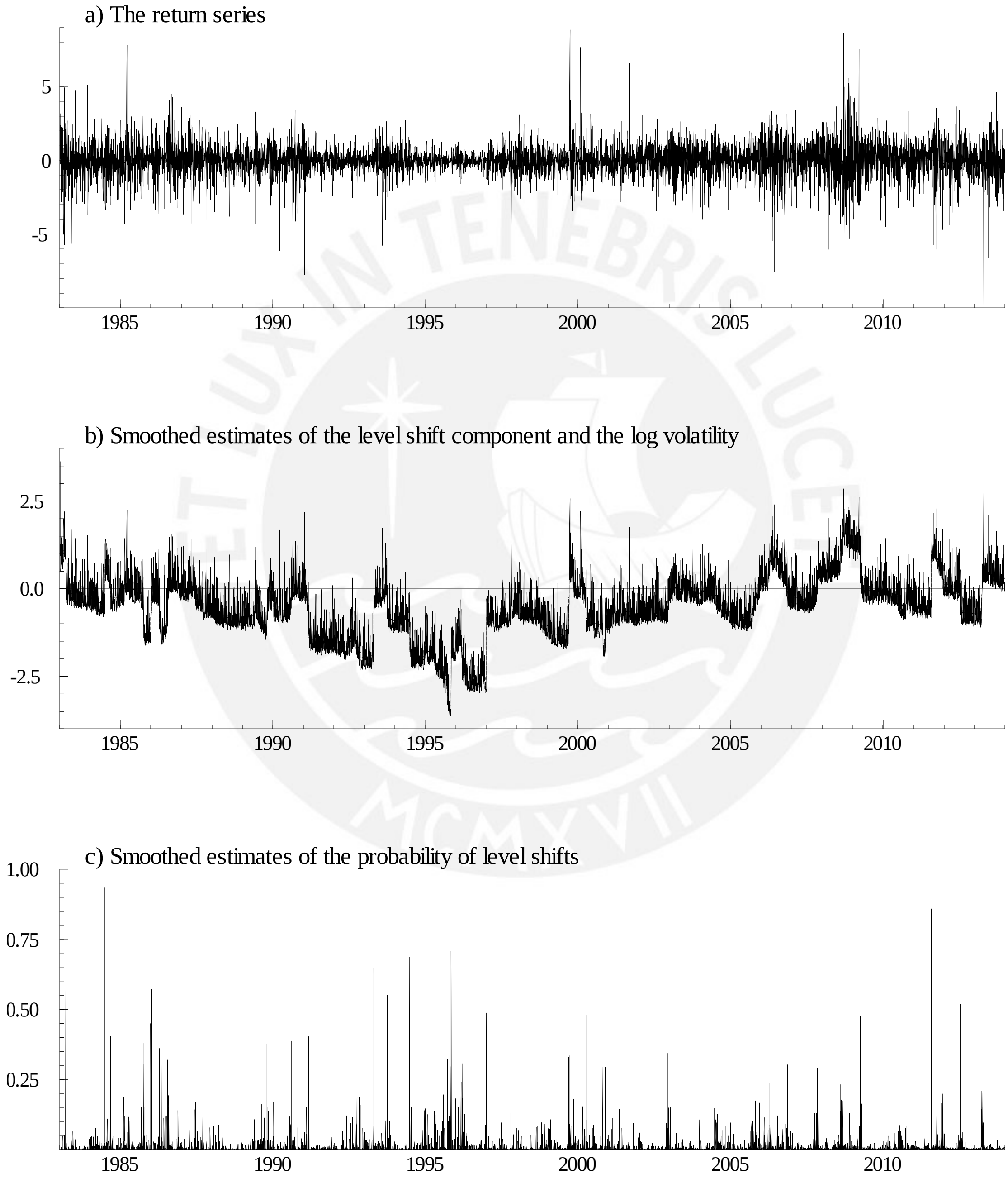

Figure 9. Posterior Estimates for Gold Index Volatility

