# Pontificia Universidad Católica del Perú

# Escuela de Posgrado

Investigation of different compliant mechanisms

Tesis para obtener el grado académico de Maestro en Ingeniería Mecánica que presenta:

*Alexander Javier Sea Ordaya*

Asesor PUCP (PUCP) : *Jorge Hernán Alencastre Miranda*

Co-Asesor Technical University of Munich (TUI) : *Lena Zentner*

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|---|---|
| DNI: 10588073 | Firma |
| ORCID: 0000-0001-8442-8255 | |

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Alexander Javier Sea Ordaya

## Abstract

The use of flexible mechanisms has been increasing in the recent years. This is due of the advantages that offer with respect to rigid mechanism. However, the method to study them become more complicated. Just to mention, the mathematical equations to investigate their behavior are of a higher level, so the assistance of calculation software is required.

For this reason an analytical model is developed to facilitate the study of compliant mechanism. To do this, a review of the theory involved in the calculation of compliant mechanism is made and applied in the development of the analytical model. In this case the equations derived from the Euler-Bernoulli Beam Theory were used. In order to have a contrast of results, a software that use the Finite Element Method was used.

After the simulation of the structures in both models, the relative errors were obtained. In the case of the simulations to obtain the displacement due to external forces, only the first node of the structures were defined as "Clamp" and the external forces were applied to the rest of the nodes. For this investigation, nine structures were simulated. To obtain a representative relative error, an average of them were calculated, which are 5.69% and 4.28% in the x and y axis respectively. Subsequently, because in all the cases, the first node is fixed, the moment at that node were obtained and compared. The average relative error found was 1.74%. After that, the simulations to obtain the forces due to the rotation of the structures were carried out. To make the simulations, the structures rotate in the range of -0.4 $rad$ and 0.4 $rad$ with steps of 0.1 $rad$. For this investigation, 4 structures were simulated and 14 parameters were defined. The maximal relative error found was 5.05%.

Finally, an study of the behavior of the normalized moment at the first node of the 4 structures when the 14 parameters vary was carried out. In this investigation it was found on one hand that the parameters $R1$ and $R2$ from the structure 3 have good influence in the curves but on the other hand, the parameters $R1$ and $\alpha$ from the structure 4 have almost no influence.

## Resumen

El uso de los mecanismos flexibles ha ido en aumento en los últimos años. Esto se debe a las ventajas que ofrece con respecto a los mecanismos rígidos. Sin embargo, los métodos para estudiarlos es mucho más complicado. Solo para mencionar, las ecuaciones matemáticas para investigar su comportamiento son de un nivel superior, por lo tanto, se necesita usar un software para realizar los cálculos.

Por esta razón se ha desarrollado un modelo analítico para facilitar el estudio de mecanismos flexibles complejos. Para ello, se realiza una revisión de la teoría implicada en el cálculo de los mecanismos flexibles y se aplica en el desarrollo del modelo analítico. En este caso, fueron usadas las ecuaciones derivadas de la teoría de la viga Euler-Bernoulli. Con el fin de contrastar los resultados, se usó un software que utiliza el método de los elementos finitos.

Después de la simulación de las estructuras en ambos modelos, se obtuvieron los errores relativos. En el caso de las simulaciones para obtener los desplazamientos debido a fuerzas externas, solo los primeros nodos de la estructuras se definieron como "Clamp" y las fuerzas externas fueron aplicadas en el resto de los nodos. Para esta investigación nueve estructuras fueron simuladas. Para obtener un error relativo representativo, se calcularon los promedio de ellos, los cuales son 5.69 % y 4.28 % en los eje x y y respectivamente. Seguidamente, debido a que en todos lo casos el primer nodo se encuentra fijo, se calcularon y compararon los momentos en dichos nodos. El error relativo promedio encontrado fue de 1.74 %. Después de eso, se llevaron a cabo las simulaciones para obtener las fuerzas debido a la rotación de la estructura. Para realizar las simulaciones, la estructura gira entre -0.4 rad y 0.4 rad con pasos de 0.1 rad. Para esta investigación, cuatro estructuras fueron simuladas y 14 parámetros fueron definidos. El máximo error relativo encontrado fue de 5.05 %.

Finalmente, se llevó a cabo un estudio del comportamiento del momento normalizado en el primer nodo de las cuatro estructuras cuando los 14 parámetros varían. En esta investigación se encontró que, por un lado, los parámetros R1 y R2 de la estructura 3 tienen una buena influencia en las curvas pero, por otro lado, los parámetros R1 y $\alpha$ de la estructura 4 casi no tienen influencia.

## Kurzfassung

Die Anwendung von nachgiebigen Mechanismen hat in den letzten Jahren zugenommen. Dies liegt an den Vorteilen, die sie im Vergleich zu starren Mechanismen bieten. Allerdings wird die Methode zu ihrer Untersuchung immer komplizierter. So sind die mathematischen Gleichungen zur Untersuchung ihres Verhaltens auf einem höheren Niveau, so dass die Unterstützung durch Berechnungssoftware erforderlich ist.

