PONTIFICIA UNIVERSIDAD CATÓLICA DEL PERÚ

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CALCULATIONS OF HEAVY MAJORANA NEUTRINO DECAY WIDTHS, IN AN EXTENSION OF THE STANDARD MODEL USING TYPE I SEESAW AND DIMENSION FIVE EFFECTIVE OPERATORS

Tesis para obtener el título profesional de Licenciado en Física

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Resumen

En el modelo estándar de física de partículas, los neutrinos son descritos en términos de fermiones no masivos con quiralidad de mano izquierda. Sin embargo, la observación de las oscilaciones de neutrinos implica que los neutrinos tienen masa. Esto incentiva la búsqueda de extensiones del modelo estándar que expliquen la masa de los neutrinos. Uno de estos modelos es el *dimension-5 seesaw portal*, en el cual se utiliza una combinación del mecanismo *seesaw* tipo 1 y operadores efectivos de dimensión cinco para modificar el lagrangiano del modelo estándar. El objetivo de este trabajo es derivar fórmulas para calcular la anchura de decaimiento de neutrinos pesados, en un modelo *dimension-5 seesaw portal* que añade tres neutrinos masivos de Majorana con quiralidad de mano derecha, con masas en el rango GeV. Los resultados obtenidos fueron graficados para analizar la importancia de cada término y se encontró que la contribución proveniente de los términos de decamientos a tres cuerpos con operadores de dimensión cinco es pequeña. Esto es debido a que su inclusión reduce la distancia que podria viajar el neutrino antes de desintegrarse en un pequeño porcentaje $< 6\%$. Por lo que si bien son necesarios para un calculo completo, se pueden excluir estos términos si es que no se requiere alta precisión.

Abstract

In the Standard Model(SM), neutrinos are described in terms of massless left handed fermions, however the observation of neutrino flavor oscillations indicates non vanishing neutrino masses. This is an incentive for new physics beyond the standard model. One approach to explain neutrino masses is known as dimension-5 seesaw portal, in which the SM Lagrangian is modified using a combination of type I Seesaw and dimension five effective operators. The objective of this work is to derive formulae to compute the three-body decay of heavy neutrinos in a dimension-5 seesaw portal that adds three right handed Majorana neutrinos with masses in the GeV range to the SM. After obtaining the various formulae we plotted the results to analyze the relevance of each term, and found that the removal of the contribution of all terms coming from the three-body decays with dimension five operators only reduce the decay length by a few percent $< 6\%$ of the total decay length. Although the inclusion of these terms is necessary for an accurate result, it is possible to exclude them form calculations if precision is not of the utmost importance.

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Chapter 1

Introduction

The objective of this work is to derive formulae to compute the three-body decay of heavy neutrinos, in a model that adds three right handed Majorana neutrinos with masses in the GeV range, to the Standard Model(SM). This work is based in the model in [1], which uses a combination of type I Seesaw and dimension five effective operators. These formulae are intended to simplify calculations of decay widths on the model. Part of the results obtained in this work are portrayed in [1].

In the Standard Model, neutrinos are described in terms of massless left handed fermions, however the observation of neutrino flavor oscillations indicates non vanishing neutrino masses. This is an incentive for new physics beyond the standard model, one way of explaining the neutrino masses is the addition of right-handed Majorana neutrino singlets, which results in the Lagrangian of the type I Seesaw model [2, 3, 4, 5].

The downsides of this approach is that, for the seesaw mechanism to produce the mass of the light neutrinos either the mass of the heavy neutrinos would be too heavy to be probed, or in the case these masses lie at the electroweak scale range, it would require small neutrino Yukawa couplings, which are too small to produce these heavy neutrino in an experiment.

Another approach is effective field theory (EFT), in which assuming there are no new particles below the electroweak scale, the effects from new physics above that scale can be incorporated into a tower of higher dimensional operators. The Standard model effective field theory (SMEFT) contains all the SM fields to construct the effective lagrangians and respects the SM gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$. The framework to describe various new physics effects when adding right-handed neutrinos with masses below TeV is the standard model effective field theory extended with right-handed neutrinos(νSMEFT), with operators known up to dimension d=9.[6, 7, 8, 9, 10].

The addition of higher dimension effective interactions to the Type-I Seesaw can drastically change the production rate and decay widths of the heavy neutrinos. This can lead to a variety of signals that can be studied at different experiments. In this work the model being used is the type I Seesaw with dimension five effective operators.

A Majorana fermion is a particular solution for the Dirac equation, that fulfills the condition of being invariant under charge conjugation ($\Psi^c = \Psi$). The charge conjugation of fermions is defined as: $\Psi^c = C\overline{\Psi}^T$, and C, the charge conjugation matrix, is a unitary matrix that satisfies:

$$C(\gamma^\mu)^T C^{-1} = -\gamma^\mu$$
$$C(\gamma^5)^T C^{-1} = \gamma^5$$

By adding right handed Majorana neutrino singlets to the standard model, one allows a Yukawa term that couples the left handed neutrinos and the right handed neutrinos to the Higgs doublet to be added to the theory. There is also a Majorana mass term for the right handed neutrinos. This extension of the standard model is known as the type I Seesaw. The Lagrangian density for this model:

$$\mathcal{L}_{Seesaw} = \mathcal{L}_{SM} + i\overline{\nu}_{Rs}\partial\!\!\!/\nu_{Rs} - (\overline{L}_a(Y_\nu^\dagger)_{as}\widetilde{\phi}\nu_{Rs} + \frac{1}{2}\overline{\nu}_{Rs}(M_N)_{ss'}\nu^c_{Rs'} + h.c.)$$

$$a = e, \mu, \tau \quad s = s_1, s_2, s_3$$

where ν_{Rs} represents the right handed neutrino singlets, L_a represents the left handed lepton doublet of flavor a. Y_ν is a matrix of complex Yukawa couplings, $\widetilde{\phi}$ denotes the hypercharge conjugated Higgs doublet and M_N is a symmetric matrix of Majorana masses. Upon electroweak symmetry breaking, the Higgs field acquires a nonzero vacuum expectation value v, and the mass term for the neutrinos would be the following:

$$\mathcal{L}_\nu^{mass} = -\frac{1}{2}\Bigg\{ \left(\frac{v}{\sqrt{2}}Y_\nu^\dagger\right)_{as}(\overline{\nu}_{La}\nu_{Rs} + \overline{\nu}^c_{Rs}\nu^c_{La}) + \left(\frac{v}{\sqrt{2}}Y_\nu\right)_{sa}(\overline{\nu}_{Rs}\nu_{La} + \overline{\nu}^c_{La}\nu^c_{Rs}) + \frac{1}{2}\overline{\nu}_{Rs}(M_N)_{ss'}\nu^c_{Rs'} + \frac{1}{2}\nu^c_{Rs}(M_N^\dagger)_{ss'}\nu_{Rs'} \Bigg\}$$

Organizing the neutrino fields in a left handed multiplet $\Psi_L \equiv (\nu_L, \nu_R^c)^T$, and defining a block matrix $\mathcal{M}_\nu$, the mass term can be expressed as:

$$\mathcal{L}_\nu^{mass} = -\frac{1}{2}\overline{\Psi}^c_L\mathcal{M}_\nu\Psi_L - \frac{1}{2}\overline{\Psi}_L\mathcal{M}_\nu^*\Psi_L^c$$

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & M_N \end{pmatrix}$$

$$m_D = \frac{v}{\sqrt{2}}Y_\nu^T$$

One can diagonalize the $\mathcal{M}$ matrix using a mixing matrix $U_{\alpha i}$, such that its eigenvalues are real. This unitary transformation also relates the interaction eigenstates $\Psi_{L\alpha}(\alpha = e, \mu, \tau, s_1, s_2, s_3)$ to the mass eigenstates $n_i(i = 1, 2, ..., 6)$ with the relation: $\Psi_{L\alpha} = U_{\alpha i} n_i$. Then the diagonalized mass would be:

$$\mathcal{L}_{\nu}^{mass} = -\frac{1}{2}\overline{n}_L^c \mathcal{M}_{\nu}^{diag} n_L - \frac{1}{2}\overline{n}_L \mathcal{M}_{\nu}^{diag} n_L^c$$

which can then be used to define six Majorana neutrino fields :

$$n \equiv n_L + n_L^c$$
$$n = n^c$$

In this case, the SM neutrinos turn into Majorana fermions, and their masses are suppressed by the large mass of the right handed neutrinos $m_{light} \approx -m_D M_N^{-1} m_D^T$. The six neutrinos couple to the bosons W,Z through the U matrix :

$$\begin{aligned}
\mathcal{L}_W =& \frac{g}{\sqrt{2}} W_\mu^- \bar{l}_a \gamma^\mu U_{ai} P_L n_i + h.c. \\
\mathcal{L}_Z =& \frac{g}{4c_W} Z_\mu \overline{n}_i \gamma^\mu (C_{ij} P_L - C_{ij}^* P_R) n_j \\
\mathcal{L}_h =& -\frac{g}{4m_W} h \overline{n}_i \left[(C_{ij} m_{n_i} + C_{ij}^* m_{n_j}) P_L + (C_{ij} m_{n_j} + C_{ij}^* m_{n_i}) P_R \right] n_j \\
C_{ij} =& \sum_a U_{ai}^* U_{aj}
\end{aligned}$$

In addition to the Seesaw terms, one can allow dimension five operators involving both left and right handed neutrinos. The terms being added would be the following:

$$\mathcal{L}_5 = \frac{(\alpha_W^\dagger)_{ab}}{\Lambda}(\overline{L}_a\widetilde{\phi})(\phi^\dagger\widetilde{L}_b^c) + \frac{(\alpha_{N\phi})_{ss'}}{\Lambda}(\phi^\dagger\phi)\overline{\nu}_{Rs}\nu_{Rs'}^c + \frac{(\alpha_{NB})_{ss'}}{\Lambda}\overline{\nu}_{Rs}\sigma^{\mu\nu}\nu_{Rs'}^c B_{\mu\nu} + h.c.$$

The first term corresponds to the Weinberg operator [11], the second operator was introduced by Anisimov [12] and Graesser [13][14], while the third one, which is a dipole operator, was identified by Aparici, Kim, Santamaria and Wudka [7]. In the expression $\sigma^{\mu\nu} = \frac{\mathrm{i}}{2}\left(\gamma^\mu\gamma^\nu - \gamma^{nu}\gamma^{mu}\right)$,α_W and $\alpha_{N\phi}$ are symmetric matrices, and α_{NB} is antisymmetric. After symmetry breaking there would be new contributions to the neutrino mass term:

$$\mathcal{L}_5^{mass} = \frac{v^2(\alpha_W^\dagger)_{ab}}{2\Lambda}\overline{\nu}_{La}\nu_{Lb}^c + \frac{v^2(\alpha_W)_{ab}}{2\Lambda}\overline{\nu}_{La}^c\nu_{Lb} + \frac{v^2(\alpha_{N\phi})_{ss'}}{2\Lambda}\overline{\nu}_{Rs}\nu_{Rs'}^c + \frac{v^2(\alpha_{N\phi}^\dagger)_{ss'}}{2\Lambda}\overline{\nu}_{Rs}^c\nu_{Rs'}$$

and the block matrix $\mathcal{M}_\nu$, would now be as follows:

$$\begin{aligned} m_{LL} =& \frac{v^2\alpha_W}{\Lambda} \\ M_{RR} =& \frac{v^2\alpha_{N\phi}}{\Lambda} + M_N \\ \mathcal{M}_\nu =& \begin{pmatrix} m_{LL} & m_D \\ m_D^T & M_{RR} \end{pmatrix} \end{aligned}$$

In addition to the new mass term contributions, the dimension five operators add new couplings, that allows for interactions of the heavy neutrino with the Higgs, photon and Z boson. which are:

$$\mathcal{L}_{51} = \frac{v}{2\Lambda}h\overline{n}_i\left[(\alpha_W'^*)_{ij}P_R + (\alpha_W')_{ij}P_L\right]n_j + \frac{1}{2\Lambda}h^2\overline{n}_i\left[(\alpha_W'^*)_{ij}P_R + (\alpha_W')_{ij}P_L\right]n_j$$

$$\mathcal{L}_{52} = \frac{v}{2\Lambda} h\overline{n}_i \left[(\alpha'^*_{N\phi})_{ij} P_R + (\alpha'_{N\phi})_{ij} P_L\right] n_j + \frac{1}{2\Lambda} h^2 \overline{n}_i \left[(\alpha'^*_{N\phi})_{ij} P_R + (\alpha'_{N\phi})_{ij} P_L\right] n_j$$
$$\mathcal{L}_{53} = \frac{1}{\Lambda} \overline{n}_i \sigma^{\mu\nu} \left[(\alpha'_{NB})_{ij} P_L - (\alpha'^*_{NB})_{ij} P_R\right] n_j (c_W F^A_{\mu\nu} - s_W F^Z_{\mu\nu})$$

$$(\alpha'_W)_{ij} = U_{ai} (\alpha_W)_{ab} U_{bj}$$
$$(\alpha'_{N\phi})_{ij} = U_{si} (\alpha_{N\phi})_{ss'} U_{s'j}$$
$$(\alpha'_{NB})_{ij} = U_{si} (\alpha_{NB})_{ss'} U_{s'j}$$

In the standard Seesaw, the total decay of a heavy neutrino with mass in the GeV range can be calculated through three-body decays[15, 16, 17, 18]. The list of decay widths is as follows:

$$\Gamma(N_j \to \nu_\alpha l^-_\beta l^+_\beta) = \frac{G_F^2 M_j^5}{96\pi^3} |U_{\alpha j}|^2 \Big[(g^l_L g^l_R + \delta^\beta_\alpha g^l_R) I_2(x_\nu, x_\beta, x_\beta)$$
$$+ \big((g^l_L)^2 + (g^l_R)^2 + \delta^\beta_\alpha (1 + 2g^l_L)\big) I_1(x_\nu, x_\beta, x_\beta)\Big]$$
$$\Gamma(N_j \to l^-_\alpha l^+_\beta \nu_\beta) = \frac{G_F^2 M_j^5}{192\pi^3} |U_{aj}|^2 I_1(x_b, x_\nu, x_a)$$
$$\sum_\beta \Gamma(N_j \to \nu_\alpha \nu_\beta \overline{\nu}_\beta) = \frac{G_F^2 M_j^5}{96\pi^3} |U_{\alpha j}|^2$$
$$\Gamma(N_j \to \nu_\alpha q\overline{q}) = \frac{G_F^2 M_j^5}{32\pi^3} |U_{\alpha j}|^2 (g^q_L g^q_R I_2(x_\nu, x_q, x_q) + ((g^q_L)^2 + (g^q_R)^2) I_1(x_\nu, x_q, x_q))$$
$$\Gamma(N_j \to l^-_\alpha u\overline{d}) = \frac{G_F^2 M_j^5}{64\pi^3} |V_{ud}|^2 |U_{\alpha j}|^2 I_1(x_l, x_u, x_d)$$

Here $x_i = m_i/M_j$ are scaled masses, $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2b$ is the Källén function. The SM neutral coupling of leptons and quarks are:

$$g_L^l = -1/2 + \mathrm{Sin}^2\theta_W \qquad g_L^u = 1/2 - (2/3)\mathrm{Sin}^2\theta_W \qquad g_L^d = -1/2 + (1/3)\mathrm{Sin}^2\theta_W$$

$$g_R^l = \mathrm{Sin}^2\theta_W \qquad g_R^u = -(2/3)\mathrm{Sin}^2\theta_W \qquad g_R^d = (1/3)\mathrm{Sin}^2\theta_W$$

The kinematical functions:

$$\begin{aligned}
I_1(x,y,z) =& 12\int_{(x+y)^2}^{(1-z)^2} \frac{\mathrm{d}s}{s}(s-x^2-y^2)(1+z^2-s)\lambda^{1/2}(s,x^2,y^2)\lambda^{1/2}(1,s,z^2) \\
I_2(x,y,z) =& 24yz\int_{(y+z)^2}^{(1-x)^2} \frac{\mathrm{d}s}{s}(1+x^2-s)\lambda^{1/2}(s,y^2,z^2)\lambda^{1/2}(1,s,x^2)
\end{aligned}$$

and the total decay of the heavy neutrino N_j is:

$$\begin{aligned}
\Gamma_{N_j} = \sum_{\alpha,\beta,q,u,d} \big[& 2\Gamma(N_j \to l_\alpha^- u\bar{d}) + 2\Gamma(N_j \to l_\alpha^- l_\beta^+ \nu_\beta) \\
& + \Gamma(N_j \to \nu_\alpha q\bar{q}) + \Gamma(N_j \to \nu_\alpha l_\beta^- l_\beta^+) + \Gamma(N_j \to \nu_\alpha \nu_\beta \overline{\nu}_\beta)\big]
\end{aligned}$$

Given that we are working in a mass range for the heavy neutrino below the Higgs mass, the Anisimov-Graesser operator plays no role in the determination of the lifetime. When including the dipole operator a new decay channel into on shell $\nu\gamma$, and the three-body decays are modified. For the new three-body partial decay widths, the relevant couplings of the neutrinos are:

$$\begin{aligned}
\mathcal{L}_W =& \frac{g}{\sqrt{2}} W_\mu^- \bar{l}_a \gamma^\mu U_{ai} P_L n_i + h.c. \\
\mathcal{L}_Z =& \frac{g}{4c_W} Z_\mu \overline{n}_i \gamma^\mu (C_{ij}P_L - C_{ij}^* P_R) n_j \\
& - \frac{s_W}{\Lambda}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)\overline{n}_i \sigma^{\mu\nu}\left[(\alpha'_{NB})_{ij}P_L - (\alpha'^*_{NB})_{ij}P_R\right] n_j
\end{aligned}$$

$$\mathcal{L}_\gamma = \frac{c_W}{\Lambda}(\partial_\mu A_\nu - \partial_\nu A_\mu)\overline{n}_i \sigma^{\mu\nu}\left[(\alpha'_{NB})_{ij} P_L - (\alpha'^{*}_{NB})_{ij} P_R\right] n_j$$

Chapter 2

Formulas for three-body decays

In this Chapter, the decay widths for various possible amplitudes are derived. The new expressions will be functions only of the masses of the particles involved in the process, and thus can be easily calculated. In general the decay width of a particle 1 decaying into three particles $2, 3, 4$ is given by:

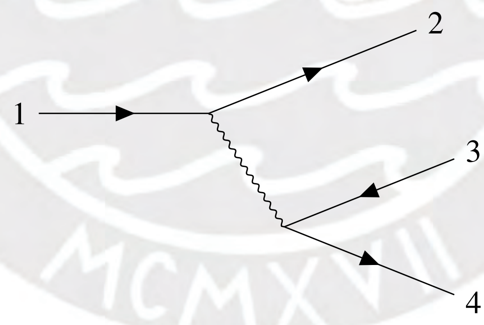


Figure 1: Feynman diagram for a three-body decay.

$$\Gamma = \frac{S_f}{(2\pi)^5} \int \frac{\mathrm{d}^3\vec{p}_2}{2E_1 2E_2} \int \frac{\mathrm{d}^3\vec{p}_3}{2E_3} \int \frac{\mathrm{d}^3\vec{p}_4}{2E_4} \delta^4(p_1 - p_2 - p_3 - p_4) \left\langle |\mathcal{M}|^2 \right\rangle \tag{2.1}$$

In this expression S_f is a statistical factor depending on the number of identical particles in the final state. E_i is the energy, p_i the four-momentum, and $\vec{p}_i$ the momentum of the ith particle. $\mathcal{M}$ is the amplitude of the process, and $\langle|\mathcal{M}|^2\rangle$ is the average over all initial spins configurations and the sum over all final spins configurations of the amplitude squared. In order to perform the required integral, the explicit expression of the amplitude is required. In this case the relevant amplitudes are the following:

$$
\begin{aligned}
\langle|\mathcal{M}_1|^2\rangle &\simeq \frac{(p_1\cdot p_2)^n}{q^m}(p_3\cdot p_4), & \langle|\mathcal{M}_2|^2\rangle &\simeq \frac{(p_1\cdot p_2)^n}{q^m}(p_1\cdot p_3)(p_1\cdot p_4),\\
\langle|\mathcal{M}_3|^2\rangle &\simeq \frac{(p_1\cdot p_2)^n}{q^m}(p_2\cdot p_3)(p_2\cdot p_4), & \langle|\mathcal{M}_4|^2\rangle &\simeq \frac{(p_1\cdot p_2)^n}{q^m}(p_1\cdot p_3)(p_2\cdot p_4),\\
\langle|\mathcal{M}_5|^2\rangle &\simeq \frac{(p_1\cdot p_2)^n}{q^m}
\end{aligned}
$$

where m is a natural number, n is an even natural number, and $q^\mu = (p_1 - p_2)^\mu = (p_3 + p_4)^\mu$ is the momentum of the propagator.