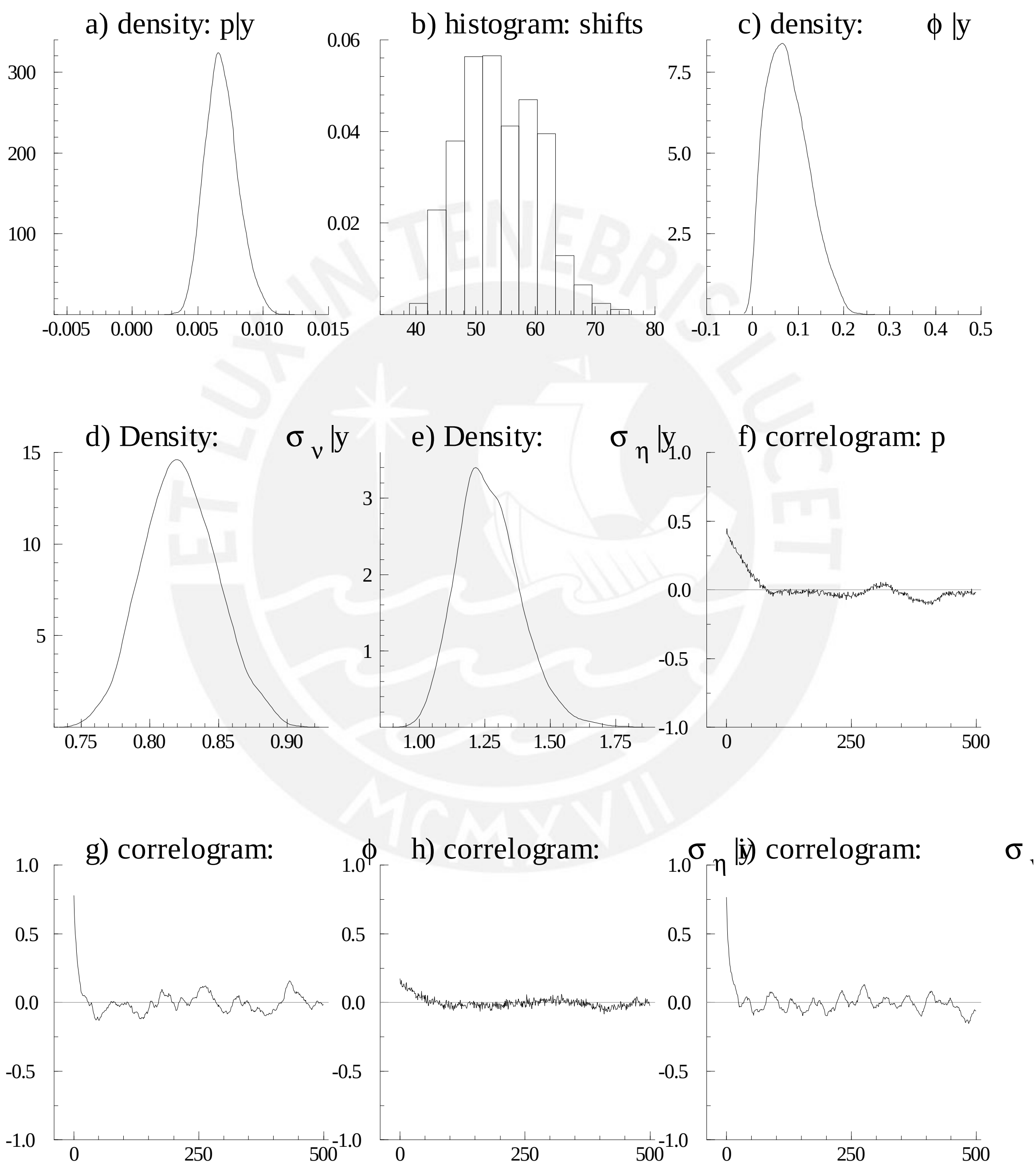

Figure 10. Results for Oil Index Volatility

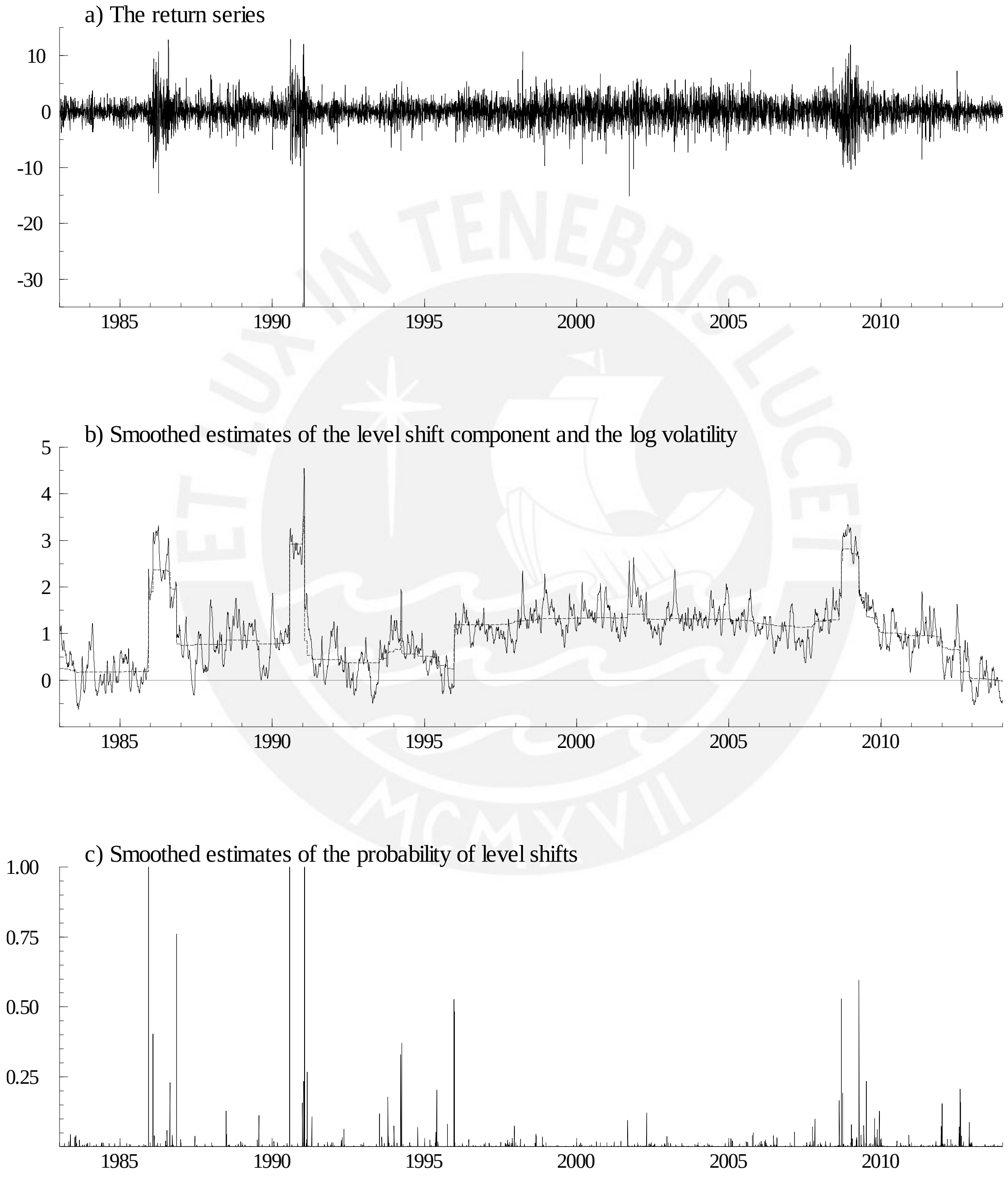