Aus diesem Grund wird ein analytisches Modell entwickelt, um die Untersuchung von nachgiebigen Mechanismen zu erleichtern. Zu diesem Zweck wird ein Überblick über die Theorie zur Berechnung des nachgiebigen Mechanismus gegeben und bei der Entwicklung des analytischen Modells angewendet. In diesem Fall wurden die Gleichungen, die aus der Euler-Bernoulli-Balkentheorie abgeleitet, verwendet. Um einen Vergleich der Ergebnisse zu ermöglichen, wurde eine Software verwendet, die die Finite-Elemente-Methode anwendet.

Nach der Simulation der Strukturen in beiden Modellen wurden die relativen Fehler ermittelt. Bei den Simulationen zur Ermittlung der Verschiebung aufgrund externer Kräfte wurde nur der erste Knoten der Strukturen als "Clamp" definiert und die externen Kräfte wurden auf die anderen Knoten angewendet. Für diese Untersuchung wurden neun Strukturen simuliert. Um einen repräsentativen relativen Fehler zu erhalten, wurde ein Mittelwert berechnet, der 5,69% und 4,28% in der x- bzw. y-Achse beträgt. Weil in allen Fällen der erste Knotenpunkt fixiert ist, wurden die Momente an diesem Knotenpunkt ermittelt und verglichen. Der durchschnittliche relative Fehler beträgt 1,74%. Danach wurden die Simulationen zur Ermittlung der Kräfte bei der Rotation der Strukturen durchgeführt. Für die Simulationen wurden die Strukturen in einem Bereich von -0,4 $rad$ und 0,4 $rad$ mit Schritten von 0,1 $rad$ gedreht. Für diese Untersuchung wurden 4 Strukturen simuliert und 14 Parameter definiert. Der maximal ermittelte relative Fehler beträgt 5,05%.

Schließlich wurde eine Studie über das Verhalten des normalisierten Moments am ersten Knoten der 4 Strukturen durchgeführt, während die 14 Parameter verändert. In dieser Untersuchung wurde einerseits festgestellt, dass die Parameter $R1$ und $R2$ aus der Struktur 3 einen guten Einfluss auf die Kurven haben, andererseits haben die Parameter $R1$ und $\alpha$ aus der Struktur 4 fast keinen Einfluss.

# Contents

## List of Abbreviations

| | |
|---|---|
| **CAD** | Computer-Aided-Design |
| **FEM** | Finite-Elemente-Methode |
| **GUI** | Graphical User Interface |
| **TBT** | Thimoshenko Beam Theory |
| **EBBT** | Euler-Bernoulli Beam Theory |
| **PRBM** | Pseudo Rigid-Body Model |
| **SSBLM** | Stephenson six-bar Linkage Mechanism |
| **SEA** | Series Elastic Actuator |
| **SVP** | Saint-Venant's Principle |
| **CM** | Compliant mechanism |
| **VSC** | Visual Studio Code |

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## List of Symbols

| Symbols | Definition | Unit |
|---|---|---|
| $E$ | Elastic modulus | $N/mm^2$ |
| $F$ | Generic Force | N |
| $F_R$ | Resultant force | N |
| $F_x$ | Force in x-direction | N |
| $F_y$ | Force in y-direction | N |
| $\tilde{F}$ | Normalized Force | - |
| $\triangle F_x$ | Force in x-direction in nodefeature matrix | N |
| $\triangle F_y$ | Force in y-direction in nodefeature matrix | N |
| $g$ | Acceleration of the gravity | $N/m^2$ |
| $h$ | Height | mm |
| $I$ | Moment of inertia | $mm^4$ |
| $I_x$ | Moment of inertia with respect to x-axis | $mm^4$ |
| $I_y$ | Moment of inertia with respect to y-axis | $mm^4$ |
| $I_z$ | Moment of inertia with respect to z-axis | $mm^4$ |
| $L$ | Beam's length | mm |
| $m/s$ | Meters per second | m/s |
| $M_e$ | External Bending moment at the last node of the structure | N mm |
| $M_z$ | Bending moment with respect to z-axis | N mm |
| $\tilde{M}$ | Normalized moment | - |
| $w$ | Width | mm |
| $W$ | Weight of an external mass or generic force | N |
| $X$ | Position of the node in x direction | m |
| $Y$ | Position of the node in y direction | m |
| $X_0$ | Initial position of the node in x direction | mm |
| $Y_0$ | Initial position of the node in y direction | mm |
| $\triangle X$ | Displacement of the node in x direction | mm |
| $\triangle Y$ | Displacement of the node in y direction | mm |
| $\alpha_F$ | Angle of the force with respect to the x - axis | rad |
| $\delta_a$ | Deformation of the beam due to axial force | mm |
| $\delta_b$ | Deformation of the beam due to bending force | mm |
| $\epsilon$ | General strain | - |
| $\theta_3$ | Angle rotated around z-axis | $^\circ$ |
| $\kappa$ | Curvature of a beam | $mm^-1$ |
| $\nu$ | Poisson's ratio | - |
| $\rho$ | Radius of curvature | mm |
| $\sigma$ | General stress | $N/mm^2$ |