To calculate the decay width, we follow the technique presented in the Greiner[19]. We first evaluate a Lorentz covariant subsection of the expression that contains the integral over two momenta and the delta function. For the amplitudes listed, these subsections are $I_{\alpha\beta}$ for $\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3, \mathcal{M}_4$ and $I'_{\alpha\beta}$ for $\mathcal{M}_5$, with:

$$
\begin{aligned}
I_{\alpha\beta} &= \int (p_4)_\alpha (p_3)_\beta \delta^4(p_2 + p_3 + p_4 - p_1)\frac{\mathrm{d}^3\vec{p}_4}{2E_4}\frac{\mathrm{d}^3\vec{p}_3}{2E_3}\\
I'_{\alpha\beta} &= \int \eta_{\alpha\beta}\delta^4(p_2 + p_3 + p_4 - p_1)\frac{\mathrm{d}^3\vec{p}_4}{2E_4}\frac{\mathrm{d}^3\vec{p}_3}{2E_3}
\end{aligned}
$$

First, for the value of $I_{\alpha\beta}$, as the momenta $\vec{p}_3, \vec{p}_4$ are being integrated, only the second rank tensors $\eta_{\alpha\beta}$ and $q_\alpha q_\beta$ can occur in the result. So $I_{\alpha\beta}$ can be expressed in the form:

$$I_{\alpha\beta} = Aq^2\eta_{\alpha\beta} + Bq_\alpha q_\beta \tag{2.2}$$

From this expression, we can construct the following system of equations:

$$I_{\alpha\beta}\eta^{\alpha\beta} = (4A + B)q^2 \tag{2.3}$$

$$I_{\alpha\beta}q^\alpha q^\beta = (A + B)q^4 \tag{2.4}$$

Our objective is to find A and B. Replacing $I_{\alpha\beta}$ in Eq.(2.3):

$$(4A + B)q^2 = \int (p_4 \cdot p_3)\delta^4(p_2 + p_3 + p_4 - p_1)\frac{\mathrm{d}^3\vec{p}_4}{2E_4}\frac{\mathrm{d}^3\vec{p}_3}{2E_3}$$

We perform the calculation in a frame of reference in which q^μ consists only of a time-like component $\tilde{q}^\mu = \tilde{q}^0 = (\sqrt{q^2}, 0)$:

$$\int (p_4 \cdot p_3)\delta(E_3 + E_4 - \tilde{q}^0)\delta^3(\vec{p}_3 + \vec{p}_4)\frac{\mathrm{d}^3\vec{p}_4}{2E_4}\frac{\mathrm{d}^3\vec{p}_3}{2E_3}$$

Integrating over $d^3\vec{p}_4$, the delta function gives: $\vec{p}_3 = -\vec{p}_4 = \vec{k}$.

$$\int \left(E_3E_4 + \left\|\vec{k}\right\|^2\right)\delta(E_3 + E_4 - \tilde{q}^0)\frac{\mathrm{d}^3\vec{k}}{4E_4E_3}$$

Transforming $d^3\vec{k}$ to spherical coordinates, integrating over the solid angle and making a change of variable to $x = E_3 + E_4$, we find an expression in terms of x, m_3, m_4:

$$\int \frac{\pi}{4x^2}\left(x^2 - m_3^2 - m_4^2\right)\lambda^{1/2}(x^2, m_3^2, m_4^2)\delta(\tilde{q}^0 - x)\mathrm{d}x$$

After integrating over x, the delta function gives $x = \tilde{q}^0$. Then using the fact that $(\tilde{q}^0)^2 = q^2$, the expression is Lorentz invariant:

$$(4A + B)q^2 = \frac{\pi}{4q^2}(q^2 - m_3^2 - m_4^2)\lambda^{1/2}(q^2, m_3^2, m_4^2)\Theta(q^2 - (m_3 + m_4)^2)$$

where $\Theta(x)$ is the Heaviside step function:

$$\Theta(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

which ensures the condition from the delta function, as this result is only valid for $\tilde{q}^0 \geq E_3 + E_4 \geq m_3 + m_4$. With a similar calculation, the value for Eq.(2.4) is:

$$(A + B)q^4 = \frac{\pi}{8q^2}\left(q^4 - (m_3^2 - m_4^2)^2\right)\lambda^{1/2}(q^2, m_3^2, m_4^2)\Theta(q^2 - (m_3 + m_4)^2)$$

With these equations we find the value of A and B, then replace them in Eq.(2.2) which yields an equivalent expression for $I_{\alpha\beta}$:

$$\begin{aligned} I_{\alpha\beta} =& \frac{\pi}{24q^2}\lambda^{1/2}(q^2,m_3^2,m_4^2)\Theta\left(q^2-(m_3+m_4)^2\right) \\ & \Big[\frac{\eta_{\alpha\beta}}{q^2}\lambda(q^2,m_3^2,m_4^2)+2q_\alpha q_\beta\left(1+\frac{m_3^2+m_4^2}{q^2}-2\frac{(m_3^2-m_4^2)^2}{q^4}\right)\Big] \end{aligned} \tag{2.5}$$

Likewise, we find for $I'_{\alpha\beta}$:

$$I'_{\alpha\beta} = \frac{\pi\lambda^{1/2}(q^2,m_3^2,m_4^2)}{2q^2}\eta_{\alpha\beta}\Theta(q^2-(m_3+m_4)^2) \tag{2.6}$$

We now derive the expressions for the decay widths for each possible amplitude, which requires performing the integral over the momentum p_2, for every case. In the case of an amplitude:

$$\left\langle|\mathcal{M}_1|^2\right\rangle \simeq \frac{(p_1\cdot p_2)^n}{(p_1-p_2)^m}(p_3\cdot p_4) = \frac{(p_1\cdot p_2)^n}{q^m}\eta^{\alpha\beta}(p_3)_\alpha(p_4)_\beta$$

We see we only need $I_{\alpha\beta}$ such that the width will be then proportional to:

$$\begin{aligned} \frac{S_f}{(2\pi)^5}\int\frac{d^3\vec{p}_2}{2E_1 2E_2}\frac{(p_1\cdot p_2)^n}{q^m}\frac{\pi}{24q^2}\lambda^{1/2}(q^2,m_3^2,m_4^2)\Bigg[\frac{4}{q^2}\lambda(q^2,m_3^2,m_4^2) \\ +2q^2\left(1+\frac{m_3^2+m_4^2}{q^2}-2\frac{(m_3^2-m_4^2)^2}{q^4}\right)\Bigg]\Theta\left(q^2-(m_3+m_4)^2\right) \end{aligned}$$

where part of the integral has been replaced with the expression in Eq. (2.5), and $q^\mu = (p_1 - p_2)^\mu$.

For the rest of the calculation, it is convenient to work in the reference frame in which the decaying particle p_1 is at rest. We can also simplify the terms within square brackets above, leading to:

$$\frac{S_f}{(2\pi)^5}\int \frac{(m_1 E_2)^n \pi}{4q^{2+m}} \lambda^{1/2}(q^2, m_3^2, m_4^2)(q^2 - m_3^2 - m_4^2)\Theta\left(q^2 - (m_3+m_4)^2\right)\frac{\mathrm{d}^3\vec{p}_2}{2m_1 2E_2}$$

Expressing the $\mathrm{d}^3\vec{p}_2$ in spherical coordinates, integrating over the solid angle, then making a variable change to $s = q^2/m_1^2$, and scaling the masses respect to the mass of the decaying particle $x_a = m_a/m_1$, we have:

$$\frac{S_f}{\pi^3 2^{n+9}}\int_{(x_3+x_4)^2}^{(1-x_2)^2} (m_1)^{2n-m+3}(x_1^2 + x_2^2 - s)^n \lambda^{1/2}(x_1^2, x_2^2, s)\lambda^{1/2}(s, x_3^2, x_4^2)(s - x_3^2 - x_4^2)\frac{\mathrm{d}s}{s^{1+m/2}}$$

For the bounds of the integral, the lower bound comes from the step function in Eq.(2.5) and the upper bound comes from the restriction $E_2 \geq m_2$. Finally the width for this decay can be expressed as a function of the masses of the particles:

$$\begin{aligned}\Gamma &\simeq \frac{S_f}{(2\pi)^5}\int \frac{(p_1\cdot p_2)^n}{(p_1 - p_2)^m}(p_3\cdot p_4)(p_1 - p_2 - p_3 - p_4)\frac{\mathrm{d}^3\vec{p}_2}{2E_1 2E_2}\frac{\mathrm{d}^3\vec{p}_3}{2E_3}\frac{\mathrm{d}^3\vec{p}_4}{2E_4} \\ &\simeq \frac{S_f\, m_1^{2n-m+3}}{12\pi^3 2^{n+9}}\,\Gamma_1^{nm}\big(x_2, x_3, x_4\big)\end{aligned}$$

where $\Gamma_1^{nm}(a, b, c)$ is defined:

$$\Gamma_1^{nm}(a,b,c) \coloneqq 12 \int_{(b+c)^2}^{(1-a)^2} (1+a^2-s)^n (s-b^2-c^2) \lambda^{1/2}(1,a^2,s)\lambda^{1/2}(s,b^2,c^2) \frac{\mathrm{d}s}{s^{1+m/2}} \quad (2.7)$$

The other expressions are calculated using the same process. For a decay with an amplitude:

$$\left\langle |\mathcal{M}_2|^2 \right\rangle \simeq \frac{(p_1 \cdot p_2)^n}{(p_1 - p_2)^m}(p_1 \cdot p_3)(p_1 \cdot p_4) = \frac{(p_1 \cdot p_2)^n}{q^m}(p_1)^\alpha (p_1)^\beta (p_3)_\alpha (p_4)_\beta$$

we find we again need $I_{\alpha\beta}$, the width for its decay will be proportional to:

$$\begin{aligned} \Gamma &\simeq \frac{S_f}{(2\pi)^5} \int \frac{(p_1 \cdot p_2)^n}{q^m} (p_1)^\alpha (p_1)^\beta I_{\alpha\beta} \frac{\mathrm{d}^3 \vec{p}_2}{2E_1 2E_2} \\ &\simeq \frac{S_f\, m_1^{5+2n-m}}{12\pi^3 2^{10+n}} \, \Gamma_2^{nm}\big(x_2, x_3, x_4\big) \end{aligned}$$

The expression $\Gamma_2^{nm}(a,b,c)$ is defined as:

$$\begin{aligned} \Gamma_2^{nm}(a,b,c) \coloneqq & 4 \int_{(b+c)^2}^{(1-a)^2} (1+a^2-s)^n \lambda^{1/2}(1,a^2,s) \lambda^{1/2}(s,b^2,c^2) \\ & \left[\lambda(s,b^2,c^2) + \left(1-a^2+s\right)^2 \left(\frac{s+b^2+c^2}{2} - \frac{(b^2-c^2)^2}{s} \right) \right] \frac{\mathrm{d}s}{s^{2+m/2}} \end{aligned} \quad (2.8)$$

For amplitude terms of the next form:

$$\langle|\mathcal{M}_3|^2\rangle \simeq \frac{(p_1\cdot p_2)^n}{(p_1-p_2)^m}(p_2\cdot p_3)(p_2\cdot p_4) = \frac{(p_1\cdot p_2)^n}{q^m}(p_2)^\alpha(p_2)^\beta(p_3)_\alpha(p_4)_\beta$$

we use $I_{\alpha\beta}$ so the width for its decay will be given by:

$$\begin{aligned}\Gamma &\simeq \frac{S_f}{(2\pi)^5}\int \frac{(p_1\cdot p_2)^n}{q^m}(p_2)^\alpha(p_2)^\beta I_{\alpha\beta}\frac{\mathrm{d}^3\vec{p}_2}{2E_1 2E_2}\\ &\simeq \frac{S_f\, m_1^{5+2n-m}}{12\pi^3 2^{10+n}}\,\Gamma_3^{nm}\big(x_2,x_3,x_4\big)\end{aligned}$$

The expression $\Gamma_3^{nm}(a,b,c)$ is defined as:

$$\begin{aligned}\Gamma_3^{nm}(a,b,c) \coloneqq& 4\int_{(b+c)^2}^{(1-a)^2}(1+a^2-s)^n\lambda^{1/2}(1,a^2,s)\lambda^{1/2}(s,b^2,c^2)\\ &\left[\frac{a^2}{s}\lambda(s,b^2,c^2)+\frac{(1-a^2-s)^2}{2}\left(1+\frac{b^2+c^2}{s}-2\frac{(b^2-c^2)^2}{s^2}\right)\right]\frac{\mathrm{d}s}{s^{1+m/2}}\end{aligned} \tag{2.9}$$

For an amplitude of the form:

$$\langle|\mathcal{M}_4|^2\rangle \simeq \frac{(p_1\cdot p_2)^n}{(p_1-p_2)^m}(p_1\cdot p_3)(p_2\cdot p_4) = \frac{(p_1\cdot p_2)^n}{q^m}(p_1)^\alpha(p_2)^\beta(p_3)_\alpha(p_4)_\beta$$

we use $I_{\alpha\beta}$ so the width for its decay will be given by:

$$\begin{aligned}\Gamma &\simeq \frac{S_f}{(2\pi)^5}\int \frac{(p_1\cdot p_2)^n}{q^m}(p_1)^\alpha(p_2)^\beta I_{\alpha\beta}\frac{\mathrm{d}^3\vec{p}_2}{2E_1 2E_2}\\ &\simeq \frac{S_f\, m_1^{5+2n-m}}{12\pi^3 2^{11+n}}\,\Gamma_4^{nm}\big(x_2,x_3,x_4\big)\end{aligned}$$

The expression $\Gamma_4^{nm}(a,b,c)$ is defined as:

$$\Gamma_4^{nm}(a,b,c) := 4\int_{(b+c)^2}^{(1-a)^2} (1+a^2-s)^{n+1}\lambda^{1/2}(1,a^2,s)\lambda^{1/2}(s,b^2,c^2) \left[\lambda(s,b^2,c^2) + \frac{((1-a^2)^2-s^2)}{(1+a^2-s)}\left(s+b^2+c^2-2\frac{(b^2-c^2)^2}{s}\right)\right]\frac{\mathrm{d}s}{s^{2+m/2}} \tag{2.10}$$

For an amplitude of the form:

$$\left\langle|\mathcal{M}_5|^2\right\rangle \simeq \frac{(p_1\cdot p_2)^n}{(p_1-p_2)^m} = \frac{(p_1\cdot p_2)^n}{4q^m}\eta^{\alpha\beta}\eta_{\alpha\beta}$$

we finally require $I'_{\alpha\beta}$,the width for its decay will be given by:

$$\begin{aligned}\Gamma &\simeq \frac{S_f}{(2\pi)^5}\int \frac{(p_1\cdot p_2)^n}{4q^m}\eta^{\alpha\beta}I'_{\alpha\beta}\frac{\mathrm{d}^3\vec{p}_2}{2E_1 2E_2} \\ &\simeq \frac{S_f\, m_1^{2n+3-m}}{24\pi^3 2^{n+8}}\,\Gamma_5^{nm}\big(x_2,x_3,x_4\big)\end{aligned}$$

The expression $\Gamma_5^{nm}(a,b,c)$ is defined as:

$$\Gamma_5^{nm}(a,b,c) := 24bc\int_{(b+c)^2}^{(1-a)^2}(1+a^2-s)^n\lambda^{1/2}(1,a^2,s)\lambda^{1/2}(s,b^2,c^2)\frac{\mathrm{d}s}{s^{1+m/2}} \tag{2.11}$$

To show the behavior of the $\Gamma_{1-5}^{nm}(a,b,c)$ functions, we plot the results for different choices of parameters in Figures 2 and 3. Many of the heavy neutrino decay channels produce one light neutrino and a pair of fermions with equal mass. For this reason the arguments of Γ_{1-5}^{nm} in the plots are taken as $(0,x,x)$. The parameters n and m range through their three lowest values.

After comparing the panels we can conclude that an increase in m, drastically increases the value of the function $\Gamma_{1-5}^{nm}(a,b,c)$, while for bigger n the result decreases slightly. The result of the functions grow as the value of x is reduced, with the exception of Γ_5^{nm} for $m = 0, 2$. In this case the functions present a maximum value for $x \in [0.1, 0.5]$.