Figure 11. Posterior Estimates for Oil Index Volatility

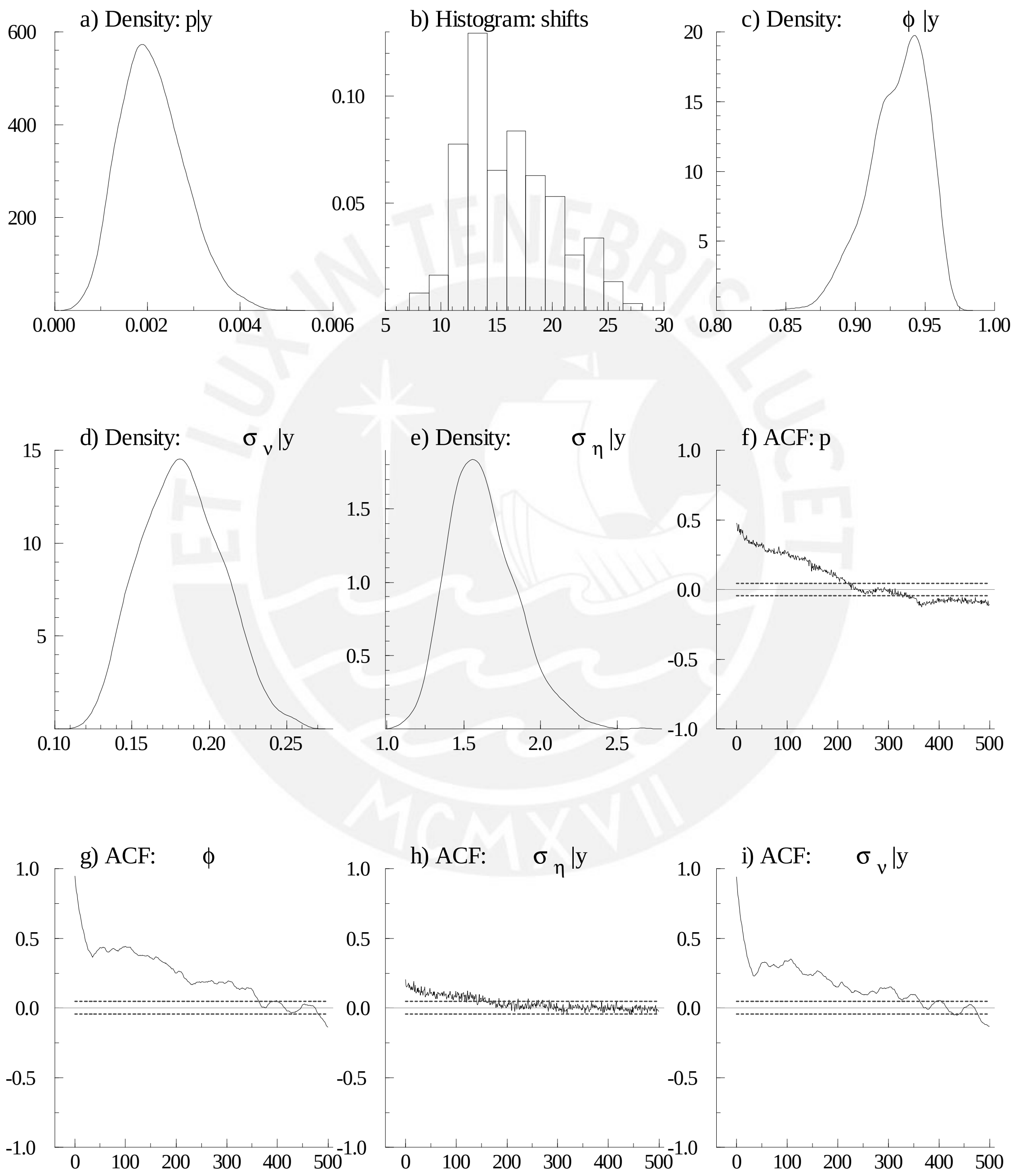