# 1 Introduction

## 1.1 Motivation

According to Howell [13], a Compliant mechanism (CM) consists of elements that have the ability to bend in order to fulfill a specific task. This definition is simple but at the same time very extensive. The use of flexible elements can be found in any field of engineering and even in everyday life. Moreover, the mathematical calculations required to determine the deformations or stresses derived from external forces and the specific boundary conditions are generally more complex than in the case of using rigid elements. Therefore, software assistance is required to perform the analytical solutions. The development of an analytical model is important because it parametrizes the different variables that influence the behavior of the CM. In this way, changing one parameter is enough to obtain a new mechanism and new results. Also, the use of an analytical model reduces the time to obtain data, if we compare it with another method such as the Finite-Elemente-Methode (FEM) which requires more computational resources. Therefore, it facilitates the calculation process to optimize a compliant mechanism.

## 1.2 Specification of the task

As it can be seen in the Fig. 1.1, in the nature there are several examples of compliant elements. For instance, the vertebral column has to be compliant in order to let us move. The elephant's trunks, with which they feed, collect object and make sounds, have to be compliant as well. The body of the eels has to be compliant in order to be able to swim. Even insect's wings have to be compliant in order to let them fly. In addition to that, in the plant kingdom, it is important for algae and flowers to be compliant, otherwise they could be damaged by sea or air currents, respectively.

However, for the purposes of this work, we will consider compliant elements that have applications in engineering, more specifically, those that have the ability to support external stresses, storing the energy that is applied through those external forces, and then releasing it. This is the same way a torsional spring works, however, it is intended to obtain a nonlinear-behavior, in order to take the benefits that nonlinearity can provide [14, 15, 22]. When these elements come together, they are called CM.

For this reason, flexure hinges will not be taken into account since they are used to replace rigid revolute joints [20]. Flexure hinges are also ideal for high precision applications, but at the present work we will focus in the description of the displacement-rotation characteristics at certain points in the CM due to external forces and moments. In addition to that, because most of the applications for CM are made in a plane, the analytical model that will be performed in the present work will also be developed in the plane.