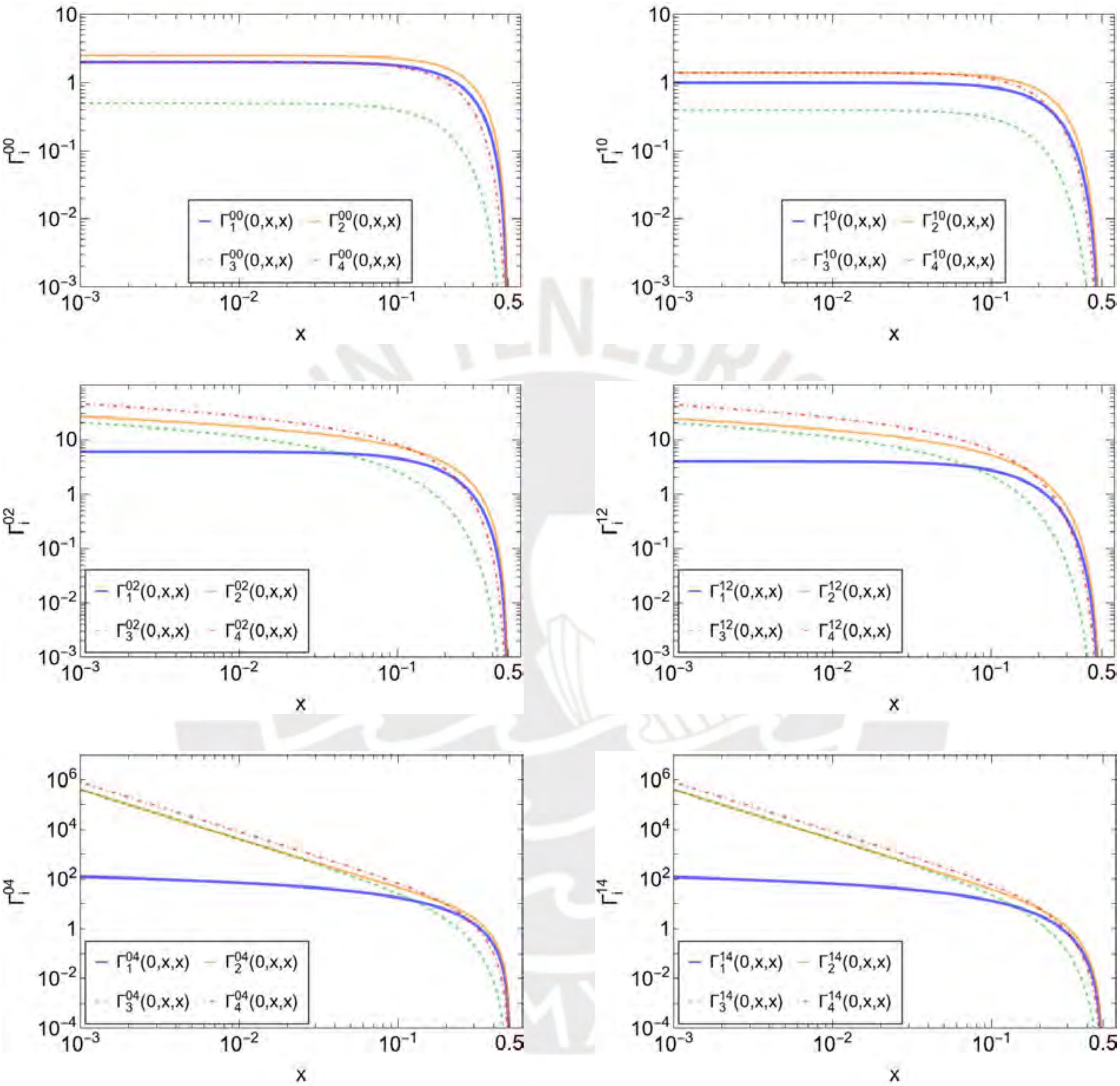


Figure 2: Behavior of the $\Gamma_{1-4}^{nm}(0, x, x)$ functions. For the left panels $n = 0$, and on the right $n = 1$. From top to bottom m varies from 0 to 4. Each panel shows the result for Γ_1 to Γ_4 for the same choice of nm

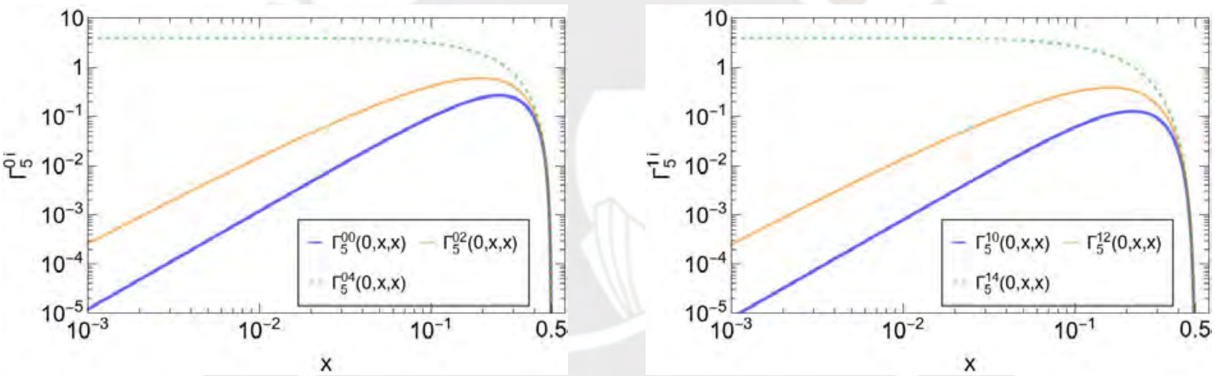


Figure 3: Behavior of the $\Gamma_5^{nm}(0, x, x)$ functions. For the left panels $n = 0$, and on the right $n = 1$. Each panel shows the results for $m = 0, 2, 4$

Chapter 3

Decay widths

In this Chapter, we detail the three-body partial decay widths for the heavy neutrino. The channels are separated in three categories based on the contents of the final state. The first category includes all the partial widths that have quarks as a product, this will be the channel $\Gamma(N_j \to Xqq)$. If the decay produces two charged leptons, it is accounted for in the second category, the channel $\Gamma(N_j \to \nu ll)$. The last category consists of the decay to only light neutrinos, the channel $\Gamma(N_j \to \nu\nu\nu)$.

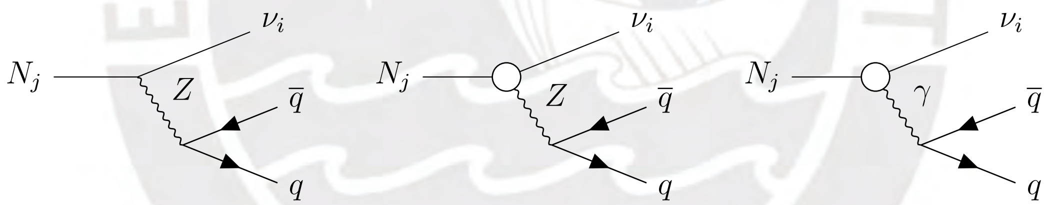
For the calculation of the partial widths, we use a four-fermion interaction with the gauge bosons integrated out. For energies below the mass of the propagator $q^2 \gg M^2$, the virtual particle is "pointlike", so the four fermions are directly interacting with each other, and the propagator is approximated to $\mathrm{i}\eta_{\mu\nu}/M^2$. The calculations of the amplitudes was performed by the other authors of [1], and were checked with the assistance of $\mathbf{FeynCalc\,9.3.1}$ [20, 21, 22]

3.1 Decay to Quarks

The decays of the heavy neutrino that have quarks in the final state are of two types: $N_j \to l\overline{q}q$, and $N_j \to \nu_i\overline{q}q$. The first case is mediated by a virtual W boson, and the addition of the effective operators does not modify it. So the focus will be on the second type. For the decay $N_j \to l\overline{q}q$, this only has one possible diagram which is present in the standard Seesaw. In the expression, V_{ud} is the appropriate element of the CKM matrix, its decay width:

$$\Gamma(N_j \to l_a q_u \overline{q}_d) = \frac{G_f^2 m_N^5}{64\pi^3} |V_{ud}|^2 |U_{aj}|^2 \; \Gamma_5^{10}(x_{l_a}, x_{q_u}, x_{q_d}) \tag{3.1}$$

For the case of the decay channel: $N_j \to \nu_i \overline{q} q$ in the standard Seesaw model there is only one diagram at tree level. This is mediated by a virtual Z boson and is labeled as $\mathcal{M}_{Z_{SM}}$. When including the effective operators two extra diagrams are possible. First: $\mathcal{M}_{Z_{eff}}$ is also mediated by a virtual Z boson, via the new interaction. The effective coupling also adds a contribution with virtual photons which is labeled $\mathcal{M}_{\gamma_{eff}}$. The diagrams with their respective amplitudes are the following:



$$\mathrm{i}\mathcal{M}_{Z_{SM}} = -\left(\frac{g}{2C_W}\right)^2 \left(\mathrm{i}\frac{2\eta_{\mu\rho}}{M_Z^2}\right) [\overline{u}_q \gamma^\rho (g_L^q P_L + g_R^q P_R) v_{\overline{q}}] \, [\overline{u}_{\nu_i} \gamma^\mu (C_{ij} P_L - C_{ij}^* P_R) u_{N_j}]$$

$$\mathrm{i}\mathcal{M}_{Z_{eff}} = \left[\frac{4S_W g \eta_{\mu\rho}}{C_W M_Z^2}\right] q_\nu \, [\overline{u}_q \gamma^\rho (g_L^q P_L + g_R^q P_R) v_{\overline{q}}] \left[\overline{u}_{\nu_i} \sigma^{\mu\nu} \left(\frac{(\alpha'_{NB})_{ij}}{\Lambda} P_L - \frac{(\alpha'_{NB})_{ij}^*}{\Lambda} P_R\right) u_{N_j}\right]$$

$$\mathrm{i}\mathcal{M}_{\gamma_{eff}} = \left[\frac{4C_W \mathrm{e} Q_q \eta_{\mu\rho}}{q^2}\right] q_\nu \, [\overline{u}_q \gamma^\rho v_{\overline{q}}] \left[\overline{u}_{\nu_i} \sigma^{\mu\nu} \left(\frac{(\alpha'_{NB})_{ij}}{\Lambda} P_L - \frac{(\alpha'_{NB})_{ij}^*}{\Lambda} P_R\right) u_{N_j}\right]$$

Figure 4: Tree level Feynman diagrams for heavy neutrino decay into final states containing quarks, with their respective amplitudes. From left to right $\mathcal{M}_{Z_{SM}}$, $\mathcal{M}_{Z_{eff}}$ and $\mathcal{M}_{\gamma_{eff}}$

For the amplitudes in Figure 4 : $S_W = \mathrm{Sin}\theta_W$, $C_W = \mathrm{Cos}\theta_W$, Q_q refers to the electric charge of the quark and q in the expressions is the momentum of the propagator. The total amplitude is:

$$\left\langle|\mathcal{M}|^2\right\rangle = \left\langle|\mathcal{M}_{Z_{SM}}|^2\right\rangle + \left\langle\left|\mathcal{M}_{Z_{eff}}\right|^2\right\rangle + \left\langle\left|\mathcal{M}_{\gamma_{eff}}\right|^2\right\rangle + \left\langle\mathcal{M}_{Z_{SM}}\mathcal{M}^*_{Z_{eff}} + \mathcal{M}^*_{Z_{SM}}\mathcal{M}_{Z_{eff}}\right\rangle$$
$$+ \left\langle\mathcal{M}_{Z_{SM}}\mathcal{M}^*_{\gamma_{eff}} + \mathcal{M}^*_{Z_{SM}}\mathcal{M}_{\gamma_{eff}}\right\rangle + \left\langle\mathcal{M}_{Z_{eff}}\mathcal{M}^*_{\gamma_{eff}} + \mathcal{M}^*_{Z_{eff}}\mathcal{M}_{\gamma_{eff}}\right\rangle$$

The expressions for each term of the total amplitude are the following:

$$\begin{aligned}\left\langle|\mathcal{M}_{Z_{SM}}|^2\right\rangle =& \frac{3g^4\left|C_{ij}\right|^2}{C_W^4 M_Z^4}\Big[4m_q^2 g_L^q g_R^q (p_N\cdot p_{\nu_i})\\ &+ 2\left((g_L^q)^2 + (g_R^q)^2\right)\left((p_N\cdot p_{\overline{q}})(p_{\nu_i}\cdot p_q) + (p_N\cdot p_q)(p_{\nu_i}\cdot p_{\overline{q}})\right)\Big]\end{aligned}$$

$$\begin{aligned}\left\langle\left|\mathcal{M}_{Z_{eff}}\right|^2\right\rangle =& \frac{-192g^2 S_W^2}{C_W^2 M_Z^4}\left|\frac{(\alpha'_{NB})_{ij}}{\Lambda}\right|^2\Big\{2m_q^2 g_L^q g_R^q (p_N\cdot p_{\nu_i})(2(p_N\cdot p_{\nu_i}) - 3m_N^2)\\ &- \left((g_L^q)^2 + (g_R^q)^2\right)\Big[(p_N\cdot p_{\nu_i})\Big(m_N^2(p_q\cdot p_{\overline{q}}) - 2(p_N\cdot p_{\nu_i})(p_q\cdot p_{\overline{q}})\\ &+ 2\big((p_{\nu_i}\cdot p_{\overline{q}}) + (p_N\cdot p_{\overline{q}})\big)\big((p_{\nu_i}\cdot p_q) + (p_N\cdot p_q)\big)\Big)\\ &- 4m_N^2(p_{\nu_i}\cdot p_q)(p_{\nu_i}\cdot p_{\overline{q}})\Big]\Big\}\end{aligned}$$

$$\begin{aligned}\left\langle\left|\mathcal{M}_{\gamma_{eff}}\right|^2\right\rangle =& \frac{-384\mathrm{e}^2 Q_q^2 C_W^2}{q^4}\left|\frac{(\alpha'_{NB})_{ij}}{\Lambda}\right|^2\Big\{(p_N\cdot p_{\nu_i})^2\big(2(p_q\cdot p_{\overline{q}}) + 2m_q^2\big)\\ &+ 4m_N^2(p_{\nu_i}\cdot p_q)(p_{\nu_i}\cdot p_{\overline{q}}) - (p_N\cdot p_{\nu_i})\Big[m_N^2(p_q\cdot p_{\overline{q}})\\ &+ 3m_N^2 m_q^2 + 2\big((p_N\cdot p_q) + (p_{\nu_i}\cdot p_q)\big)\big((p_N\cdot p_{\overline{q}}) + (p_{\nu_i}\cdot p_{\overline{q}})\big)\Big]\Big\}\end{aligned}$$

$$\left\langle \mathcal{M}_{Z_{SM}}\mathcal{M}^*_{Z_{eff}}\right\rangle = \frac{-24g^3 m_N S_W}{C_W^3 M_Z^4}\text{Re}\left[C_{ij}\frac{(\alpha'_{NB})_{ij}}{\Lambda}\right]\Big\{6m_q^2 g_L^q g_R^q (p_N\cdot p_{\nu_i}) + \left((g_L^q)^2+(g_R^q)^2\right)\Big[(p_N\cdot p_{\nu_i})(p_q\cdot p_{\overline{q}}) + (p_N\cdot p_{\overline{q}})(p_{\nu_i}\cdot p_q) + (p_N\cdot p_q)(p_{\nu_i}\cdot p_{\overline{q}}) - 2(p_{\nu_i}\cdot p_{\overline{q}})(p_{\nu_i}\cdot p_q)\Big]\Big\}$$

$$\left\langle \mathcal{M}_{Z_{SM}}\mathcal{M}^*_{\gamma_{eff}}\right\rangle = \frac{-24\text{e}Q_q g^2 m_N\,(g_L^q+g_R^q)}{q^2 C_W M_Z^2}\text{Re}\left[C_{ij}\frac{(\alpha'_{NB})_{ij}}{\Lambda}\right]\Big((p_N\cdot p_{\nu_i})(p_q\cdot p_{\overline{q}}) - 2(p_{\nu_i}\cdot p_{\overline{q}})(p_{\nu_i}\cdot p_q) + 3m_q^2(p_N\cdot p_{\nu_i}) + (p_N\cdot p_{\overline{q}})(p_{\nu_i}\cdot p_q) + (p_N\cdot p_q)(p_{\nu_i}\cdot p_{\overline{q}})\Big)$$

$$\left\langle \mathcal{M}_{Z_{eff}}\mathcal{M}^*_{\gamma_{eff}}\right\rangle = \frac{-192\text{e}Q_q g S_W\,(g_L^q+g_R^q)}{q^2 M_Z^2}\left|\frac{(\alpha'_{NB})_{ij}}{\Lambda}\right|^2\Big\{(p_N\cdot p_{\nu_i})^2\big(2(p_q\cdot p_{\overline{q}})+2m_q^2\big) + 4m_N^2(p_{\nu_i}\cdot p_{\overline{q}})(p_{\nu_i}\cdot p_q) - (p_N\cdot p_{\nu_i})\Big[m_N^2(p_q\cdot p_{\overline{q}}) + 3m_N^2 m_q^2 + 2\big((p_{\nu_i}\cdot p_{\overline{q}})+(p_N\cdot p_{\overline{q}})\big)\big((p_{\nu_i}\cdot p_q)+(p_N\cdot p_q)\big)\Big]\Big\}$$

The expression for the decay width of $N_j \to \nu_i q_x \overline{q}_x$ is split in parts, to keep track of the origin of each contribution. We calculate the width to be:

$$\Gamma(N_j\to\nu_i q_x\overline{q}_x) = \Gamma^{Z_{SM}}_{\nu q\overline{q}} + \Gamma^{Z_{eff}}_{\nu q\overline{q}} + \Gamma^{\gamma_{eff}}_{\nu q\overline{q}} + \Gamma^{Z_{eff}+\gamma_{eff}}_{\nu q\overline{q}} + \Gamma^{Z_{SM}+Z_{eff}}_{\nu q\overline{q}} + \Gamma^{Z_{SM}+\gamma_{eff}}_{\nu q\overline{q}} \tag{3.2}$$

$$\Gamma^{Z_{SM}}_{\nu q\overline{q}} = \frac{G_f^2 m_N^5}{32\pi^3}\,\|C_{ij}\|^2\left(g_L^q g_R^q\,\Gamma_5^{10}(0,x_q,x_q) + ((g_L^q)^2+(g_R^q)^2)\Gamma_1^{10}(0,x_q,x_q)\right) \tag{3.3}$$

$$\Gamma^{Z_{eff}}_{\nu q\overline{q}} = -\frac{G_f m_N^7 S_W^2}{8\sqrt{2}\pi^3 M_Z^2}\left\|\frac{(\alpha'_{NB})_{ij}}{\Lambda}\right\|^2\Big(2g_L^q g_R^q\Big(\Gamma_5^{20}(0,x_q,x_q) - 3\Gamma_5^{10}(0,x_q,x_q)\Big) + ((g_L^q)^2+(g_R^q)^2)\Big(\Gamma_1^{20}(0,x_q,x_q) + 4\Gamma_3^{00}(0,x_q,x_q) - \Gamma_2^{10}(0,x_q,x_q) - \Gamma_3^{10}(0,x_q,x_q) - \Gamma_4^{10}(0,x_q,x_q) - \Gamma_1^{10}(0,x_q,x_q)\Big)\Big) \tag{3.4}$$

$$\Gamma^{\gamma_{eff}}_{\nu q\bar{q}} = \frac{m_N^3 C_W^2 Q_q^2 e^2}{32\pi^3} \left\| \frac{(\alpha'_{NB})_{ij}}{\Lambda} \right\|^2 \Big(-\Gamma_1^{24}(0,x_q,x_q) - \Gamma_5^{24}(0,x_q,x_q) - 4\Gamma_3^{04}(0,x_q,x_q) + \Gamma_1^{14}(0,x_q,x_q) + 3\Gamma_5^{14}(0,x_q,x_q) + \Gamma_2^{14}(0,x_q,x_q) + \Gamma_3^{14}(0,x_q,x_q) + \Gamma_4^{14}(0,x_q,x_q) \Big) \tag{3.5}$$

$$\Gamma^{Z_{eff}+\gamma_{eff}}_{\nu q\bar{q}} = \frac{m_N^5 Q_q e g S_W}{32\pi^3 M_Z^2} \left\| \frac{(\alpha'_{NB})_{ij}}{\Lambda} \right\|^2 (g_L^q + g_R^q) \Big(-\Gamma_1^{22}(0,x_q,x_q) - \Gamma_5^{22}(0,x_q,x_q) - 4\Gamma_3^{02}(0,x_q,x_q) + \Gamma_1^{12}(0,x_q,x_q) + 3\Gamma_5^{12}(0,x_q,x_q) + \Gamma_2^{12}(0,x_q,x_q) + \Gamma_3^{12}(0,x_q,x_q) + \Gamma_4^{12}(0,x_q,x_q) \Big) \tag{3.6}$$

$$\Gamma^{Z_{SM}+Z_{eff}}_{\nu q\bar{q}} = -\frac{G_f m_N^6 g S_W}{32\sqrt{2}\pi^3 C_W M_Z^2} Re\left[\frac{(\alpha'_{NB})_{ij}}{\Lambda} C_{ij} \right] \Big(\big((g_L^q)^2 + (g_R^q)^2\big) \big(\Gamma_4^{00}(0,x_q,x_q) - 2\Gamma_3^{00}(0,x_q,x_q) + \Gamma_1^{10}(0,x_q,x_q)\big) + g_L^q g_R^q 6\Gamma_5^{10}(0,x_q,x_q) \Big) \tag{3.7}$$

$$\Gamma^{Z_{SM}+\gamma_{eff}}_{\nu q\bar{q}} = -\frac{G_f m_N^4 C_W Q_q e}{32\sqrt{2}\pi^3} Re\left[\frac{(\alpha'_{NB})_{ij}}{\Lambda} C_{ij} \right] (g_L^q + g_R^q) \Big(\Gamma_1^{12}(0,x_q,x_q) - 3\Gamma_5^{12}(0,x_q,x_q) + \Gamma_4^{02}(0,x_q,x_q) - 2\Gamma_3^{02}(0,x_q,x_q) \Big) \tag{3.8}$$

With this formulae, one can define the total decay width of the heavy neutrino to quarks $\Gamma(N_j \to Xqq)$. The factor 2 in the first term is to account for its charge conjugate,and the sum is performed over the three light neutrinos, all lepton flavors and all quarks lighter than m_N.

$$\Gamma(N_j \to Xqq) = \sum_{i,a,u,d} 2\Gamma(N_j \to l_a^+ q_u \bar{q}_d) + \Gamma(N_j \to \nu_i q_u \bar{q}_u) + \Gamma(N_j \to \nu_i q_d \bar{q}_d) \tag{3.9}$$

In order to visualize the effect of adding effective operators to the model, and to compare the results obtained through this formulae relative to the standard Seesaw, Figure 5 shows the partial width of a heavy neutrino to final states containing quarks. For this plot N_5 is taken as the lightest heavy neutrino. The plot shows the width for several values of $(\alpha'_{NB})_{56}/\Lambda$, as well as the width for the standard Seesaw. To account for quark hadronization in the final state, the three-body decay to quarks is being multiplied by an appropriate QCD correction function $(1 + \Delta_{QCD})$[17].

$$1 + \Delta_{QCD} = \frac{\Gamma(\tau \to \nu_\tau + hadrons)}{\Gamma_{tree}(\tau \to \nu_\tau \overline{u} q)}$$
$$\Delta_{QCD} = \frac{\alpha_s}{\pi} + 5.2\frac{\alpha_s^2}{\pi^2} + 26.4\frac{\alpha_s^3}{\pi^3}$$

For $(\alpha'_{NB})_{56}/\Lambda \leq 10^{-6}$ GeV^{-1} the results obtained from this formulae are indistinguishable from those of the standard Seesaw. For higher values of $(\alpha'_{NB})_{56}/\Lambda$ it is clear that the partial width into quarks can be significantly enhanced compared to the standard Seesaw. This is the result of the virtual photon contribution, which is not suppressed by the Z mass. However this contribution is proportional to m_N^3 while the standard Seesaw contributions are proportional to m_N^5 which means that at high mass the latter become more relevant. This can be observed in the plot as the slope for $(\alpha'_{NB})_{56}/\Lambda = 10^{-2}, 10^{-4}$ GeV^{-1} is steeper than the slope for the standard Seesaw.