Figure 12. Results for Agriculture Index Volatility

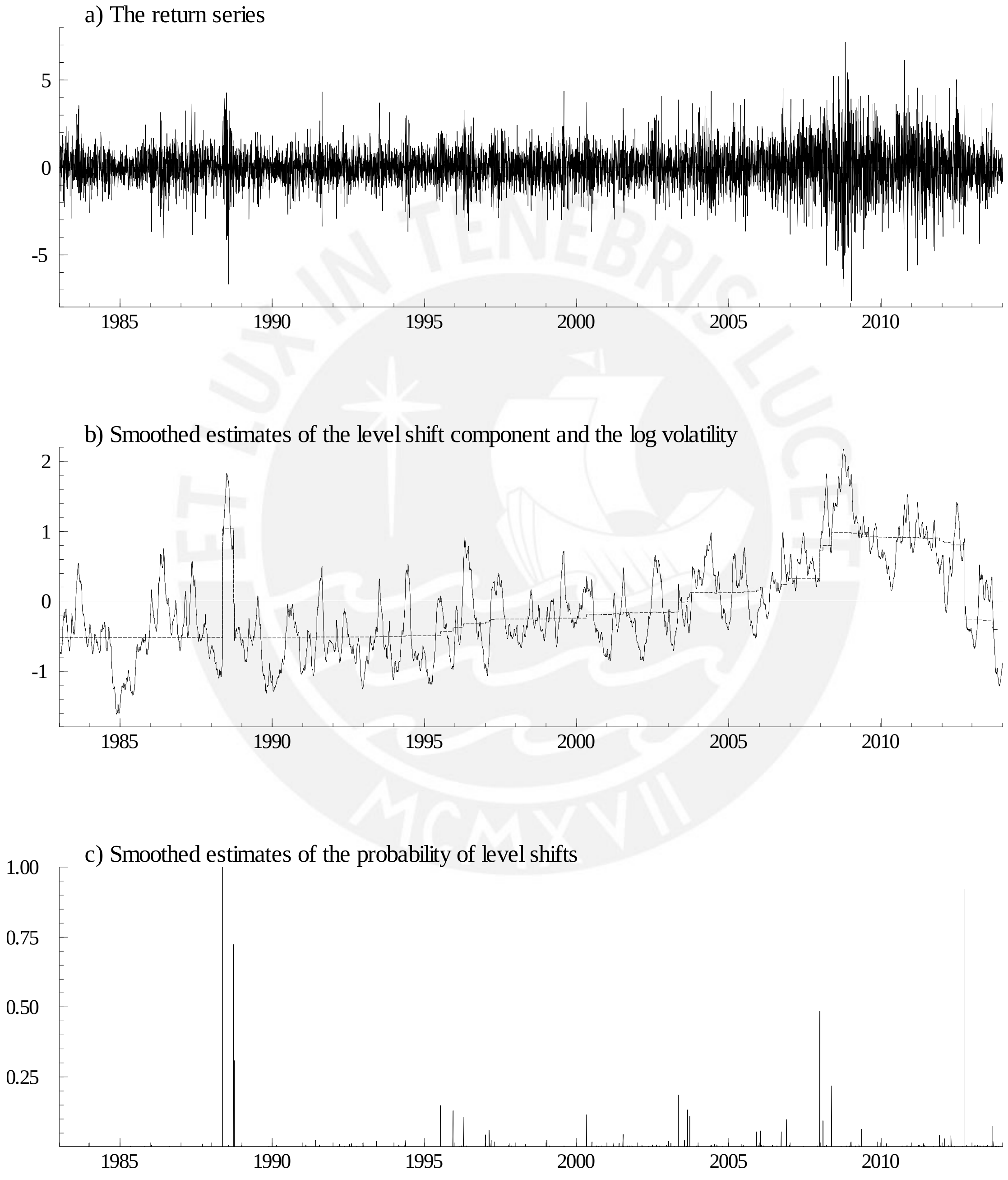