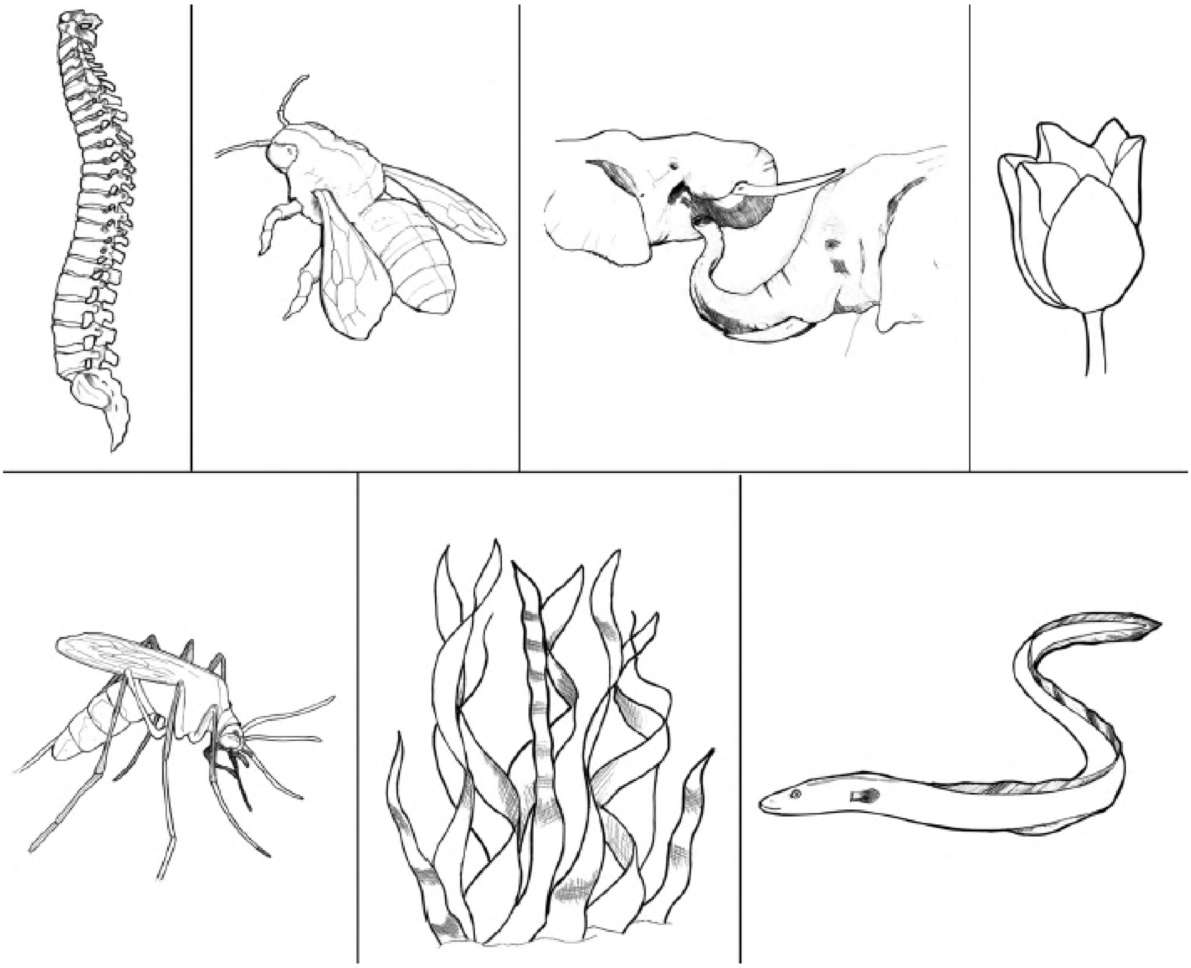


Figure 1.1: Example of compliant elements in the nature [13]

## 1.3 Delimitation of the topic

The present research work is focused on the study of CM. In order to achieve this, a study was carried out firstly on the existing applications of flexible mechanisms, specially on those that are able to store energy. After that, a study of the theory corresponding to the prediction of the behavior of the CM under certain loads was carried out, in order to be able to design the analytical model. It is important that the analytical result are contrasted by another method, therefore, a FEM model must be performed through the ANSYS software. In Summary, the tasks performed in the present work are as follows:

- Research on the topics of compliant mechanism

- Selection of the best fitting mechanism for a given characteristic
- Build a FEM-model in Ansys and an analytical model in Julia.
- Variation of the parameters to match the given characteristic

### 1.4 Clarification of terms

The following terms are going to be used throughout this work so a short explanation is presented:

- Beams are the elements that conform a CM. This elements have a constant area and material. If it is not indicated, they are considered to be straight. Furthermore, they are considered homogeneous and isotropic, therefore, they have a constant modulus of elasticity.
- Nodes are the joints of the beams. They are important because the boundary conditions are defined on them, that means, that the external forces are only applied at the nodes. In addition to that, the imperfections due to manufacturing are not considered in the calculation process.
- Structures are the composition of beams, nodes and the boundary conditions. In its initial state, the structures can not be subjected to stress.
- External forces are referred both for forces and moments. When it is not indicated, they are supposed to be combined.

# 2 State of art

In this chapter, analytical models and last investigations that describe the behavior of compliant mechanism in engineering are going to be presented. Also it is intended to show that for some specific applications, compliant mechanisms have a better performance than linear-behavior springs.
An investigation about improving energy efficiency through nonlinear elements was carried out in [22]. Making a study at different walking speeds of the bipedal robot, it was demonstrated, that it is preferred to use a compliant mechanism as an energy store device than a lineal torsional spring. A bipedal robot must be autonomous, that is, it must carry its own energy source, e.g., an electrical battery. However, the battery has a limited energy capacity, therefore, the bipedal robot has to be efficient with its energy use. When this is achieved, this will result in first place in a longer distance traveled with only one charge. In addition to that, the energy that is saved in the robot's gait can be used in other important actuators. Additionally, having a better energy performance also means less environmental impact. All this result in a better performance for the task it was built.

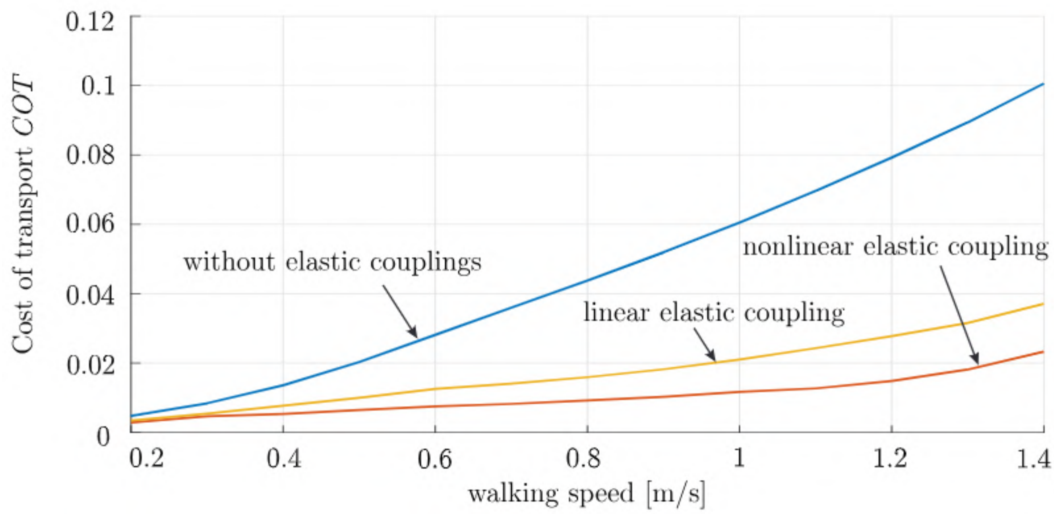



Figure 2.1: Cost of transport for different walking speeds [22]