In Figure 6 we compare the different contributions to the decay $\Gamma_{N_j \to Xqq}$ for $(\alpha'_{NB})_{56}/\Lambda = 10^{-4}$. In this plot $\Gamma^{int} = \Gamma_{\nu q\bar{q}}^{Z_{eff}+\gamma_{eff}} + \Gamma_{\nu q\bar{q}}^{Z_{SM}+Z_{eff}} + \Gamma_{\nu q\bar{q}}^{Z_{SM}+\gamma_{eff}}$. For low masses, the highest contribution comes from $\Gamma_{\nu q\bar{q}}^{\gamma_{eff}}$, making almost 100% of the decay, as the mass of the heavy neutrino becomes larger the impact of this channel wanes down to 30%.

For larger masses, the main contribution is from $\Gamma_{lud}^{W_{SM}}$ which can be up to 50% of the total decay. The contribution of $\Gamma_{\nu qq}^{Z_{SM}}$ goes up to 20%. The contributions from $\Gamma_{\nu qq}^{Z_{eff}}$ and Γ^{int} are very small in comparison to the other contributions, making up less than 1% of the total decay.

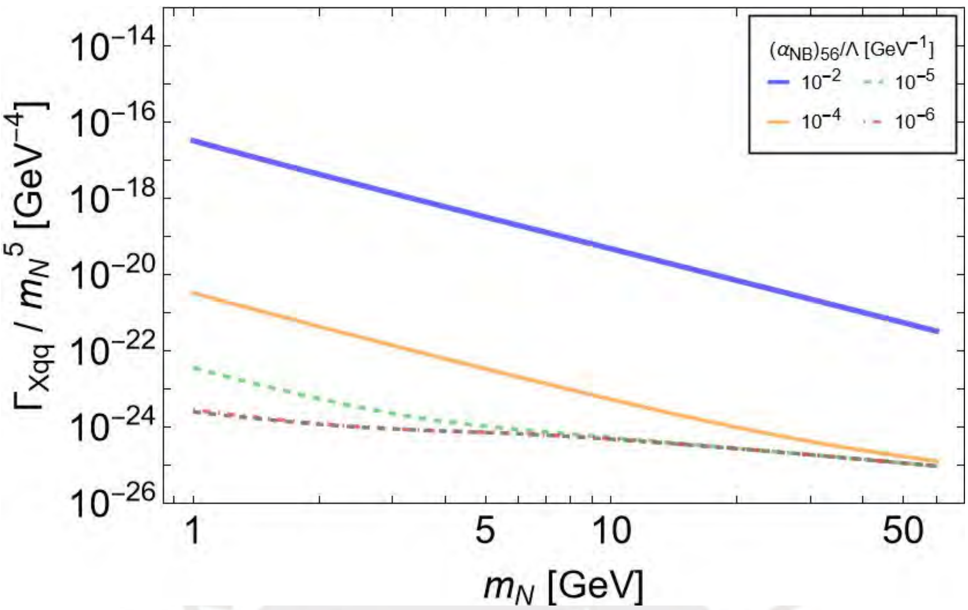


Figure 5: Heavy neutrino partial width into final states that include quarks, normalized with respect to m_N^5, the widths plotted corresponding to $(\alpha'_{NB})_{56}/\Lambda = 10^{-2}, 10^{-4}, 10^{-5}, 10^{-6}$ GeV^{-1} are being shown with thick blue, thin orange, dotted red and dotted green lines respectively. In addition the gray dashed line indicate the standard Seesaw three-body decay to quarks.

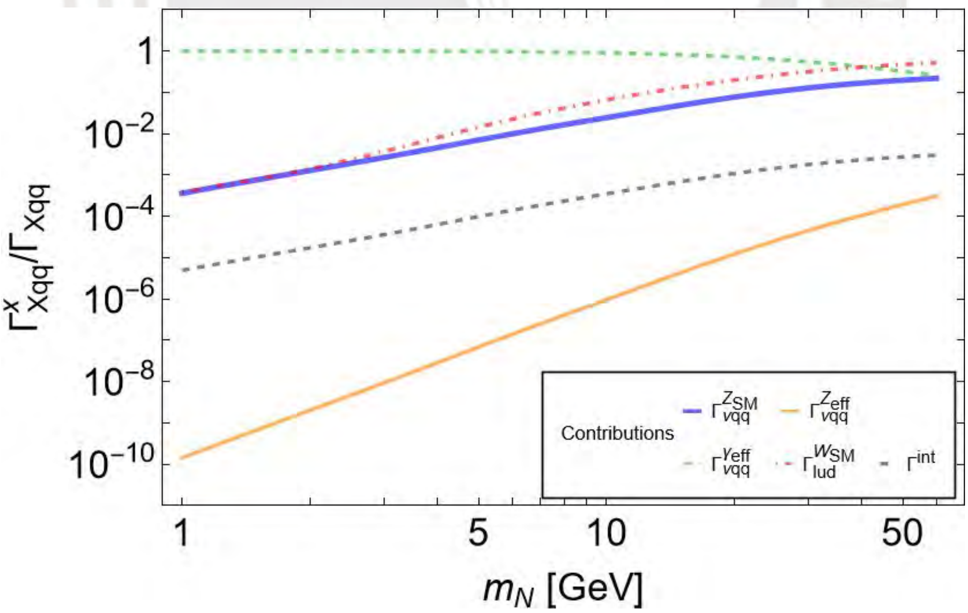
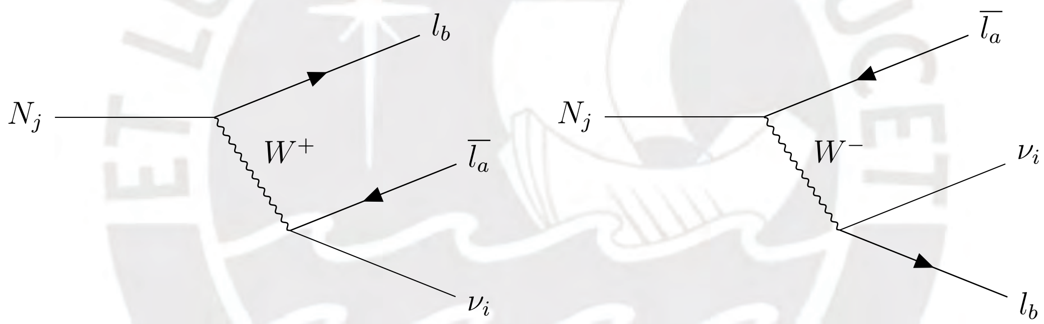


Figure 6: Contribution of different origin, normalized with respect to the full Γ_{Xqq}. The contribution shown, are for $(\alpha'_{NB})_{56}/\Lambda = 10^{-4}$. The contributions for $N \to \nu qq$ are separated into $\Gamma^{Z_{SM}}_{\nu q\bar{q}}$ for the standard Seesaw decay, for the channels that have an effective dipole, $\Gamma^{Z_{eff}}_{\nu q\bar{q}}$ is mediated by a virtual Z boson and $\Gamma^{\gamma_{eff}}_{\nu q\bar{q}}$ is mediated by a virtual photon. Γ^{int} is the sum of all the interference terms. The curve labeled$\Gamma^{W_{SM}}_{lud}$ is the total decay $N \to lqq$.

3.2 Decay to charged Leptons

For the decay to two charged leptons, it makes a difference whether the leptons are of the same flavor or not. When the charged leptons have different flavors, there are two diagrams possible. Both are mediated by a virtual W boson. In this case the addition of the effective operators does not change the decay width. The diagrams with their respective amplitudes are shown in Figure 7, and the width would be:

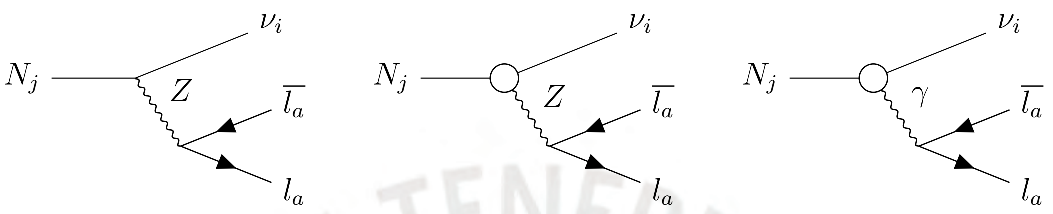
$$\Gamma(N_j \to \nu_i l_b \bar{l}_a) = \frac{G_f^2 m_N^5}{192\pi^3}\left(\|U_{ai}\|^2 \|U_{bj}\|^2 + \|U_{bi}\|^2 \|U_{aj}\|^2\right)\Gamma_1^{10}(x_{l_a}, x_{l_b}, 0) \tag{3.10}$$



$$\begin{aligned}
\mathrm{i}\mathcal{M}_{W_1} &= \left(\frac{\mathrm{i}g^2\eta_{\mu\rho}}{2m_W^2}\right)\left[\overline{u}_{\nu_i}\gamma^{\rho}U_{ai}^{*}P_L v_{l_a^+}\right]\left[\overline{u}_{l_b}\gamma^{\mu}U_{aj}P_L u_{N_j}\right] \\
\mathrm{i}\mathcal{M}_{W_2} &= -\left(\frac{\mathrm{i}g^2\eta_{\mu\rho}}{2m_W^2}\right)\left[\overline{u}_{l_b}\gamma^{\rho}U_{ai}P_L v_{\nu_i}\right]\left[\overline{v}_{N_j}\gamma^{\mu}U_{aj}^{*}P_L v_{l_a^+}\right]
\end{aligned}$$

Figure 7: Tree level Feynman diagrams for the decay of a heavy neutrino into charged leptons through the charged current. On the left, diagram for $\mathcal{M}_{W_1}$, and on the right for $\mathcal{M}_{W_2}$

When the flavor of the charged leptons is the same, the process is more complex. For the decay: $N_j \to \nu_i \overline{l_a} l_a$ there are five possible tree level diagrams, two mediated by the Z boson, two mediated via the W boson and one via a photon. The diagrams labeled $\mathcal{M}_{Z_{SM}}$, $\mathcal{M}_{Z_{eff}}$ decay through a Z boson. The difference lies in the interaction between the neutrinos. The diagrams labeled $\mathcal{M}_{W_1}$, $\mathcal{M}_{W_2}$ decay via the W boson and are the charge conjugate of each other. In the case of the decay mediated by the photon, which is called $\mathcal{M}_{\gamma_{eff}}$, the interaction between the neutrinos is via the effective operator. The diagrams with their respective amplitudes are the following:



$$\mathrm{i}\mathcal{M}_{Z_{SM}} = -\left(\frac{g}{2C_W}\right)^2\left(\mathrm{i}\frac{2\eta_{\mu\rho}}{M_Z^2}\right)\left[\overline{u}_{l_a}\gamma^\rho(g_L^l P_L + g_R^l P_R)v_{\overline{l}_a}\right]\left[\overline{u}_{\nu_i}\gamma^\mu(C_{ij}P_L - C_{ij}^* P_R)u_{N_j}\right]$$

$$\mathrm{i}\mathcal{M}_{Z_{eff}} = \left[\frac{4S_W g\eta_{\mu\rho}}{C_W M_Z^2}\right] q_\nu \left[\overline{u}_{l_a}\gamma^\rho(g_L^l P_L + g_R^l P_R)v_{\overline{l}_a}\right]\left[\overline{u}_{\nu_i}\sigma^{\mu\nu}\left(\frac{(\alpha'_{NB})_{ij}}{\Lambda}P_L - \frac{(\alpha'_{NB})_{ij}^*}{\Lambda}P_R\right)u_{N_j}\right]$$

$$\mathrm{i}\mathcal{M}_{\gamma_{eff}} = \left(\frac{4C_W\eta_{\mu\rho}}{\mathrm{e}\,q^2}\right) q_\nu \left[\overline{u}_{l_a}\gamma^\rho v_{\overline{l}_a}\right]\left[\overline{u}_{\nu_i}\sigma^{\mu\nu}\left(\frac{(\alpha'_{NB})_{ij}}{\Lambda}P_L - \frac{(\alpha'_{NB})_{ij}^*}{\Lambda}P_R\right)u_{N_j}\right]$$

Figure 8: Tree level Feynman diagrams for the decay of a heavy neutrino into charged leptons through the neutral current. From left to right $\mathcal{M}_{Z_{SM}}$,$\mathcal{M}_{Z_{eff}}$,$\mathcal{M}_{\gamma_{eff}}$.

The total amplitude is:

$$\begin{aligned}
\left\langle|\mathcal{M}|^2\right\rangle = &\left\langle|\mathcal{M}_{Z_{SM}}|^2\right\rangle + \left\langle|\mathcal{M}_{W_{SM}}|^2\right\rangle + \left\langle\left|\mathcal{M}_{Z_{eff}}\right|^2\right\rangle + \left\langle\left|\mathcal{M}_{\gamma_{eff}}\right|^2\right\rangle \\
&+ \left\langle\mathcal{M}_{Z_{SM}}\mathcal{M}^*_{Z_{eff}} + \mathcal{M}^*_{Z_{SM}}\mathcal{M}_{Z_{eff}}\right\rangle + \left\langle\mathcal{M}_{Z_{SM}}\mathcal{M}^*_{W_{SM}} + \mathcal{M}^*_{Z_{SM}}\mathcal{M}_{W_{SM}}\right\rangle \\
&+ \left\langle\mathcal{M}_{Z_{eff}}\mathcal{M}^*_{W_{SM}} + \mathcal{M}^*_{Z_{eff}}\mathcal{M}_{W_{SM}}\right\rangle + \left\langle\mathcal{M}_{\gamma_{eff}}\mathcal{M}^*_{W_{SM}} + \mathcal{M}^*_{\gamma_{eff}}\mathcal{M}_{W_{SM}}\right\rangle \\
&+ \left\langle\mathcal{M}_{Z_{SM}}\mathcal{M}^*_{\gamma_{eff}} + \mathcal{M}^*_{Z_{SM}}\mathcal{M}_{\gamma_{eff}}\right\rangle + \left\langle\mathcal{M}_{Z_{eff}}\mathcal{M}^*_{\gamma_{eff}} + \mathcal{M}^*_{Z_{eff}}\mathcal{M}_{\gamma_{eff}}\right\rangle
\end{aligned}$$

Where $\mathcal{M}_{W_{SM}} = \mathcal{M}_{W_1} + \mathcal{M}_{W_2}$. The expression or each term of the total amplitude are the following:

$$\left\langle|\mathcal{M}_{Z_{SM}}|^2\right\rangle = \frac{2g^4\left|C_{ij}\right|^2}{C_W^4 M_Z^4}\Big[2m_{l_a}^2 g_L^l g_R^l (p_N\cdot p_{\nu_i}) + \left(\left(g_L^l\right)^2+\left(g_R^l\right)^2\right)\left((p_N\cdot p_{\bar{l}_a})(p_{\nu_i}\cdot p_{l_a})+(p_N\cdot p_{l_a})(p_{\nu_i}\cdot p_{\bar{l}_a})\right)\Big]$$

$$\left\langle|\mathcal{M}_{W_{SM}}|^2\right\rangle = \frac{2g^4}{m_W^4}\left|U_{ai}\right|^2\left|U_{aj}\right|^2\left((p_N\cdot p_{\bar{l}_a})(p_{\nu_i}\cdot p_{l_a})+(p_N\cdot p_{l_a})(p_{\nu_i}\cdot p_{\bar{l}_a})\right)$$

$$\left\langle\left|\mathcal{M}_{Z_{eff}}\right|^2\right\rangle = \frac{-64g^2S_W^2}{C_W^2M_Z^4}\left|\frac{(\alpha'_{NB})_{ij}}{\Lambda}\right|^2\Big\{g_L^l g_R^l m_{l_a}^2 (p_N\cdot p_{\nu_i})\big(4(p_N\cdot p_{\nu_i})-6m_N^2\big) + \left(\left(g_L^l\right)^2+\left(g_R^l\right)^2\right)\Big[(p_N\cdot p_{\nu_i})\Big(-m_N^2(p_{l_a}\cdot p_{\bar{l}_a})+2(p_N\cdot p_{\nu_i})(p_{l_a}\cdot p_{\bar{l}_a}) + \left((p_{\nu_i}\cdot p_{\bar{l}_a})+(p_N\cdot p_{\bar{l}_a})\right)\left((p_{\nu_i}\cdot p_{l_a})+(p_N\cdot p_{l_a})\right)\Big) + 4m_N^2(p_{\nu_i}\cdot p_{l_a})(p_{\nu_i}\cdot p_{\bar{l}_a})\Big]\Big\}$$

$$\left\langle\left|\mathcal{M}_{\gamma_{eff}}\right|^2\right\rangle = \frac{-128\mathrm{e}^2C_W^2}{q^4}\left|\frac{(\alpha'_{NB})_{ij}}{\Lambda}\right|^2\Big\{2(p_N\cdot p_{\nu_i})^2((p_{l_a}\cdot p_{\bar{l}_a})+m_{l_a}^2) + 4m_N^2(p_{\nu_i}\cdot p_{l_a})(p_{\nu_i}\cdot p_{\bar{l}_a})-(p_N\cdot p_{\nu_i})\Big[m_N^2(p_{l_a}\cdot p_{\bar{l}_a}) + 3m_N^2m_{l_a}^2+2\left((p_N\cdot p_{l_a})+(p_{\nu_i}\cdot p_{l_a})\right)\left((p_N\cdot p_{\bar{l}_a})+(p_{\nu_i}\cdot p_{\bar{l}_a})\right)\Big]\Big\}$$

$$\left\langle\mathcal{M}_{Z_{SM}}\mathcal{M}^*_{W_{SM}}\right\rangle = \frac{2g^4}{C_W^2M_W^2M_Z^2}\mathrm{Re}\left[U_{ai}U_{aj}C_{ij}\right]\Big((p_N\cdot p_{\nu_i})g_R^l m_{l_a}^2 + g_L^l\left((p_N\cdot p_{l_a})(p_{\nu_i}\cdot p_{\bar{l}_a})+(p_N\cdot p_{\bar{l}_a})(p_{\nu_i}\cdot p_{l_a})\right)\Big)$$