Figure 13. Posterior Estimates for Agriculture Index Volatility

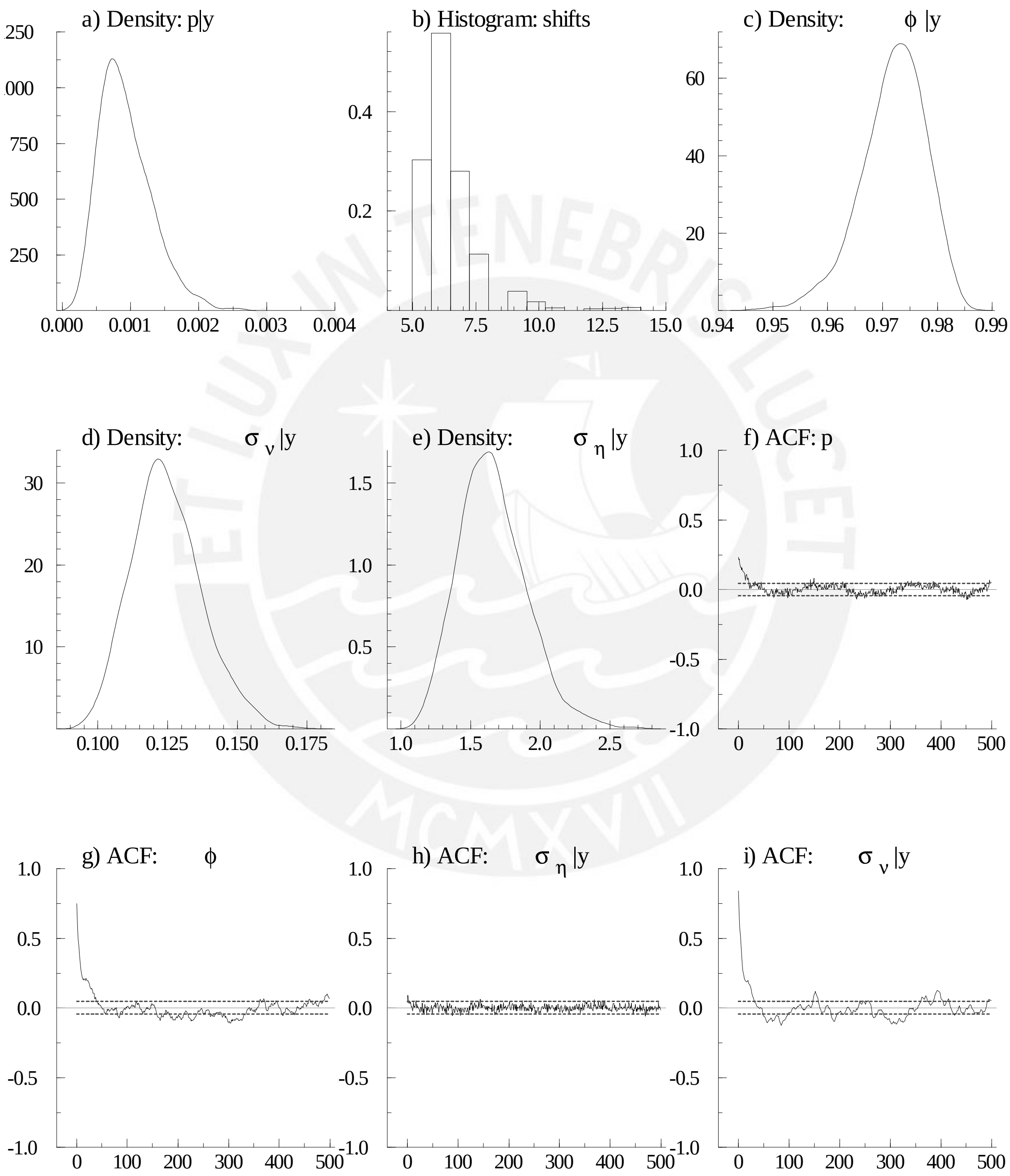