A way to measure energy consumption is the cost of transportation $COT = E/mgL$, which is defined here as the energy supplied by the motors divided by the mass of the robot and the length traveled, where $E$ is the electrical energy supplied by the motors, $m$ is the mass of the system, $g$ corresponds to the gravity acceleration and $L$ corresponds to the distance traveled. With this, the amount of energy that the electric motors must recover in each cycle will be measured. In Fig. 2.1 it is clearly seen, that in the range of 0.2 $m/s$ and 1.4 $m/s$ the use of

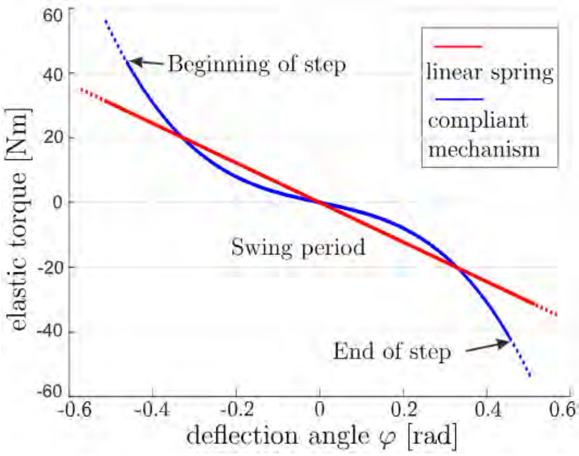
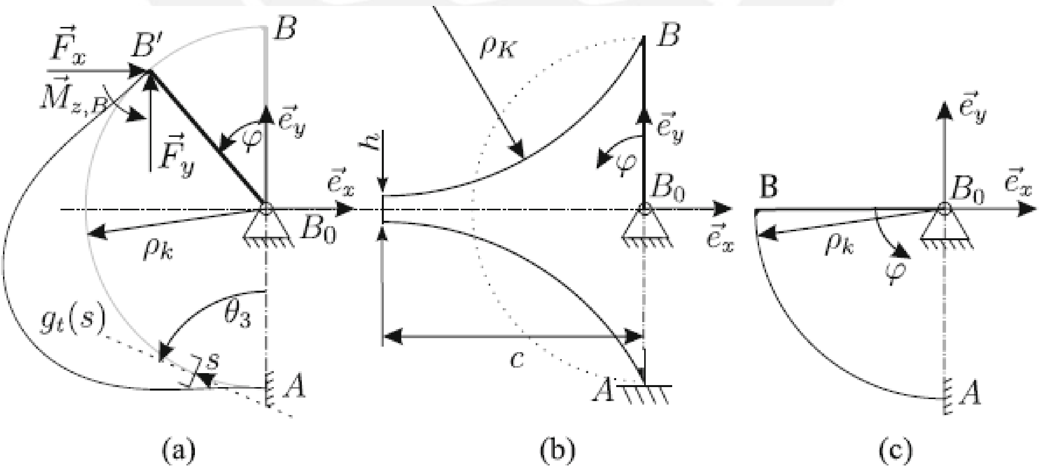



Figure 2.2: Springs Characteristics [23]

nonlinear elastic coupling has a better performance than the use of a linear elastic coupling, that means that the latter needs more energy consumption.
In [22] it was determined too that an average of 78% energy is saved by installing a nonlinear coupling system, while a linear coupling system saves only an average of 62% energy. The comparison between both types of behavior can be seen in Fig. 2.2. In that figure it is noticeable, that in certain deflection angle, a lower absolute value of torque is required by the compliant mechanism than the linear spring. When the deflection angle exceeds that limit, the advantages of that compliant mechanism are lost. So, in order to reduce the energy consumption, a compliant mechanism with the blue line type behavior is preferred instead of a linear torsional spring in a certain working range.




(a) a semicircle, (b) a semirhombus and (c) a quartercircle

Figure 2.3: The compliant mechanism that are modeled in [31]

In [31], the concept of database in order to create a different kind of compliant mechanism was introduced. That means, that different kinds of beams and their boundary conditions