$$\left\langle\mathcal{M}_{Z_{SM}}\mathcal{M}^*_{Z_{eff}}\right\rangle = \frac{-8g^3m_NS_W}{C_W^3M_Z^4}\mathrm{Re}\left[C_{ij}\frac{(\alpha'_{NB})_{ij}}{\Lambda}\right]\Big[6m_{l_a}^2 g_L^l g_R^l(p_N\cdot p_{\nu_i}) + \left(\left(g_L^l\right)^2+\left(g_R^l\right)^2\right)\Big((p_N\cdot p_{\bar{l}_a})(p_{\nu_i}\cdot p_{l_a})+(p_N\cdot p_{\nu_i})(p_{l_a}\cdot p_{\bar{l}_a}) + (p_N\cdot p_{l_a})(p_{\nu_i}\cdot p_{\bar{l}_a})-2(p_{\nu_i}\cdot p_{\bar{l}_a})(p_{\nu_i}\cdot p_{l_a})\Big)\Big]$$

$$\left\langle \mathcal{M}_{W_{SM}}\mathcal{M}^*_{Z_{eff}} \right\rangle = \frac{-8g^3 S_W m_N}{C_W M_W^2 M_Z^2}\text{Re}\left[U_{aj}U_{ij}\frac{(\alpha'_{NB})_{ij}}{\Lambda}\right]\Big\{g_L^l\Big[(p_{\nu_i}\cdot p_{l_a})(p_N\cdot p_{\bar{l}_a}) + (p_{\nu_i}\cdot p_{\bar{l}_a})(p_N\cdot p_{l_a}) - 2(p_{\nu_i}\cdot p_{l_a})(p_{\nu_i}\cdot p_{\bar{l}_a})\Big] + (p_N\cdot p_{\nu_i})\left(g_L^l(p_{l_a}\cdot p_{\bar{l}_a}) + 3m_{l_a}^2 g_R^l\right)\Big\}$$

$$\left\langle \mathcal{M}_{Z_{SM}}\mathcal{M}^*_{\gamma_{eff}} \right\rangle = \frac{8\text{e}g^2 m_N\left(g_L^l + g_R^l\right)}{q^2 C_W M_Z^2}\text{Re}\left[C_{ij}\frac{(\alpha'_{NB})_{ij}}{\Lambda}\right]\Big((p_N\cdot p_{\nu_i})((p_{l_a}\cdot p_{\bar{l}_a}) + 3m_{l_a}^2) + (p_N\cdot p_{\bar{l}_a})(p_{\nu_i}\cdot p_{l_a}) + (p_N\cdot p_{l_a})(p_{\nu_i}\cdot p_{\bar{l}_a}) - 2(p_{\nu_i}\cdot p_{\bar{l}_a})(p_{\nu_i}\cdot p_{l_a})\Big)$$

$$\left\langle \mathcal{M}_{Z_{eff}}\mathcal{M}^*_{\gamma_{eff}} \right\rangle = \frac{64\text{e}g S_W\left(g_L^l + g_R^l\right)}{q^2 M_Z^2}\left|\frac{(\alpha'_{NB})_{ij}}{\Lambda}\right|^2\Big\{(p_N\cdot p_{\nu_i})^2(2(p_{l_a}\cdot p_{\bar{l}_a}) + 2m_{l_a}^2) + 4m_N^2(p_{\nu_i}\cdot p_{\bar{l}_a})(p_{\nu_i}\cdot p_{l_a}) + (p_N\cdot p_{\nu_i})\Big[-m_N^2(p_{l_a}\cdot p_{\bar{l}_a}) - 3m_N^2 m_{l_a}^2 - 2\left((p_{\nu_i}\cdot p_{\bar{l}_a}) + (p_N\cdot p_{\bar{l}_a})\right)\left((p_{\nu_i}\cdot p_{l_a}) + (p_N\cdot p_{l_a})\right)\Big]\Big\}$$

$$\left\langle \mathcal{M}_{W_{SM}}\mathcal{M}^*_{\gamma_{eff}} \right\rangle = \frac{8\text{e}g^2 C_W m_N}{q^2 m_W^2}\text{Re}\left[U_{aj}U_{ai}\frac{(\alpha'_{NB})_{ij}}{\Lambda}\right]\Big((p_N\cdot p_{\nu_i})\left((p_{l_a}\cdot p_{\bar{l}_a}) + 3m_{l_a}^2\right) + (p_{\nu_i}\cdot p_{l_a})(p_N\cdot p_{\bar{l}_a}) + (p_{\nu_i}\cdot p_{\bar{l}_a})(p_N\cdot p_{l_a}) - 2(p_{\nu_i}\cdot p_{l_a})(p_{\nu_i}\cdot p_{\bar{l}_a})\Big)$$

In order to distinguish the different contributions to the decay width, the expression is divided in parts. The width of the decay $N_j \to \nu_i l_a \bar{l}_a$ can be expressed as:

$$\Gamma(N_j \to \nu_i l_a \bar{l}_a) = \Gamma_{\nu l\bar{l}}^{Z_{SM}} + \Gamma_{\nu l\bar{l}}^{W_{SM}} + \Gamma_{\nu l\bar{l}}^{Z_{eff}} + \Gamma_{\nu l\bar{l}}^{\gamma_{eff}} + \Gamma_{\nu l\bar{l}}^{Z_{SM}+W_{SM}} + \Gamma_{\nu l\bar{l}}^{Z_{SM}+Z_{eff}} + \Gamma_{\nu l\bar{l}}^{Z_{SM}+\gamma_{eff}} + \Gamma_{\nu l\bar{l}}^{W_{SM}+Z_{eff}} + \Gamma_{\nu l\bar{l}}^{W_{SM}+\gamma_{eff}} + \Gamma_{\nu l\bar{l}}^{Z_{eff}+\gamma_{eff}} \quad (3.11)$$

The expression for each of the different contributions would be:

$$\Gamma_{\nu l\bar{l}}^{W_{SM}} = \frac{G_f^2 m_N^5}{96\pi^3}\left\|U_{ai}\right\|^2\left\|U_{aj}\right\|^2\,\Gamma_1^{10}(x_{l_a}, x_{l_a}, 0) \quad (3.12)$$

$$\Gamma^{Z_{SM}+W_{SM}}_{\nu l\bar{l}} = \frac{G_f^2 m_N^5}{96\pi^3} Re\left[U_{ai}U^*_{aj}C_{ij}\right]\Big(2g^l_L\Gamma^{10}_1(x_{l_a},x_{l_a},0)+g^l_R\Gamma^{10}_5(0,x_{l_a},x_{l_a})\Big) \tag{3.13}$$

$$\begin{aligned}\Gamma^{W_{SM}+Z_{eff}}_{\nu l\bar{l}} =& -\frac{G_f m_N^6 g S_W}{96\sqrt{2}\pi^3 C_W M_Z^2} Re\left[\frac{(\alpha'_{NB})_{ij}U_{aj}U^*_{ai}}{\Lambda}\right]\Big(3g^l_R\Gamma^{10}_5(0,x_{l_a},x_{l_a})\\ &+g^l_L\big(\Gamma^{10}_1(0,x_{l_a},x_{l_a})+\Gamma^{00}_4(0,x_{l_a},x_{l_a})-2\Gamma^{00}_2(0,x_{l_a},x_{l_a})\big)\Big)\end{aligned} \tag{3.14}$$

$$\begin{aligned}\Gamma^{W_{SM}+\gamma_{eff}}_{\nu l\bar{l}} =& \frac{eG_f m_N^4 C_W}{96\sqrt{2}\pi^3} Re\left[\frac{(\alpha'_{NB})_{ij}U_{aj}U^*_{ai}}{\Lambda}\right]\Big(3\Gamma^{12}_5(0,x_{l_a},x_{l_a})+\Gamma^{12}_1(0,x_{l_a},x_{l_a})\\ &+\Gamma^{02}_4(0,x_{l_a},x_{l_a})-2\Gamma^{02}_2(0,x_{l_a},x_{l_a})\Big)\end{aligned} \tag{3.15}$$

Some of the terms present similitude with terms defined in the section for quarks. In these cases, one can use the widths defined for quarks, but interchanging the values of mass, electric charge and factors g^x_L, g^x_R of the quarks for the respective values of leptons. The factor $1/3$ accounts for color.

$$\begin{aligned}\Gamma^{Z_{SM}}_{\nu l\bar{l}} &\simeq \frac{1}{3}\Gamma^{Z_{SM}}_{\nu q\bar{q}} & \Gamma^{Z_{SM}+\gamma_{eff}}_{\nu l\bar{l}} &\simeq \frac{1}{3}\Gamma^{Z_{SM}+\gamma_{eff}}_{\nu q\bar{q}}\\ \Gamma^{Z_{eff}}_{\nu l\bar{l}} &\simeq \frac{1}{3}\Gamma^{Z_{eff}}_{\nu q\bar{q}} & \Gamma^{Z_{SM}+\gamma_{eff}}_{\nu l\bar{l}} &\simeq \frac{1}{3}\Gamma^{Z_{SM}+\gamma_{eff}}_{\nu q\bar{q}}\\ \Gamma^{\gamma_{eff}}_{\nu l\bar{l}} &\simeq \frac{1}{3}\Gamma^{\gamma_{eff}}_{\nu q\bar{q}} & \Gamma^{Z_{eff}+\gamma_{eff}}_{\nu l\bar{l}} &\simeq \frac{1}{3}\Gamma^{Z_{eff}+\gamma_{eff}}_{\nu q\bar{q}}\end{aligned}$$

With the formulae for the decay width with two charged lepton in the final state, one can define the total decay width of the heavy neutrino to two charged leptons $\Gamma(N_j \to \nu ll)$, where the sum is performed over the light neutrinos and all flavors of the charged leptons. The Dirac delta in the second term ensures that it will only account for different flavor leptons.

$$\Gamma(N_j \to \nu ll) = \sum_{i,a,b}\Gamma(N_j \to \nu_i l_a \bar{l}_a) + (1-\delta^b_a)\Gamma(N_j \to \nu_i l_b \bar{l}_a) \tag{3.16}$$

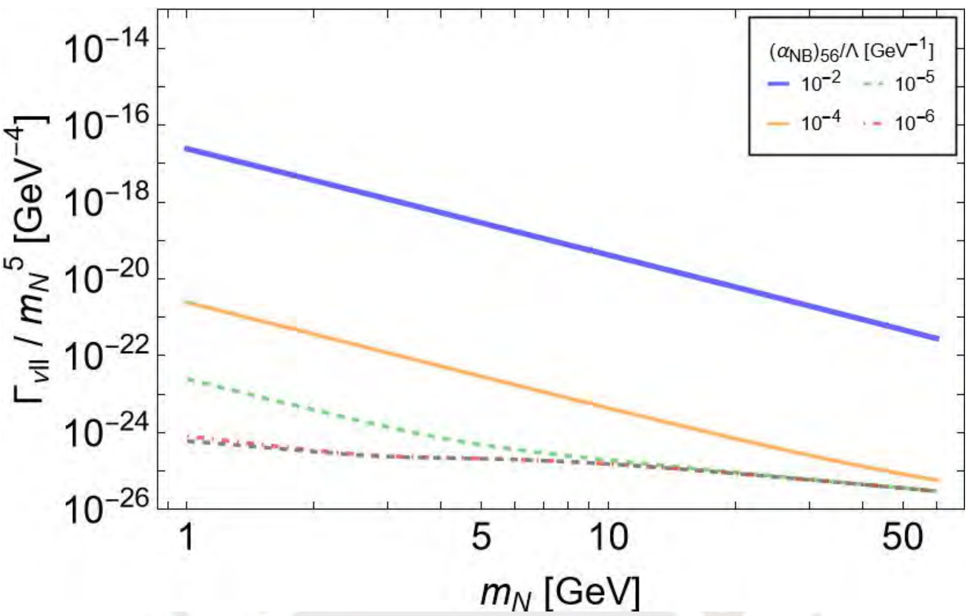


Figure 9: Heavy neutrino partial width into final states that include charged leptons, normalized wit respect to m_N^5, the widths plotted correspond to $(\alpha'_{NB})_{56}/\Lambda = 10^{-2}, 10^{-4}, 10^{-5}, 10^{-6}$ GeV^{-1}, and are being shown with thick blue, thin orange, dotted red and dotted green lines respectively. The gray dashed line indicate the standard Seesaw three-body decay to charged leptons.

In order to visualize the effect of adding effective operators to the model, and to compare the results obtained through this formulae relative to the standard Seesaw, Figure 9 shows the partial width of a heavy neutrino to final states containing charged leptons. For this plot N_5 is taken as the lightest heavy neutrino. The plot shows the width for several values of $(\alpha'_{NB})_{56}/\Lambda$, as well as the width for the standard Seesaw.

For $(\alpha'_{NB})_{56}/\Lambda/ \leq 10^{-6}$ GeV^{-1} the results obtained through this formulae are indistinguishable from those of the standard Seesaw. Similar to the partial width to quarks, for higher values of $(\alpha'_{NB})_{56}/\Lambda$ this partial width can be enhanced considerably from the baseline of the standard Seesaw width. This enhancement comes from the contribution of the virtual photon processes. In this case this contribution is also proportional to m_N^3, so at higher masses it becomes less relevant than the standard Seesaw contributions, which are proportional to m_N^5.

In Figure 10 we compare the different contributions to the decay $\Gamma_{N_j \to \nu ll}$ for $(\alpha'_{NB})_{56}/\Lambda = 10^{-4}$ GeV^{-1}. In this case for masses lower than 10 GeV the channel $\Gamma_{\nu ll}^{\gamma_{eff}}$ is the largest, then for heavier masses it goes down to 50% at ~ 60 GeV. For large masses, the contribution from $\Gamma^W_{\nu l_1 l_2}$

can take 35% and $\Gamma_{\nu ll}^{W_{SM}}$ around 15%. The contribution from $\Gamma_{\nu ll}^{Z_{eff}}$ is very small in comparison to the other contributions. The interference Γ^{int}, shown in the bottom panel of Figure 10 can take up to -10% for large masses. In contrast to the quark case the term is negative, which comes from the difference in electric charge between the quarks and the charged leptons.

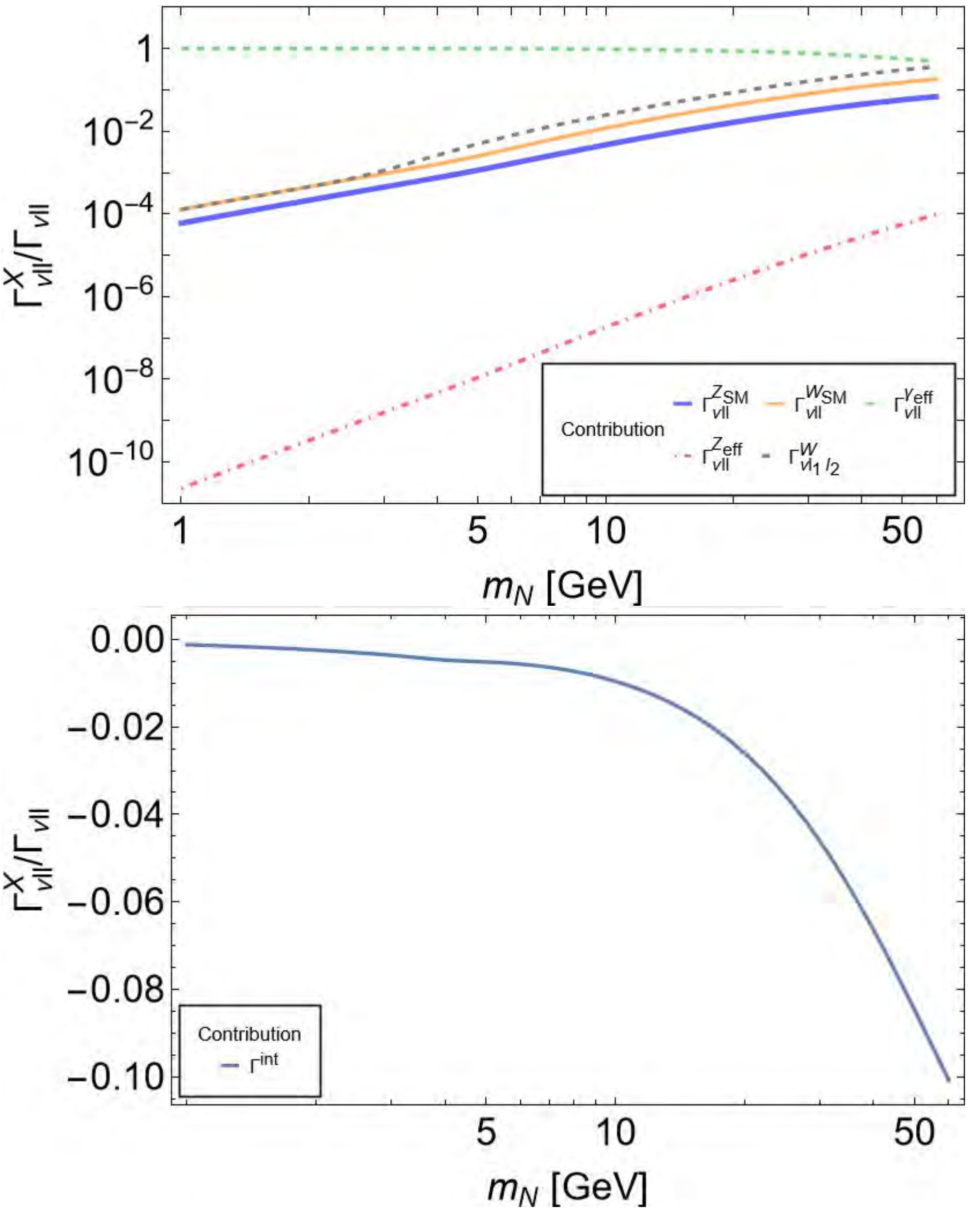


Figure 10: Contribution of different origin, normalized with respect to the complete $\Gamma_{\nu ll}$. The contribution shown, are for $(\alpha'_{NB})_{56}/\Lambda = 10^{-4}$. Top : the contributions for $\Gamma(N_j \to \nu_i l_a \bar{l}_a)$ are separated into the standard Seesaw decays: $\Gamma^{Z_{SM}}_{\nu ll}$, $\Gamma^{W_{SM}}_{\nu ll}$, and the channels that have an effective dipole: $\Gamma^{Z_{eff}}_{\nu ll}$, $\Gamma^{\gamma_{eff}}_{\nu ll}$. The curve labeled $\Gamma^{W}_{\nu l_1 l_2}$ is the total decay $\Gamma(N_j \to \nu_i l_b \bar{l}_a)$. In the bottom image Γ^{int} is the sum of all interference terms of $\Gamma(N_j \to \nu_i l_a \bar{l}_a)$.