Figure 14. Results for Livestock Index Volatility

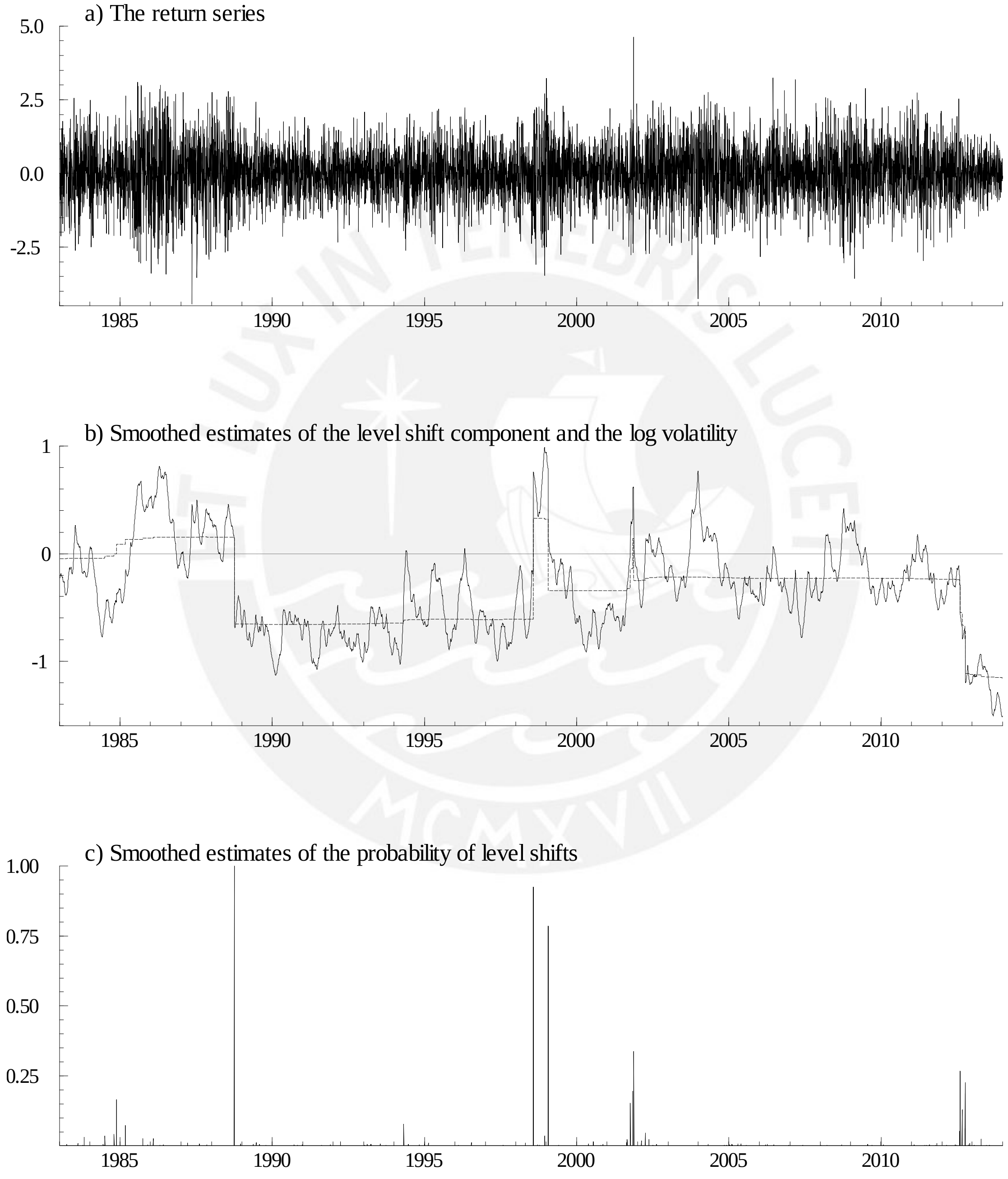