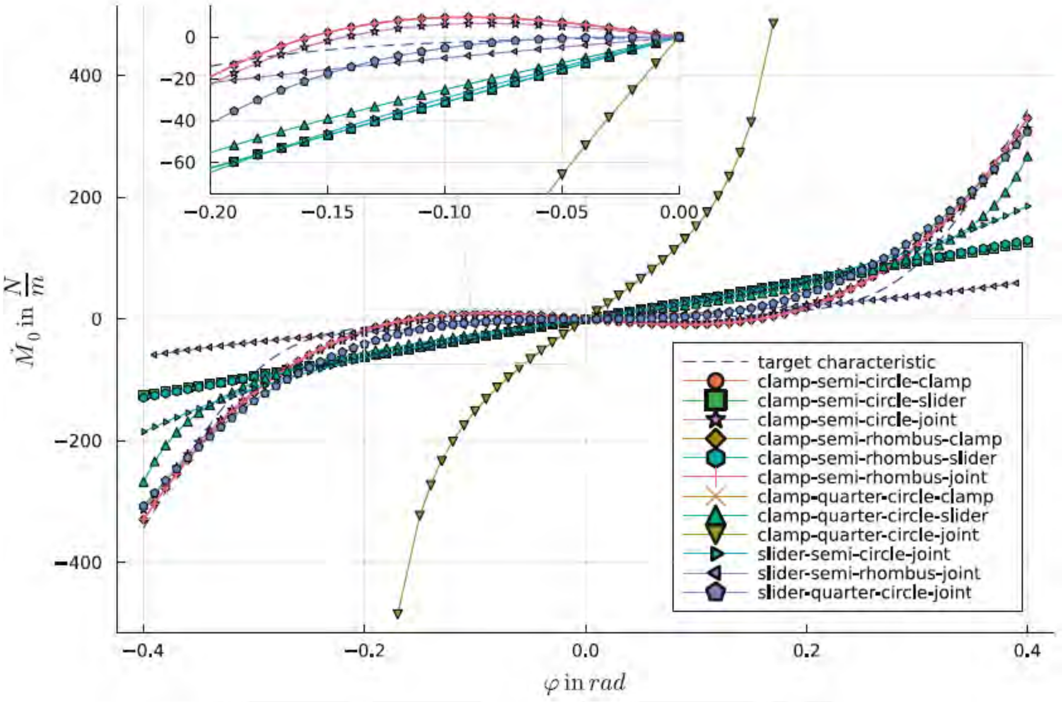



Figure 2.4: The characteristic curve of 12 different kinds of compliant mechanisms compared to the optimal target characteristic [31]

are modeled. As depicted in Fig. 2.3, three different kinds of compliant mechanism are modeled. The first compliant mechanism refers to semicircle curved beam, the second compliant mechanism refers to a semi-rhombus, which is composed of two curved beams and a short beam, and finally, the third compliant refers to a quarter of circle. In all the cases, the boundary conditions are interchangeable between fixed supports, hinged supports or sliders, and they are deflected due to external forces and moments. Depending on what type of boundary condition is used, different kind of results are obtained. It is important to select correctly the boundary condition that best fit to the physical conditions in a real application. In order to determine the displacement due to the external forces and moments, an analytical model based on the Euler-Bernoulli Beam Theory (EBBT) for large deformation is used. As clearly depicted in 2.4, 12 new compliant mechanisms, which have been created by changing the boundary conditions, have been simulated and put in the same graphic in order to compare their behavior to an optimal target characteristic. Those results were compared latter using a FEM model in Ansys. The maximal relative errors found were 7,5% for the torque and 6,5% for the displacement.
In [24], an existing planar Stephenson six-bar Linkage Mechanism (SSBLM) (Fig. 2.5) is going to be optimized by the combination and use of large deflections compliant joints, topology optimization and fuse filament deposition modelling technique, which is an specific type of additive manufacturing technique. In this case, the compliant joints were used because of the several advantages they have, just to mention, the reduction of weight, friction and back-lash. Moreover, the topology optimization was used to reduced the inertia of the mechanism.

After the application of this approach, a new SSBLM was built. This new model can be seen in Fig. 2.6. It is important to mention that the PRBM was used, in order to assure that the virtual joints coincide with the real ones. In this new SSBLM several advantages were found. First of all, the weight of the components were reduced (from 40% to 93%). Not only because of the use of additive manufacturing materials like Carbon Fiber Reinforced Nylon instead of aluminum, but also because the total number of elements were reduced. For instance, 12 ball-bearings were eliminated. In addition to that, the friction because of the ball bearings were eliminated. As a result of the optimization, the reduction of the required motor torque was achieved. It was reported that the Root Mean Square was reduced by 97% and the peak motor torque value was reduced by 96%.

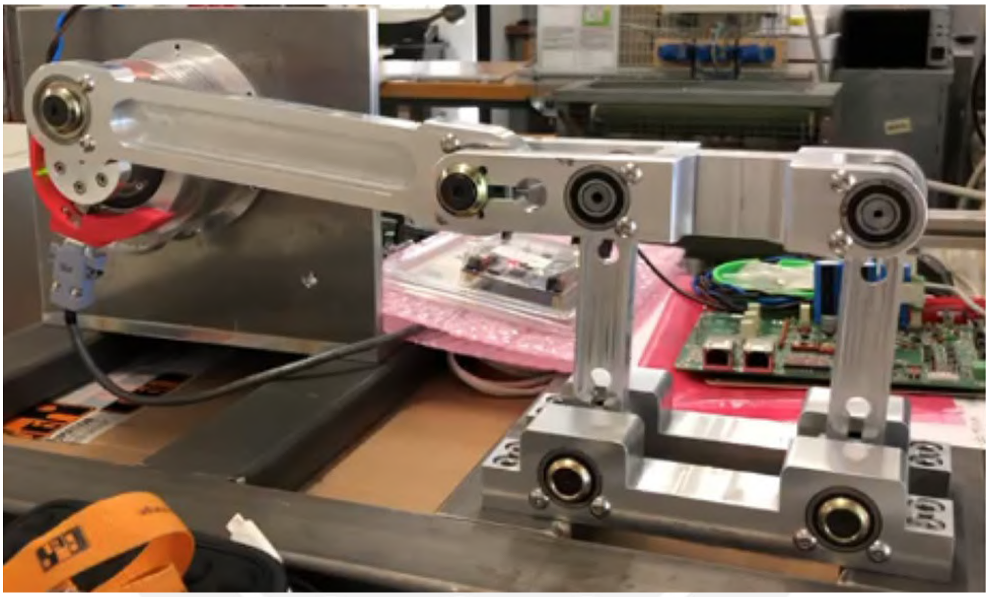
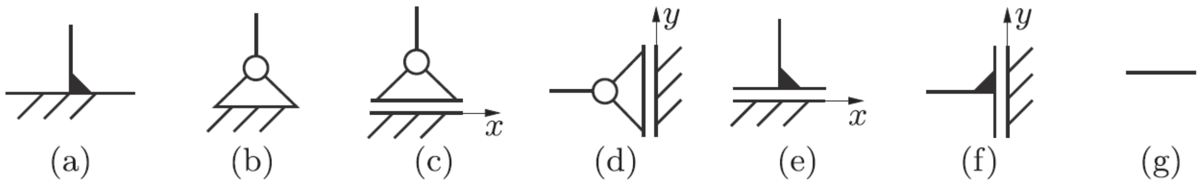