3.3 Decay to neutrinos

For the decay to only light neutrinos $N_j \to \nu_i \nu_m \nu_n$, there are nine tree level diagrams possible. Many of these only differ from each other in the specific light neutrinos in each vertex. For the calculations the sub-indexes are used to differentiate each light neutrino, however the important result is the total decay width to neutrinos $N_j \to \nu\nu\nu$. So in the end a sum over all the light neutrino states is performed. The amplitudes are being separated depending in the interactions present

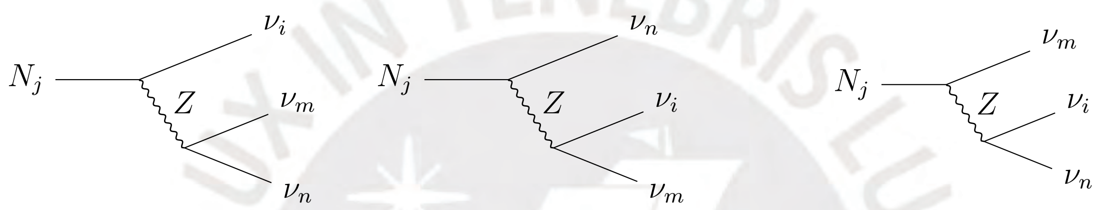


Figure 11: Diagrams for the decay to only light neutrinos. From left to right $\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3$

The diagrams from $\mathcal{M}_1$ to $\mathcal{M}_3$ only include interactions present in the Seesaw model without the dimension five operators. These SM-like contributions are written as $\mathcal{M}_{SM} = \mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3$ where:

$$
\begin{aligned}
\mathrm{i}\mathcal{M}_1 &= -\left(\frac{g}{2C_W}\right)^2 \left(\mathrm{i}\frac{\eta_{\mu\rho}}{M_Z^2}\right) \left[\overline{u}_{\nu_n}\gamma^\rho (C_{nm}P_L - C_{nm}^* P_R) v_{\nu_m}\right] \left[\overline{u}_{\nu_i}\gamma^\mu (C_{ij}P_L - C_{ij}^* P_R) u_{N_j}\right] \\
\mathrm{i}\mathcal{M}_2 &= \left(\frac{g}{2C_W}\right)^2 \left(\mathrm{i}\frac{\eta_{\mu\rho}}{M_Z^2}\right) \left[\overline{u}_{\nu_i}\gamma^\rho (C_{im}P_L - C_{im}^* P_R) v_{\nu_m}\right] \left[\overline{u}_{\nu_n}\gamma^\mu (C_{nj}P_L - C_{nj}^* P_R) u_{N_j}\right] \\
\mathrm{i}\mathcal{M}_3 &= -\left(\frac{g}{2C_W}\right)^2 \left(\mathrm{i}\frac{\eta_{\mu\rho}}{M_Z^2}\right) \left[\overline{u}_{\nu_n}\gamma^\rho (C_{ni}P_L - C_{ni}^* P_R) v_{\nu_i}\right] \left[\overline{u}_{\nu_m}\gamma^\mu (C_{mj}P_L - C_{mj}^* P_R) u_{N_j}\right]
\end{aligned}
$$

For $\mathcal{M}_4, \mathcal{M}_5, \mathcal{M}_6$ the coupling to the heavy neutrino, represented by a blob in the diagrams, contains the dipole operator.

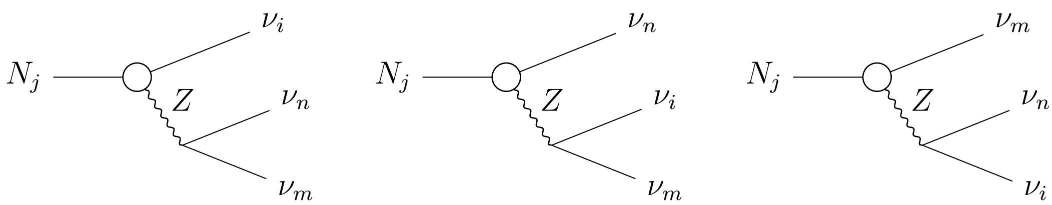


Figure 12: Diagrams for the decay to only light neutrinos. From left to right $\mathcal{M}_4, \mathcal{M}_5, \mathcal{M}_6$

$$\begin{aligned}
\mathrm{i}\mathcal{M}_4 = &-\left(\frac{4S_W g}{\Lambda C_W}\right)\left(\frac{\eta_{\mu\rho}}{M_Z^2}\right)\left(p_{N_j} - p_{\nu_i}\right)_\lambda \left[\overline{u}_{\nu_n}\gamma^\rho(C_{nm}P_L - C^*_{nm}P_R)v_{\nu_m}\right] \\
&\left[\overline{u}_{\nu_i}\sigma^{\mu\lambda}((\alpha'_{NB})_{ij}P_L - (\alpha'_{NB})^*_{ij}P_R)u_{N_j}\right] \\
\mathrm{i}\mathcal{M}_5 = &\left(\frac{4S_W g}{\Lambda C_W}\right)\left(\frac{\eta_{\mu\rho}}{M_Z^2}\right)\left(p_{N_j} - p_{\nu_n}\right)_\lambda \left[\overline{u}_{\nu_i}\gamma^\rho(C_{im}P_L - C^*_{im}P_R)v_{\nu_m}\right] \\
&\left[\overline{u}_{\nu_n}\sigma^{\mu\lambda}((\alpha'_{NB})_{nj}P_L - (\alpha'_{NB})^*_{nj}P_R)u_{N_j}\right] \\
\mathrm{i}\mathcal{M}_6 = &-\left(\frac{4S_W g}{\Lambda C_W}\right)\left(\frac{\eta_{\mu\rho}}{M_Z^2}\right)\left(p_{N_j} - p_{\nu_m}\right)_\lambda \left[\overline{u}_{\nu_n}\gamma^\rho(C_{ni}P_L - C^*_{ni}P_R)v_{\nu_i}\right] \\
&\left[\overline{u}_{\nu_m}\sigma^{\mu\lambda}((\alpha'_{NB})_{mj}P_L - (\alpha'_{NB})^*_{mj}P_R)u_{N_j}\right]
\end{aligned}$$

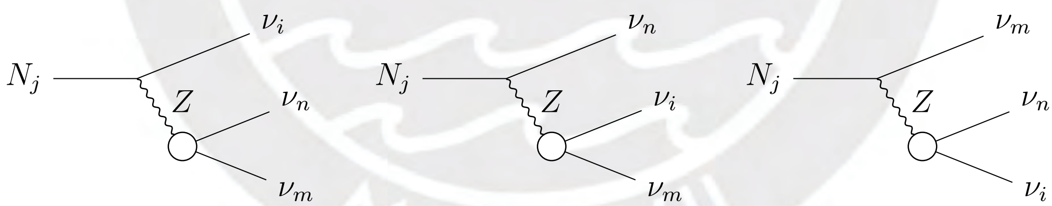


Figure 13: Diagrams for the decay to only light neutrinos. From left to right $\mathcal{M}_7, \mathcal{M}_8, \mathcal{M}_9$

For $\mathcal{M}_7, \mathcal{M}_8, \mathcal{M}_9$ there is also a coupling with the dipole operator,but it is between the light neutrinos.

$$\begin{aligned}
\mathrm{i}\mathcal{M}_7 = &-\left(\frac{4S_W g}{\Lambda C_W}\right)\left(\frac{\eta_{\mu\rho}}{M_Z^2}\right)\left(p_{N_j} - p_{\nu_i}\right)_\lambda \left[\overline{u}_{\nu_n}\sigma^{\rho\lambda}((\alpha'_{NB})_{nm}P_L - (\alpha'_{NB})^*_{nm}P_R)v_{\nu_m}\right] \\
&\left[\overline{u}_{\nu_i}\gamma^\mu(C_{ij}P_L - C^*_{ij}P_R)u_{N_j}\right]
\end{aligned}$$

$$
\begin{aligned}
\mathrm{i}\mathcal{M}_8 =& \left(\frac{4S_W g}{\Lambda C_W}\right)\left(\frac{\eta_{\mu\rho}}{M_Z^2}\right)\left(p_{N_j}-p_{\nu_n}\right)_\lambda \left[\overline{u}_{\nu_i}\sigma^{\rho\lambda}((\alpha'_{NB})_{im}P_L-(\alpha'_{NB})^*_{im}P_R)v_{\nu_m}\right] \\
&\left[\overline{u}_{\nu_n}\gamma^\mu(C_{nj}P_L-C^*_{nj}P_R)u_{N_j}\right] \\
\mathrm{i}\mathcal{M}_9 =& -\left(\frac{4S_W g}{\Lambda C_W}\right)\left(\frac{\eta_{\mu\rho}}{M_Z^2}\right)\left(p_{N_j}-p_{\nu_m}\right)_\lambda \left[\overline{u}_{\nu_n}\sigma^{\rho\lambda}((\alpha'_{NB})_{ni}P_L-(\alpha'_{NB})^*_{ni}P_R)v_{\nu_i}\right] \\
&\left[\overline{u}_{\nu_m}\gamma^\mu(C_{mj}P_L-C^*_{mj}P_R)u_{N_j}\right]
\end{aligned}
$$

All the amplitudes that contain an effective dipole are grouped in $\mathcal{M}_{eff}=\mathcal{M}_4+\mathcal{M}_5+\mathcal{M}_6+\mathcal{M}_7+\mathcal{M}_8+\mathcal{M}_9$. For the total averaged amplitude squared, the SM only terms are:

$$
\begin{aligned}
\left\langle|\mathcal{M}_{SM}|^2\right\rangle =& \frac{g^4}{M_Z^4C_W^4}\Big\{(p_N\cdot p_{\nu_n})(p_{\nu_i}\cdot p_{\nu_m})\Big[\left\|C_{ij}\right\|^2\left\|C_{nm}\right\|^2 \\
&+\left\|C_{mj}\right\|^2\left\|C_{ni}\right\|^2+2\mathrm{Re}(C_{ij}C^*_{mj}C^*_{nm}C_{ni})\Big] \\
&+(p_N\cdot p_{\nu_m})(p_{\nu_i}\cdot p_{\nu_n})\Big[\left\|C_{ij}\right\|^2\left\|C_{nm}\right\|^2+\left\|C_{nj}\right\|^2\left\|C_{im}\right\|^2+2\mathrm{Re}(C_{ij}C^*_{nj}C_{nm}C^*_{im})\Big] \\
&+(p_N\cdot p_{\nu_i})(p_{\nu_m}\cdot p_{\nu_n})\Big[\left\|C_{nj}\right\|^2\left\|C_{im}\right\|^2+\left\|C_{mj}\right\|^2\left\|C_{ni}\right\|^2+\mathrm{Re}(C_{nj}C^*_{mj}C^*_{im}C^*_{ni})\Big]\Big\}
\end{aligned}
$$

The effective dipole terms are:

$$
\begin{aligned}
\langle|\mathcal{M}_{eff}|^2\rangle = \frac{32S_W^2 g^2}{C_W^2 M_Z^4}\Bigg\{ \left\|\frac{(\alpha'_{NB})_{nm}}{\Lambda}\right\|^2 \|C_{ij}\|^2 \Big[-4m_N^2(p_{\nu_i}\cdot p_{\nu_m})(p_{\nu_i}\cdot p_{\nu_n}) \\
+ (p_N\cdot p_{\nu_i})\Big(4(p_N\cdot p_{\nu_m})(p_N\cdot p_{\nu_n}) + 4(p_{\nu_i}\cdot p_{\nu_m})(p_{\nu_i}\cdot p_{\nu_n}) - m_N^2(p_{\nu_m}\cdot p_{\nu_n})\Big)\Big] \\
+ \left\|\frac{(\alpha'_{NB})_{im}}{\Lambda}\right\|^2 \|C_{nj}\|^2 \Big[-4m_N^2(p_{\nu_n}\cdot p_{\nu_m})(p_{\nu_n}\cdot p_{\nu_i}) - m_N^2(p_N\cdot p_{\nu_n})(p_{\nu_m}\cdot p_{\nu_i}) \\
+ 4(p_N\cdot p_{\nu_n})(p_N\cdot p_{\nu_m})(p_N\cdot p_{\nu_i}) + 4(p_N\cdot p_{\nu_n})(p_{\nu_n}\cdot p_{\nu_m})(p_{\nu_n}\cdot p_{\nu_i})\Big] \\
+ \left\|\frac{(\alpha'_{NB})_{ni}}{\Lambda}\right\|^2 \|C_{mj}\|^2 \Big[-4m_N^2(p_{\nu_m}\cdot p_{\nu_i})(p_{\nu_m}\cdot p_{\nu_n}) - m_N^2(p_N\cdot p_{\nu_m})(p_{\nu_i}\cdot p_{\nu_n}) \\
+ 4(p_N\cdot p_{\nu_m})(p_N\cdot p_{\nu_i})(p_N\cdot p_{\nu_n}) + 4(p_N\cdot p_{\nu_m})(p_{\nu_m}\cdot p_{\nu_i})(p_{\nu_m}\cdot p_{\nu_n})\Big] \\
+ \left\|\frac{(\alpha'_{NB})_{ij}}{\Lambda}\right\|^2 \|C_{nm}\|^2 \Big[2(p_N\cdot p_{\nu_i})(p_{\nu_i}\cdot p_{\nu_m})\Big((p_{\nu_i}\cdot p_{\nu_n}) + (p_N\cdot p_{\nu_m})\Big) \\
-4m_N^2(p_{\nu_i}\cdot p_{\nu_m})(p_{\nu_i}\cdot p_{\nu_n}) + (p_N\cdot p_{\nu_i})(p_{\nu_m}\cdot p_{\nu_n})\big(m_N^2 - 2(p_N\cdot p_{\nu_i})\big) \\
+ 2(p_N\cdot p_{\nu_i})(p_N\cdot p_{\nu_n})\Big((p_{\nu_i}\cdot p_{\nu_m}) + (p_N\cdot p_{\nu_m})\Big)\Big] \\
+ \left\|\frac{(\alpha'_{NB})_{nj}}{\Lambda}\right\|^2 \|C_{im}\|^2 \Big[2(p_N\cdot p_{\nu_n})(p_{\nu_n}\cdot p_{\nu_m})\Big((p_{\nu_n}\cdot p_{\nu_i}) + (p_N\cdot p_{\nu_m})\Big) \\
-4m_N^2(p_{\nu_n}\cdot p_{\nu_m})(p_{\nu_n}\cdot p_{\nu_i}) + (p_N\cdot p_{\nu_n})(p_{\nu_m}\cdot p_{\nu_i})\big(m_N^2 - 2(p_N\cdot p_{\nu_n})\big) \\
+ 2(p_N\cdot p_{\nu_n})(p_N\cdot p_{\nu_i})\Big((p_{\nu_n}\cdot p_{\nu_m}) + (p_N\cdot p_{\nu_m})\Big)\Big] \\
+ \left\|\frac{(\alpha'_{NB})_{mj}}{\Lambda}\right\|^2 \|C_{ni}\|^2 \Big[2(p_N\cdot p_{\nu_m})(p_{\nu_m}\cdot p_{\nu_i})\Big((p_{\nu_m}\cdot p_{\nu_n}) + (p_N\cdot p_{\nu_i})\Big) \\
-4m_N^2(p_{\nu_m}\cdot p_{\nu_i})(p_{\nu_m}\cdot p_{\nu_n}) + (p_N\cdot p_{\nu_m})(p_{\nu_i}\cdot p_{\nu_n})\big(m_N^2 - 2(p_N\cdot p_{\nu_m})\big) \\
+ 2(p_N\cdot p_{\nu_m})(p_N\cdot p_{\nu_n})\Big((p_{\nu_m}\cdot p_{\nu_i}) + (p_N\cdot p_{\nu_i})\Big)\Big]
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{2}\mathrm{Re}\left[\frac{(\alpha'_{NB})_{ij}}{\Lambda}\frac{(\alpha'_{NB})^{*}_{nj}}{\Lambda}C^{*}_{nm}C_{im}\right]\Big[(p_N\cdot p_{\nu_n})(p_{\nu_i}\cdot p_{\nu_m})\big(m_N^2+2(p_N\cdot p_{\nu_n})\big)\\
&\quad-4(p_N\cdot p_{\nu_i})(p_N\cdot p_{\nu_n})(p_N\cdot p_{\nu_m})+(p_N\cdot p_{\nu_m})(p_{\nu_i}\cdot p_{\nu_n})\big(5m_N^2-2(p_{\nu_i}\cdot p_{\nu_n})\big)\\
&\quad+(p_N\cdot p_{\nu_i})(p_{\nu_m}\cdot p_{\nu_n})\big(m_N^2+(p_N\cdot p_{\nu_i})\big)-4m_N^2(p_{\nu_i}\cdot p_{\nu_m})(p_{\nu_i}\cdot p_{\nu_n})\\
&\quad-2(p_N\cdot p_{\nu_i})(p_N\cdot p_{\nu_n})\Big((p_{\nu_i}\cdot p_{\nu_m})+(p_{\nu_m}\cdot p_{\nu_n})\Big)\\
&\quad-2(p_N\cdot p_{\nu_n})(p_{\nu_i}\cdot p_{\nu_n})\Big((p_N\cdot p_{\nu_m})-(p_{\nu_i}\cdot p_{\nu_m})\Big)\\
&\quad-2(p_N\cdot p_{\nu_i})(p_{\nu_i}\cdot p_{\nu_n})\Big((p_N\cdot p_{\nu_m})-(p_{\nu_m}\cdot p_{\nu_n})\Big)\Big]\\
&+\frac{1}{2}\mathrm{Re}\left(\frac{(\alpha'_{NB})_{ij}}{\Lambda}\frac{(\alpha'_{NB})^{*}_{mj}}{\Lambda}C^{*}_{ni}C_{nm}\right)\Big[(p_N\cdot p_{\nu_m})(p_{\nu_i}\cdot p_{\nu_n})\big(m_N^2+2(p_N\cdot p_{\nu_m})\big)\\
&\quad-4(p_N\cdot p_{\nu_i})(p_N\cdot p_{\nu_m})(p_N\cdot p_{\nu_n})+(p_N\cdot p_{\nu_n})(p_{\nu_i}\cdot p_{\nu_m})\big(5m_N^2-2(p_{\nu_i}\cdot p_{\nu_m})\big)\\
&\quad+(p_N\cdot p_{\nu_i})(p_{\nu_n}\cdot p_{\nu_m})\big(m_N^2+2(p_N\cdot p_{\nu_i})\big)-4m_N^2(p_{\nu_i}\cdot p_{\nu_n})(p_{\nu_i}\cdot p_{\nu_m})\\
&\quad-2(p_N\cdot p_{\nu_i})(p_N\cdot p_{\nu_m})\Big((p_{\nu_i}\cdot p_{\nu_n})+(p_{\nu_n}\cdot p_{\nu_m})\Big)\\
&\quad-2(p_N\cdot p_{\nu_m})(p_{\nu_i}\cdot p_{\nu_m})\Big((p_N\cdot p_{\nu_n})-(p_{\nu_i}\cdot p_{\nu_n})\Big)\\
&\quad-2(p_N\cdot p_{\nu_i})(p_{\nu_i}\cdot p_{\nu_m})\Big((p_N\cdot p_{\nu_n})-(p_{\nu_n}\cdot p_{\nu_m})\Big)\Big]\\
&+\frac{1}{2}\mathrm{Re}\left(\frac{(\alpha'_{NB})_{mj}}{\Lambda}\frac{(\alpha'_{NB})^{*}_{nj}}{\Lambda}C^{*}_{ni}C^{*}_{im}\right)\Big[(p_N\cdot p_{\nu_n})(p_{\nu_m}\cdot p_{\nu_i})\big(m_N^2+2(p_N\cdot p_{\nu_n})\big)\\
&\quad-4(p_N\cdot p_{\nu_m})(p_N\cdot p_{\nu_n})(p_N\cdot p_{\nu_i})+(p_N\cdot p_{\nu_i})(p_{\nu_m}\cdot p_{\nu_n})\big(5m_N^2-2(p_{\nu_m}\cdot p_{\nu_n})\big)\\
&\quad+(p_N\cdot p_{\nu_m})(p_{\nu_i}\cdot p_{\nu_n})\big(m_N^2+2(p_N\cdot p_{\nu_m})\big)-4m_N^2(p_{\nu_m}\cdot p_{\nu_i})(p_{\nu_m}\cdot p_{\nu_n})\\
&\quad-2(p_N\cdot p_{\nu_m})(p_N\cdot p_{\nu_n})\Big((p_{\nu_m}\cdot p_{\nu_i})+(p_{\nu_i}\cdot p_{\nu_n})\Big)\\
&\quad-2(p_N\cdot p_{\nu_n})(p_{\nu_m}\cdot p_{\nu_n})\Big((p_N\cdot p_{\nu_i})-(p_{\nu_m}\cdot p_{\nu_i})\Big)\\
&\quad-2(p_N\cdot p_{\nu_m})(p_{\nu_m}\cdot p_{\nu_n})\Big((p_N\cdot p_{\nu_i})-(p_{\nu_i}\cdot p_{\nu_n})\Big)\Big]\\
&-\frac{1}{2}\mathrm{Re}\left(\frac{(\alpha'_{NB})_{im}}{\Lambda}\frac{(\alpha'_{NB})^{*}_{nm}}{\Lambda}C^{*}_{nj}C_{ij}\right)\Big[4m_N^2(p_{\nu_i}\cdot p_{\nu_n})\Big((p_{\nu_i}\cdot p_{\nu_m})+(p_{\nu_m}\cdot p_{\nu_n})\Big)\\
&\quad-(p_N\cdot p_{\nu_n})(p_{\nu_i}\cdot p_{\nu_m})\big(8(p_{\nu_i}\cdot p_{\nu_n})-m_N^2\big)-5m_N^2(p_N\cdot p_{\nu_m})(p_{\nu_i}\cdot p_{\nu_n})\\
&\quad-(p_N\cdot p_{\nu_i})(p_{\nu_m}\cdot p_{\nu_n})\big(8(p_{\nu_i}\cdot p_{\nu_n})-m_N^2\big)+8(p_N\cdot p_{\nu_m})(p_{\nu_i}\cdot p_{\nu_n})^2\Big]
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{1}{2}\mathrm{Re}\left(\frac{(\alpha'_{NB})_{ni}}{\Lambda}\frac{(\alpha'_{NB})^*_{nm}}{\Lambda}C^*_{mj}C_{ij}\right)\Big[4m_N^2(p_{\nu_i}\cdot p_{\nu_n})\Big((p_{\nu_i}\cdot p_{\nu_m})+(p_{\nu_n}\cdot p_{\nu_m})\Big) \\
&\quad - (p_N\cdot p_{\nu_m})(p_{\nu_i}\cdot p_{\nu_n})\big(8(p_{\nu_i}\cdot p_{\nu_m})-m_N^2\big) - 5m_N^2(p_N\cdot p_{\nu_n})(p_{\nu_i}\cdot p_{\nu_m}) \\
&\quad - (p_N\cdot p_{\nu_i})(p_{\nu_n}\cdot p_{\nu_m})\big(8(p_{\nu_i}\cdot p_{\nu_m})-m_N^2\big) + 8(p_N\cdot p_{\nu_n})(p_{\nu_i}\cdot p_{\nu_m})^2\Big] \\
&+ \frac{1}{2}\mathrm{Re}\left(\frac{(\alpha'_{NB})_{im}}{\Lambda}\frac{(\alpha'_{NB})^*_{ni}}{\Lambda}C^*_{nj}C_{mj}\right)\Big[4m_N^2(p_{\nu_m}\cdot p_{\nu_n})\Big((p_{\nu_m}\cdot p_{\nu_i})+(p_{\nu_i}\cdot p_{\nu_n})\Big) \\
&\quad + (p_N\cdot p_{\nu_n})(p_{\nu_m}\cdot p_{\nu_i})\big(8(p_{\nu_m}\cdot p_{\nu_n})-m_N^2\big) - 5m_N^2(p_N\cdot p_{\nu_i})(p_{\nu_m}\cdot p_{\nu_n}) \\
&\quad + (p_N\cdot p_{\nu_m})(p_{\nu_i}\cdot p_{\nu_n})\big(8(p_{\nu_m}\cdot p_{\nu_n})-m_N^2\big) + 8(p_N\cdot p_{\nu_i})(p_{\nu_m}\cdot p_{\nu_n})^2\Big] \\
&- \frac{1}{2}\mathrm{Re}\left(\frac{(\alpha'_{NB})_{ij}}{\Lambda}\frac{(\alpha'_{NB})^*_{im}}{\Lambda}C^*_{nj}C_{nm}\right)\Big[m_N^2(p_{\nu_i}\cdot p_{\nu_n})\Big(2(p_{\nu_i}\cdot p_{\nu_m})+5(p_N\cdot p_{\nu_m})\Big) \\
&\quad - \Big((p_N\cdot p_{\nu_n})(p_{\nu_i}\cdot p_{\nu_m})-(p_N\cdot p_{\nu_i})(p_{\nu_m}\cdot p_{\nu_n})\Big)\big(4(p_N\cdot p_{\nu_i})-m_N^2\big) \\
&\quad - 4(p_N\cdot p_{\nu_i})(p_N\cdot p_{\nu_m})\Big(2(p_N\cdot p_{\nu_n})+(p_{\nu_i}\cdot p_{\nu_n})\Big)\Big] \\
&+ \frac{1}{2}\mathrm{Re}\left(\frac{(\alpha'_{NB})_{ij}}{\Lambda}\frac{(\alpha'_{NB})^*_{ni}}{\Lambda}C^*_{mj}C^*_{nm}\right)\Big[m_N^2(p_{\nu_i}\cdot p_{\nu_m})\Big(2(p_{\nu_i}\cdot p_{\nu_n})+5(p_N\cdot p_{\nu_n})\Big) \\
&\quad - \Big((p_N\cdot p_{\nu_m})(p_{\nu_i}\cdot p_{\nu_n})-(p_N\cdot p_{\nu_i})(p_{\nu_n}\cdot p_{\nu_m})\Big)\big(4(p_N\cdot p_{\nu_i})-m_N^2\big) \\
&\quad - 4(p_N\cdot p_{\nu_i})(p_N\cdot p_{\nu_n})\Big(2(p_N\cdot p_{\nu_m})+(p_{\nu_i}\cdot p_{\nu_m})\Big)\Big] \\
&- \frac{1}{2}\mathrm{Re}\left(\frac{(\alpha'_{NB})_{nm}}{\Lambda}\frac{(\alpha'_{NB})^*_{nj}}{\Lambda}C^*_{im}C_{ij}\right)\Big[m_N^2(p_{\nu_n}\cdot p_{\nu_i})\Big(2(p_{\nu_n}\cdot p_{\nu_m})+5(p_N\cdot p_{\nu_m})\Big) \\
&\quad - \Big((p_N\cdot p_{\nu_i})(p_{\nu_n}\cdot p_{\nu_m})-(p_N\cdot p_{\nu_n})(p_{\nu_m}\cdot p_{\nu_i})\Big)\big(4(p_N\cdot p_{\nu_n})-m_N^2\big) \\
&\quad - 4(p_N\cdot p_{\nu_n})(p_N\cdot p_{\nu_m})\Big(2(p_N\cdot p_{\nu_i})+(p_{\nu_n}\cdot p_{\nu_i})\Big)\Big] \\
&+ \frac{1}{2}\mathrm{Re}\left(\frac{(\alpha'_{NB})_{nj}}{\Lambda}\frac{(\alpha'_{NB})^*_{im}}{\Lambda}C^*_{nj}C_{ni}\right)\Big[m_N^2(p_{\nu_m}\cdot p_{\nu_n})\Big(2(p_{\nu_m}\cdot p_{\nu_i})+5(p_N\cdot p_{\nu_i})\Big) \\
&\quad - \Big((p_N\cdot p_{\nu_n})(p_{\nu_m}\cdot p_{\nu_i})-(p_N\cdot p_{\nu_m})(p_{\nu_i}\cdot p_{\nu_n})\Big)\big(4(p_N\cdot p_{\nu_m})-m_N^2\big) \\
&\quad - 4(p_N\cdot p_{\nu_m})(p_N\cdot p_{\nu_i})\Big(2(p_N\cdot p_{\nu_n})+(p_{\nu_m}\cdot p_{\nu_n})\Big)\Big] \\
&- \frac{1}{2}\mathrm{Re}\left(\frac{(\alpha'_{NB})_{ni}}{\Lambda}\frac{(\alpha'_{NB})^*_{nj}}{\Lambda}C_{im}C_{mj}\right)\Big[m_N^2(p_{\nu_n}\cdot p_{\nu_m})\Big(2(p_{\nu_n}\cdot p_{\nu_i})+5(p_N\cdot p_{\nu_i})\Big) \\
&\quad - \Big((p_N\cdot p_{\nu_m})(p_{\nu_n}\cdot p_{\nu_i})-(p_N\cdot p_{\nu_n})(p_{\nu_i}\cdot p_{\nu_m})\Big)\big(4(p_N\cdot p_{\nu_n})-m_N^2\big) \\
&\quad - 4(p_N\cdot p_{\nu_n})(p_N\cdot p_{\nu_i})\Big(2(p_N\cdot p_{\nu_m})+(p_{\nu_n}\cdot p_{\nu_m})\Big)\Big]
\end{aligned}
$$