Figure 15. Posterior Estimates for Livestock Index Volatility

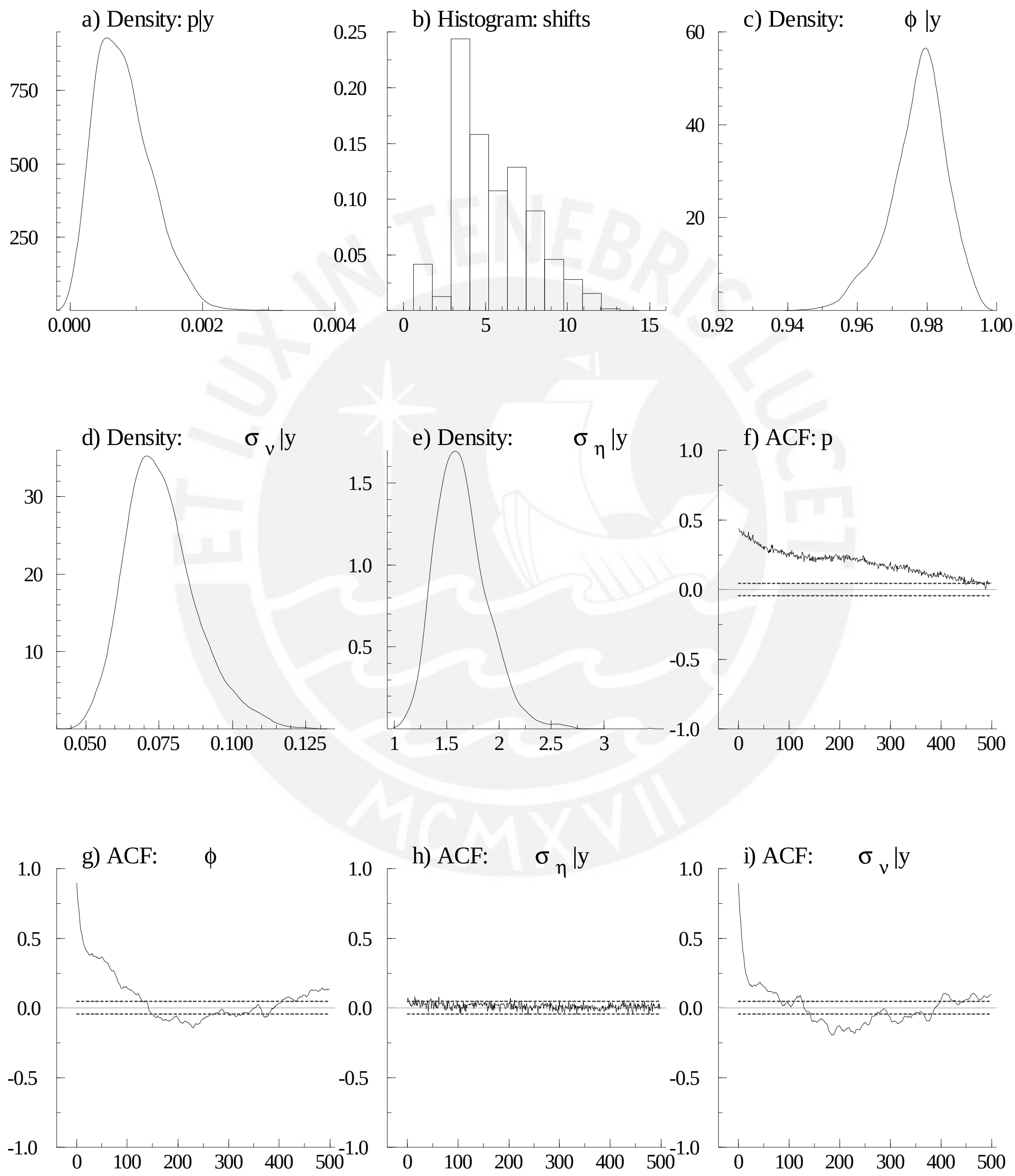