Figure 2.5: Stephenson six-bar linkage prototype [24]

Figure 2.6: Stephenson six-bar linkage PRBM [24]

An important step is the modeling of the system. Modeling is a representative simplification of a real system, in which the components, boundary conditions and other properties are preserved, so that the corresponding analysis can be carried out. It is useful because the existing mathematical tools are able to be used only the aspects that we want to study are took into consideration. For instance, some models already developed are the Elliptic Integral Solution [25], Chained-Beam-Constraint-Model [2] and PRBM [6, 26], which have their own characteristics and conditions to use. More examples of models can be seen in [10, 18]. In Fig. 2.7 the sketched representation of some boundary conditions used in engineering are illustrated. In the present work, we are going to focus on fixed support and hinges.




(a) Fixed support, (b) hinged support, (c) hinged support with slider in x -direction, (d) hinged support with slider in y-direction, (e) slider in x -direction, (f) slider in x-direction and (g) free end

Figure 2.7: Sketched representations of boundary conditions [10]

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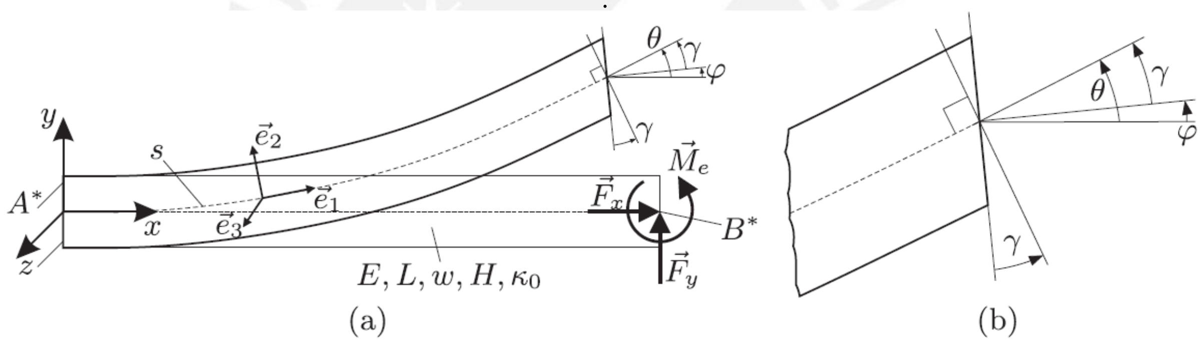



(a) Beam in its undeflected and deflected state (b) magnification of the beam's end

Figure 2.8: Deformation of the beam due to deflection and shear effects [10]

In [10], a study of planar compliant mechanisms was developed. In this study, the authors developed and integrated the theory of shear and considered the lateral contractions in EBBT for large deflections, which then can be applied individually or in combination in each element of a compliant mechanism through a Graphical User Interface (GUI). The theory of shear was developed based on Thimoshenko Beam Theory (TBT). According to this theory, it is no longer considered that the cross section remains perpendicular to the longitudinal axis. That means, that an additional deformation due to the shear forces is taken into account [21]. In this study, these effects are implemented in the system of equations 3.8, 3.9, 3.10 and , 3.11, which are going to be introduce in the section 3.1. This theory has the disadvantage that its effects are negligible when the length of the beam is ten times greater than the dimensions of the cross-section. Furthermore, the increase of accuracy than can be gained is counterbalanced by making the equations more complicated due to the introduction of new elements such as the shear angle and a cross-section correction factor. An Illustration of a deflected beam, taking