$$
+ \frac{1}{2}\mathrm{Re}\left(\frac{(\alpha'_{NB})_{nm}}{\Lambda}\frac{(\alpha'_{NB})^*_{mj}}{\Lambda}C_{ij}C_{ni}\right)\Big[m_N^2(p_{\nu_m}\cdot p_{\nu_i})\Big(2(p_{\nu_m}\cdot p_{\nu_n})+5(p_N\cdot p_{\nu_n})\Big)
$$
$$
-\Big((p_N\cdot p_{\nu_i})(p_{\nu_m}\cdot p_{\nu_n})-(p_N\cdot p_{\nu_m})(p_{\nu_n}\cdot p_{\nu_i})\Big)\big(4(p_N\cdot p_{\nu_m})-m_N^2\big)
$$
$$
-4(p_N\cdot p_{\nu_m})(p_N\cdot p_{\nu_n})\Big(2(p_N\cdot p_{\nu_i})+(p_{\nu_m}\cdot p_{\nu_i})\Big)\Big]\Bigg\}
$$

The interference terms are:

$$
\langle \mathcal{M}_{SM}\mathcal{M}^*_{eff}+\mathcal{M}^*_{SM}\mathcal{M}_{eff}\rangle = \frac{4g^3S_Wm_N}{C_W^3M_Z^4}\Bigg\{2\left\|C_{nm}\right\|^2\mathrm{Re}\left(\frac{(\alpha'_{NB})_{ij}}{\Lambda}C_{ij}\right)\Big[
$$
$$
(p_N\cdot p_{\nu_n})(p_{\nu_i}\cdot p_{\nu_m})+(p_N\cdot p_{\nu_i})(p_{\nu_m}\cdot p_{\nu_n})+\Big((p_N\cdot p_{\nu_m})-2(p_{\nu_i}\cdot p_{\nu_m})\Big)(p_{\nu_i}\cdot p_{\nu_n})\Big]
$$
$$
+2\left\|C_{im}\right\|^2\mathrm{Re}\left(\frac{(\alpha'_{NB})_{nj}}{\Lambda}C_{nj}\right)\Big[(p_N\cdot p_{\nu_i})(p_{\nu_n}\cdot p_{\nu_m})
$$
$$
+(p_N\cdot p_{\nu_n})(p_{\nu_m}\cdot p_{\nu_i})+\Big((p_N\cdot p_{\nu_m})-2(p_{\nu_n}\cdot p_{\nu_m})\Big)(p_{\nu_n}\cdot p_{\nu_i})\Big]
$$
$$
+2\left\|C_{ni}\right\|^2\mathrm{Re}\left(\frac{(\alpha'_{NB})_{mj}}{\Lambda}C_{mj}\right)\Big[(p_N\cdot p_{\nu_n})(p_{\nu_m}\cdot p_{\nu_i})
$$
$$
+(p_N\cdot p_{\nu_m})(p_{\nu_i}\cdot p_{\nu_n})+\Big((p_N\cdot p_{\nu_i})-2(p_{\nu_m}\cdot p_{\nu_i})\Big)(p_{\nu_m}\cdot p_{\nu_n})\Big]
$$
$$
+\mathrm{Re}\left(C_{ij}\frac{(\alpha'_{NB})_{nj}}{\Lambda}C_{nm}C^*_{im}\right)\Big[3(p_N\cdot p_{\nu_m})(p_{\nu_i}\cdot p_{\nu_n})
$$
$$
+(p_N\cdot p_{\nu_n})(p_{\nu_i}\cdot p_{\nu_m})-\Big(2(p_{\nu_i}\cdot p_{\nu_n})+(p_N\cdot p_{\nu_i})\Big)(p_{\nu_m}\cdot p_{\nu_n})\Big]
$$
$$
+\mathrm{Re}\left(C_{ij}\frac{(\alpha'_{NB})_{mj}}{\Lambda}C_{ni}C^*_{nm}\right)\Big[3(p_N\cdot p_{\nu_n})(p_{\nu_i}\cdot p_{\nu_m})
$$
$$
+(p_N\cdot p_{\nu_m})(p_{\nu_i}\cdot p_{\nu_n})-\Big(2(p_{\nu_i}\cdot p_{\nu_m})+(p_N\cdot p_{\nu_i})\Big)(p_{\nu_n}\cdot p_{\nu_m})\Big]
$$
$$
+\mathrm{Re}\left(C_{nj}\frac{(\alpha'_{NB})_{ij}}{\Lambda}C_{im}C^*_{nm}\right)\Big[3(p_N\cdot p_{\nu_m})(p_{\nu_n}\cdot p_{\nu_i})
$$
$$
+(p_N\cdot p_{\nu_i})(p_{\nu_n}\cdot p_{\nu_m})-\Big(2(p_{\nu_n}\cdot p_{\nu_i})+(p_N\cdot p_{\nu_n})\Big)(p_{\nu_m}\cdot p_{\nu_i})\Big]
$$
$$
+\mathrm{Re}\left(C_{mj}\frac{(\alpha'_{NB})_{nj}}{\Lambda}C_{ni}C_{im}\right)\Big[3(p_N\cdot p_{\nu_i})(p_{\nu_m}\cdot p_{\nu_n})
$$
$$
+(p_N\cdot p_{\nu_n})(p_{\nu_m}\cdot p_{\nu_i})-\Big(2(p_{\nu_m}\cdot p_{\nu_n})+(p_N\cdot p_{\nu_m})\Big)(p_{\nu_i}\cdot p_{\nu_n})\Big]
$$
$$
+\mathrm{Re}\left(C_{nj}\frac{(\alpha'_{NB})_{mj}}{\Lambda}C^*_{ni}C^*_{im}\right)\Big[3(p_N\cdot p_{\nu_i})(p_{\nu_n}\cdot p_{\nu_m})
$$

$$+ (p_N \cdot p_{\nu_m})(p_{\nu_n} \cdot p_{\nu_i}) - \Big(2(p_{\nu_n} \cdot p_{\nu_m}) + (p_N \cdot p_{\nu_n})\Big)(p_{\nu_i} \cdot p_{\nu_m})\Big]$$

$$+ \mathrm{Re}\left(C_{mj}\frac{(\alpha'_{NB})_{ij}}{\Lambda}C_{nm}C^*_{ni}\right)\Big[3(p_N \cdot p_{\nu_n})(p_{\nu_m} \cdot p_{\nu_i})$$

$$+ (p_N \cdot p_{\nu_i})(p_{\nu_m} \cdot p_{\nu_n}) - \Big(2(p_{\nu_m} \cdot p_{\nu_i}) + (p_N \cdot p_{\nu_m})\Big)(p_{\nu_n} \cdot p_{\nu_i})\Big]$$

$$+ 2\mathrm{Re}\left(C_{ij}C_{nj}C^*_{nm}\frac{(\alpha'_{NB})_{im}}{\Lambda}\right)\Big[(p_N \cdot p_{\nu_i})(p_{\nu_m} \cdot p_{\nu_n}) - (p_N \cdot p_{\nu_m})(p_{\nu_i} \cdot p_{\nu_n})\Big]$$

$$- 2\mathrm{Re}\left(C_{ij}C_{mj}C_{nm}\frac{(\alpha'_{NB})_{ni}}{\Lambda}\right)\Big[(p_N \cdot p_{\nu_i})(p_{\nu_n} \cdot p_{\nu_m}) - (p_N \cdot p_{\nu_n})(p_{\nu_i} \cdot p_{\nu_m})\Big]$$

$$+ 2\mathrm{Re}\left(C_{ij}C_{nj}C^*_{im}\frac{(\alpha'_{NB})_{nm}}{\Lambda}\right)\Big[(p_N \cdot p_{\nu_n})(p_{\nu_m} \cdot p_{\nu_i}) - (p_N \cdot p_{\nu_m})(p_{\nu_n} \cdot p_{\nu_i})\Big]$$

$$- 2\mathrm{Re}\left(C_{mj}C_{nj}C^*_{ni}\frac{(\alpha'_{NB})_{im}}{\Lambda}\right)\Big[(p_N \cdot p_{\nu_m})(p_{\nu_i} \cdot p_{\nu_n}) - (p_N \cdot p_{\nu_i})(p_{\nu_m} \cdot p_{\nu_n})\Big]$$

$$+ 2\mathrm{Re}\left(C_{mj}C_{nj}C_{im}\frac{(\alpha'_{NB})_{ni}}{\Lambda}\right)\Big[(p_N \cdot p_{\nu_n})(p_{\nu_i} \cdot p_{\nu_m}) - (p_N \cdot p_{\nu_i})(p_{\nu_n} \cdot p_{\nu_m})\Big]$$

$$- 2\mathrm{Re}\left(C_{ij}C_{mj}C_{ni}\frac{(\alpha'_{NB})_{nm}}{\Lambda}\right)\Big[(p_N \cdot p_{\nu_m})(p_{\nu_n} \cdot p_{\nu_i}) - (p_N \cdot p_{\nu_n})(p_{\nu_m} \cdot p_{\nu_i})\Big]\Bigg\}$$