Figure 16. Autocorrelations of Log Squares

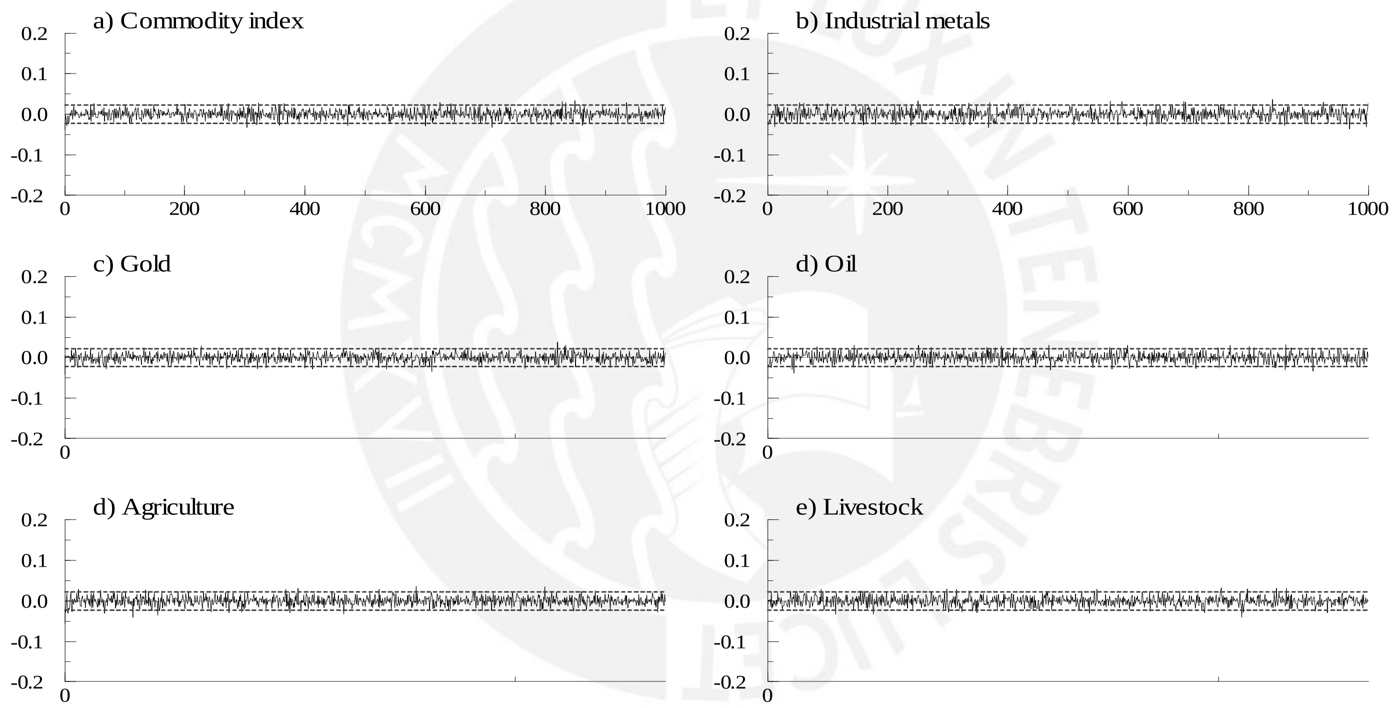

Figure 17. Normal QQ Plot