The mass of the light neutrinos is negligible, and therefore is being taken as zero. With this approximation, several simplifications occur when integrating the amplitudes, and the decay width for the process $N_j \to \nu_i \nu_m \nu_n$ would be:

$$\Gamma = \frac{G_F^2\, m_N^5\, S_{imn}}{192\pi^3}\Bigg(\|C_{ij}\|^2\,\|C_{nm}\|^2 + \|C_{mj}\|^2\,\|C_{ni}\|^2 + \|C_{nj}\|^2\,\|C_{im}\|^2$$

$$+ \mathrm{Re}(C_{ij}C_{ni}C^*_{mj}C^*_{nm}) + \mathrm{Re}(C_{ij}C^*_{nj}C_{nm}C^*_{im}) + \mathrm{Re}(C_{nj}C^*_{mj}C^*_{im}C^*_{ni})\Bigg)$$

$$+ \frac{G_F\, S_W^2\, m_N^7\, S_{imn}}{48\sqrt{2}\,\pi^3 M_Z^2}\Bigg\{0.4\,\mathrm{Re}\Big[\frac{(\alpha'_{NB})_{ij}}{\Lambda}\frac{(\alpha'_{NB})^*_{nj}}{\Lambda}C_{im}C^*_{nm}$$

$$+ \frac{(\alpha'_{NB})_{ij}}{\Lambda}\frac{(\alpha'_{NB})^*_{mj}}{\Lambda}C_{nm}C^*_{ni} + \frac{(\alpha'_{NB})_{mj}}{\Lambda}\frac{(\alpha'_{NB})^*_{nj}}{\Lambda}C^*_{ni}C^*_{im}\Big]$$

$$+ 1.6\left[\left\|\frac{(\alpha'_{NB})_{ij}}{\Lambda}\right\|^2\|C_{nm}\|^2 + \left\|\frac{(\alpha'_{NB})_{nj}}{\Lambda}\right\|^2\|C_{im}\|^2 + \left\|\frac{(\alpha'_{NB})_{mj}}{\Lambda}\right\|^2\|C_{ni}\|^2\right]$$

$$+ 0.6\left[\left\|\frac{(\alpha'_{NB})_{nm}}{\Lambda}\right\|^2\|C_{ij}\|^2 + \left\|\frac{(\alpha'_{NB})_{im}}{\Lambda}\right\|^2\|C_{nj}\|^2 + \left\|\frac{(\alpha'_{NB})_{ni}}{\Lambda}\right\|^2\|C_{mj}\|^2\right]$$

$$
\begin{aligned}
& - 0.1\,\mathrm{Re}\Big[\frac{(\alpha'_{NB})^*_{nm}}{\Lambda}C_{ij}(\frac{(\alpha'_{NB})_{im}}{\Lambda}C^*_{nj} + \frac{(\alpha'_{NB})_{ni}}{\Lambda}C^*_{mj}) - \frac{(\alpha'_{NB})_{im}}{\Lambda}\frac{(\alpha'_{NB})^*_{ni}}{\Lambda}C_{mj}C^*_{nj}\Big] \\
& + 0.6\,\mathrm{Re}\Big[\frac{(\alpha'_{NB})^*_{im}}{\Lambda}C^*_{nj}(\frac{(\alpha'_{NB})_{ij}}{\Lambda}C_{nm} - \frac{(\alpha'_{NB})_{mj}}{\Lambda}C_{ni}) - \frac{(\alpha'_{NB})_{ij}}{\Lambda}\frac{(\alpha'_{NB})^*_{ni}}{\Lambda}C^*_{mj}C^*_{nm} \\
& + \frac{(\alpha'_{NB})_{nm}}{\Lambda}C_{ij}(\frac{(\alpha'_{NB})^*_{nj}}{\Lambda}C^*_{im} - \frac{(\alpha'_{NB})^*_{mj}}{\Lambda}C_{ni}) + \frac{(\alpha'_{NB})_{ni}}{\Lambda}\frac{(\alpha'_{NB})^*_{nj}}{\Lambda}C_{im}C_{mj}\Big]\Big\} \\
& + \frac{G_F\, m_N^6\, \mathrm{e}\, S_{imn}}{192\sqrt{2}\,\pi^3\, C_W\, M_Z^2}\,\mathrm{Re}\Bigg[\frac{(\alpha'_{NB})_{nj}}{\Lambda}(C_{ij}C_{nm}C^*_{im} + C_{mj}C_{ni}C_{im}) \\
& + \frac{(\alpha'_{NB})_{mj}}{\Lambda}(C_{ij}C_{ni}C^*_{nm} + C_{nj}C^*_{im}C^*_{ni}) + \frac{(\alpha'_{NB})_{ij}}{\Lambda}(C_{nj}C_{im}C^*_{nm} + C_{mj}C_{nm}C^*_{ni}) \\
& + 2\Big(\|C_{nm}\|^2\,\frac{(\alpha'_{NB})_{ij}}{\Lambda}C_{ij} + \|C_{im}\|^2\,\frac{(\alpha'_{NB})_{nj}}{\Lambda}C_{nj} + \|C_{ni}\|^2\,\frac{(\alpha'_{NB})_{mj}}{\Lambda}C_{mj}\Big)\Bigg]
\end{aligned}
$$

$$
S_{inm} = \begin{cases} 1, & \text{if: } i \neq m \neq n \\ \frac{1}{2}, & \text{if: } (i = m) \veebar (m = n) \veebar (n = i) \\ \frac{1}{6}, & \text{if: } i = m = n. \end{cases}
$$

where S_{imn} is the statistic factor for identical particles in the final state. Making the sum over all final states $\nu_i\nu_n\nu_m$ we get that the total decay width for the process $N_j \to \nu\nu\nu$ is :

$$
\begin{aligned}
\Gamma(N_j \to \nu\nu\nu) =& \frac{G_F^2\, m_N^5}{384\pi^3}\sum_{i,n,m}\Big(\|C_{ij}\|^2\,\|C_{nm}\|^2 + \mathrm{Re}(C_{ij}C_{ni}C^*_{mj}C^*_{nm})\Big) \\
& + \frac{G_F\, S_W^2\, m_N^7}{96\sqrt{2}\,\pi^3 M_Z^2}\sum_{i,n,m}\Bigg[1.6\left\|\frac{(\alpha'_{NB})_{ij}}{\Lambda}\right\|^2\|C_{nm}\|^2 + 0.6\left\|\frac{(\alpha'_{NB})_{nm}}{\Lambda}\right\|^2\|C_{ij}\|^2 \\
& + \mathrm{Re}\Big(0.4\frac{(\alpha'_{NB})_{ij}}{\Lambda}\frac{(\alpha'_{NB})^*_{nj}}{\Lambda}C_{im}C^*_{nm} - 0.1\frac{(\alpha'_{NB})_{im}}{\Lambda}\frac{(\alpha'_{NB})^*_{nm}}{\Lambda}C_{ij}C^*_{nj} \\
& + 1.2\frac{(\alpha'_{NB})_{ij}}{\Lambda}\frac{(\alpha'_{NB})^*_{im}}{\Lambda}C_{nm}C^*_{nj}\Big)\Bigg] \\
& + \frac{G_F\, m_N^6\, \mathrm{e}}{192\sqrt{2}\,\pi^3\, C_W\, M_Z^2}\sum_{i,n,m}\mathrm{Re}\Big(\|C_{nm}\|^2\,\frac{(\alpha'_{NB})_{ij}}{\Lambda}C_{ij} + \frac{(\alpha'_{NB})_{nj}}{\Lambda}C_{ij}C_{nm}C^*_{im}\Big)
\end{aligned}
$$

The choice of parameters for this plot are the same as in Figure 5. Comparing the widths, only the result for large values of $(\alpha'_{NB})_{56}/\Lambda$ and large neutrino mass are distinguishable form the standard Seesaw.

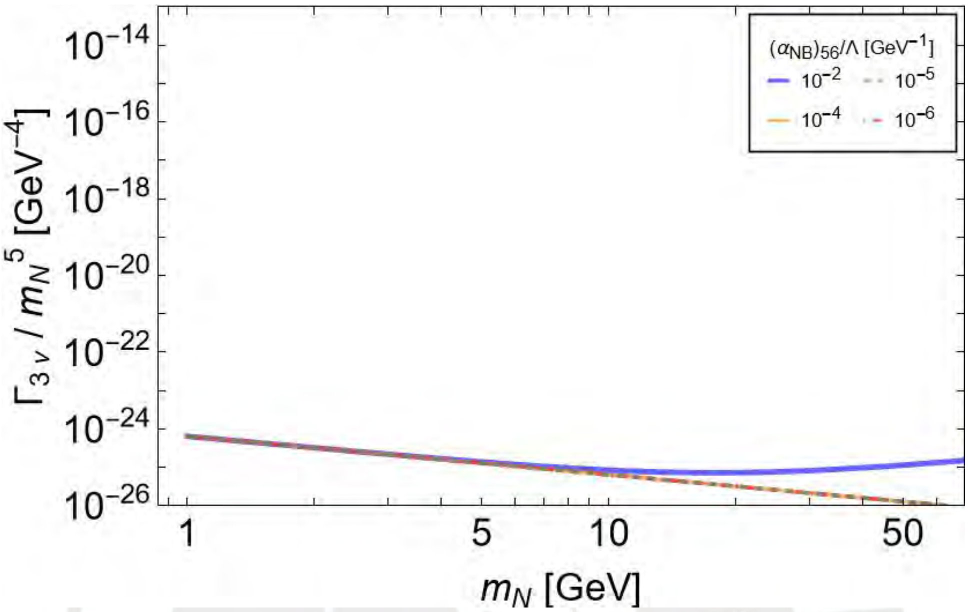


Figure 14: Heavy neutrino partial width into three light neutrinos, when including effective operators, normalized with respect to m_N^5, the widths plotted corresponding to $(\alpha'_{NB})_{56}/\Lambda = 10^{-2}, 10^{-4}, 10^{-5}, 10^{-6}$ GeV^{-1} are being shown with thick blue, thin orange, red dotted and green lines respectively. The gray dashed line indicate the standard Seesaw three-body decay to light neutrinos.

Chapter 4

Analysis and Conclusion

Besides from the three-body decays detailed previously, in this model the heavy neutrino has another channel of decay, which is the two-body decay into on shell photon, $N_j \to \nu_i \gamma$. The total decay for this processes is given by[7]:

$$\Gamma(N_j \to \nu\gamma) = \frac{2}{\pi} C_W^2 \sum_i \left| \frac{(\alpha'_{NB})_{ij}}{\Lambda} \right|^2 \tag{4.1}$$

Comparing the width of this channel, showed in Figure 15, with the ones for the three-body decays, it seems that it can dominate the total heavy neutrino decay width. To understand the balance between the widths, it is better to analyze the branching ratios of the different channels, shown in Figure 16.

As one can see on the lower right panel of Fig 16, when taking $(\alpha'_{NB})_{56}/\Lambda \geq 10^{-4}\,\text{GeV}^{-1}$ the two body decay dominates the total width, and its branching ratio is close to one. However for smaller $(\alpha'_{NB})_{56}/\Lambda$ this value can be much lower. For example for $(\alpha'_{NB})_{56}/\Lambda = 10^{-5}\,\text{GeV}^{-1}$ the branching ratio fall under 50% for $M_h \geq 20\,\text{GeV}$. And for $(\alpha'_{NB})_{56}/\Lambda = 10^{-6}\,\text{GeV}^{-1}$, this

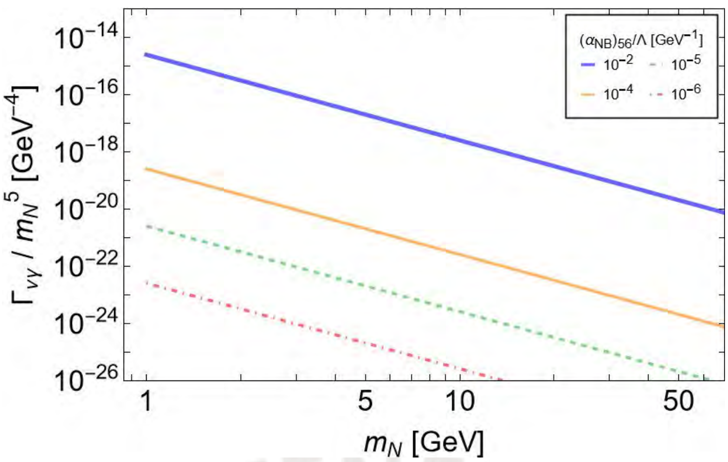


Figure 15: Heavy neutrino partial width for the two-body decay, normalized with respect to m_N^5, with the same choice of parameters as in Figure 5.

occurs for $M_h \geq 2.5\,$GeV.

For the three-body decays, the branching ratio to quarks is the largest contribution as can be seen in the upper left panel. However this contribution is still low , under 10%, for small masses of N_5. For larger values of m_N the branching ratio increase significantly, up to 50%. The branching ratio to only neutrinos is the lowest and only noticeable for $(\alpha'_{NB})_{56}/\Lambda < 10^{-4}\text{GeV}^{-1}$ this is because the partial width to neutrinos does not increase significantly with the addition of effective operators, so this branching ratio is only visible when the effects of the dimension five operators vanishes, which is the case for small $(\alpha'_{NB})_{56}/\Lambda$ or very large masses.

For all the three-body decays channels, the smaller the value of $(\alpha'_{NB})_{56}/\Lambda$ is, the bigger their contribution to the total decay is. This can be understood by considering that even though for smaller $(\alpha'_{NB})_{56}/\Lambda$ the contributions from the effective operator decrease, the standard Seesaw processes remain the same, giving a minimum contribution to the three body decays only.

Another point of comparison is the neutrino lifetime, Figure 17 presents the expected decay length $c\tau$ of the heavy neutrino in presence of the new couplings. The panel in the left show results for minimal mixing, and on the right a choice of parameters leading to an

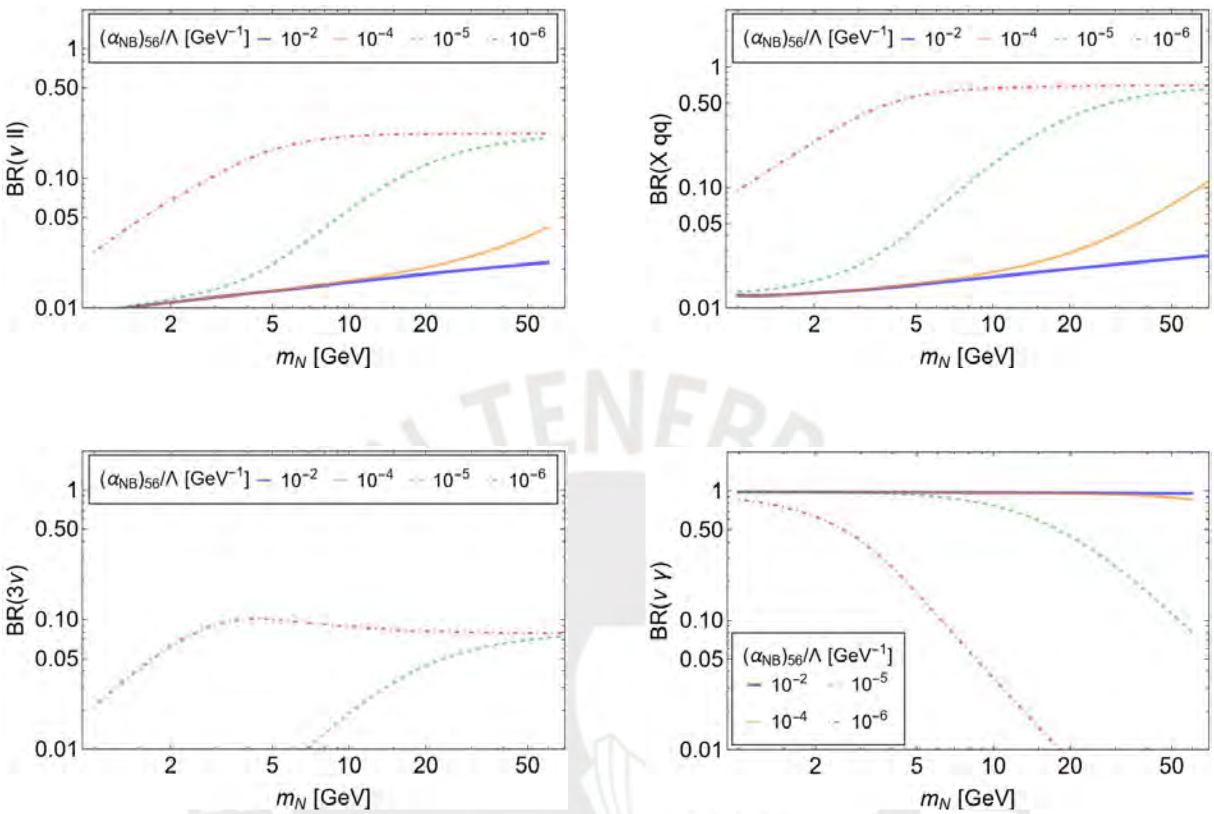


Figure 16: Heavy neutrino branching ratios. From left to right top to bottom, we show the branching ratios for $N_j \to Xqq$, $N_j \to \nu ll$, $N_j \to \nu\nu\nu$, $N_j \to \nu\gamma$. Each panel shows the branching ratio corresponding to $(\alpha'_{NB})_{56}/\Lambda = 10^{-2}, 10^{-4}, 10^{-5}, 10^{-6}$ GeV^{-1}.

enhancement of 10^3 in the sterile-light mixing squared. The gray lines represent the decay length only considering the two body decay. As expected, for $(\alpha'_{NB})_{56}/\Lambda \geq 10^{-4}\,\text{GeV}^{-1}$ one can take the lifetime just from the two body decay. In contrast, for smaller $(\alpha'_{NB})_{56}/\Lambda$ without the addition of the three-body decays, the decay length can many orders of magnitude larger. Enhancing the neutrino mixing decrease the decay length.

Lastly, Figure 18 shows the proportional contributions of the new terms in the three-body decays to the total decay length of the neutrino. Analyzing the graph, the trends that can be observed are that for a fixed mass, the deviation increases with $(\alpha'_{NB})_{56}/\Lambda$ and in all cases the inclusion of these terms leads to a reduction of the total decay length, however this reduction is only of a few percent of the total. This suggests, that although necessary for an accurate result,

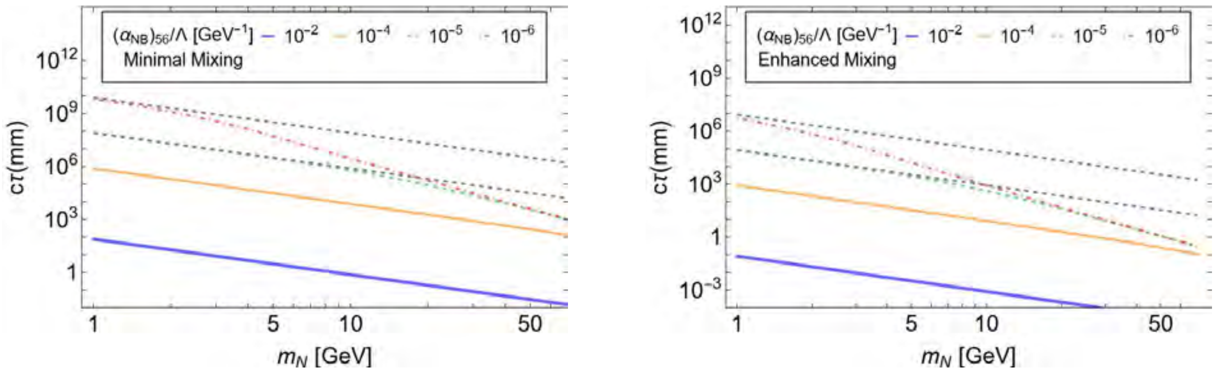


Figure 17: Heavy neutrino decay length, gray lines indicate the expected decay length when considering only two-body decays. Left: minimal mixing. Right: enhanced sterile-light mixing.

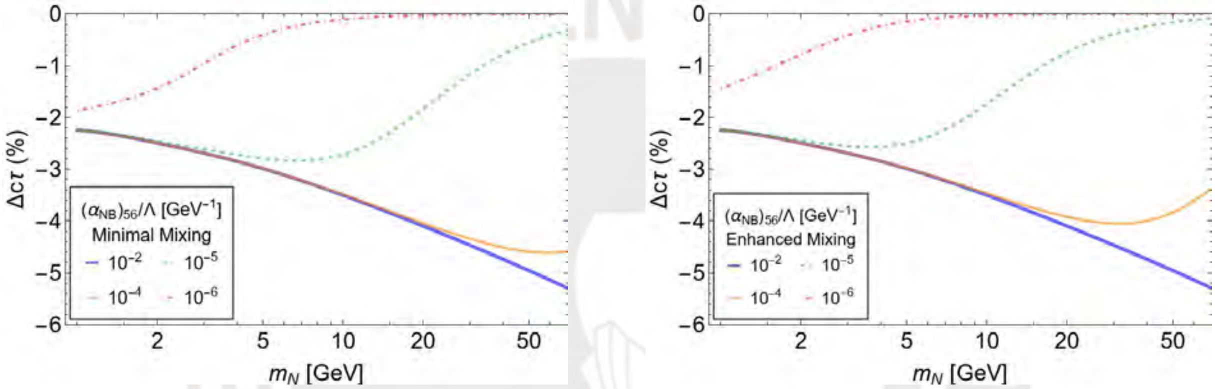


Figure 18: Difference in decay length when performing the full calculation, versus including only two-body decays and the standard Seesaw three-body decays. Left: minimal mixing. Right: enhanced sterile-light mixing.

it is possible to exclude them form calculations if precision is not of the utmost importance.

As a conclusion, in this thesis we have used a model that adds three right handed Majorana neutrinos with masses in a GeV scale to the standard model, using a type I seesaw Lagrangian, and dimension five effective operators. We then derived formulas to calculate the decay widths for different amplitudes. And using this formulas, we calculated the three-body partial decay widths of a heavy neutrino. We also compared the modified partial widths to the standard Seesaw results, for different choices of couplings. Finally we analyzed the impact of these new partial widths to the total decay length of the heavy neutrino.

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