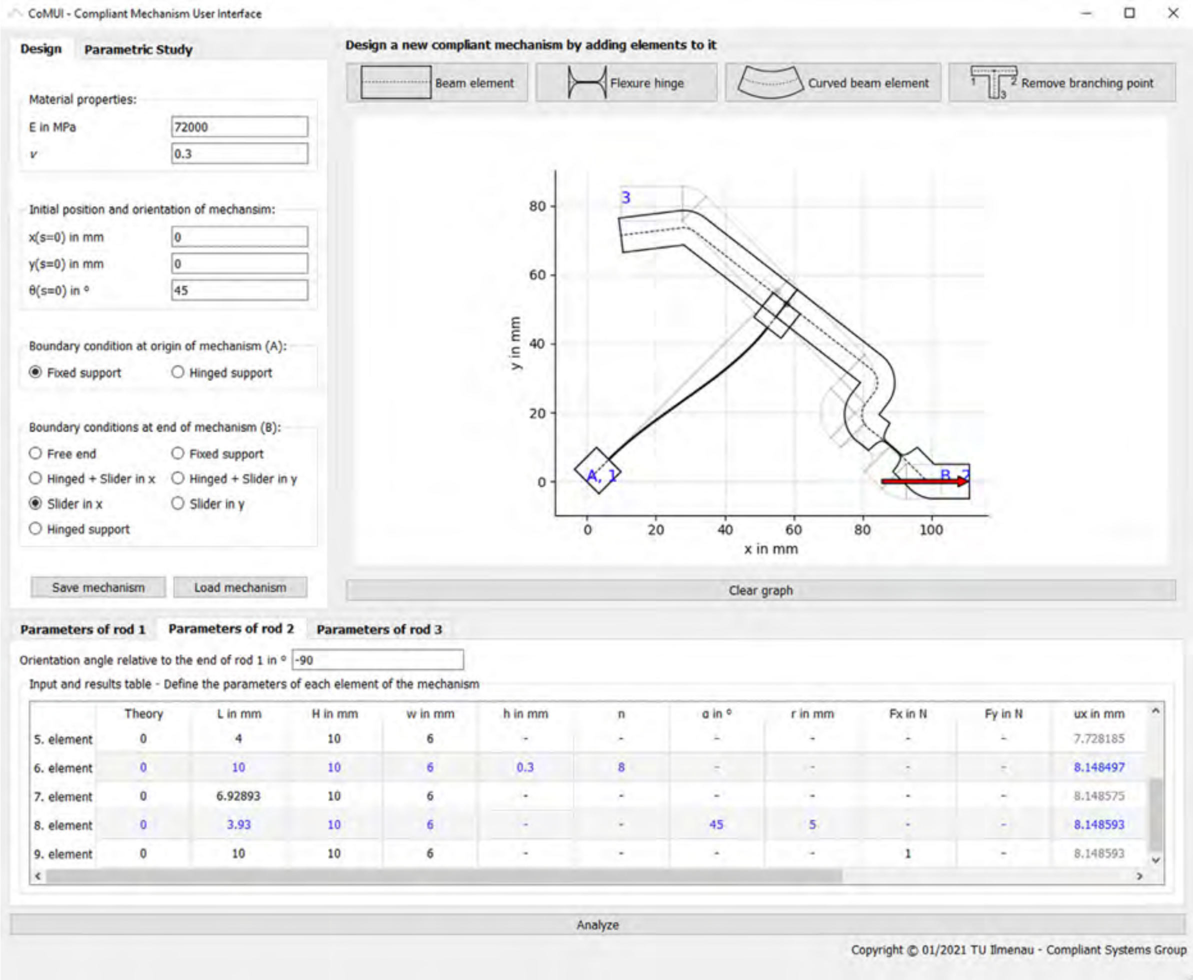



Figure 2.9: Screenshot of the Graphical Used Interface in Python [10]

into consideration the shear effects can be seen in the Fig. 2.8. Moreover, in the theory of lateral contractions, not only the stress in the applied force direction is taken into account, but also the stress in lateral direction. Here, the Hooke's Law (See Eq. 3.4) is used to predict those lateral effects. As in the previous theory, those effects are introduced in the equations 3.8, 3.9, 3.10 and , 3.11. However, this theory was developed to be applied not in beams, but in plates, which are not the object of study in this work.

Furthermore, a GUI was developed in Python for a fast analytical calculation. An screenshot of its interface is visible in Fig. 2.9. In that interface, the representation of the compliant mechanism can be seen in the middle of the screen. The value of the material properties, like the module of elasticity ($E$) and Poisson's ratio ($\nu$), the boundary conditions at the beginning and at the end of the beam and the geometrical properties, like the length ($L$), width ($w$) and height ($h$) of each beam, are needed in order to make the calculations and obtain the required outputs. The novelty of this interface is, that it is possible to change the theory that would be applied in each beam, for example, when pure bending theory is required, the number 0 must be introduced in the “Theory” field. In the case, that the theory of shear or the theory of lateral contractions are required, the numbers 1 and 2 can be used, respectively. However, this GUI only allows the use of one branch, which limits its scope to solve and simulate other types of structures. Because of verification, the results obtained by the GUI are contrasted to the result of FEM, MATLAB and, in some cases, with the results already measured from previous works [8, 19].

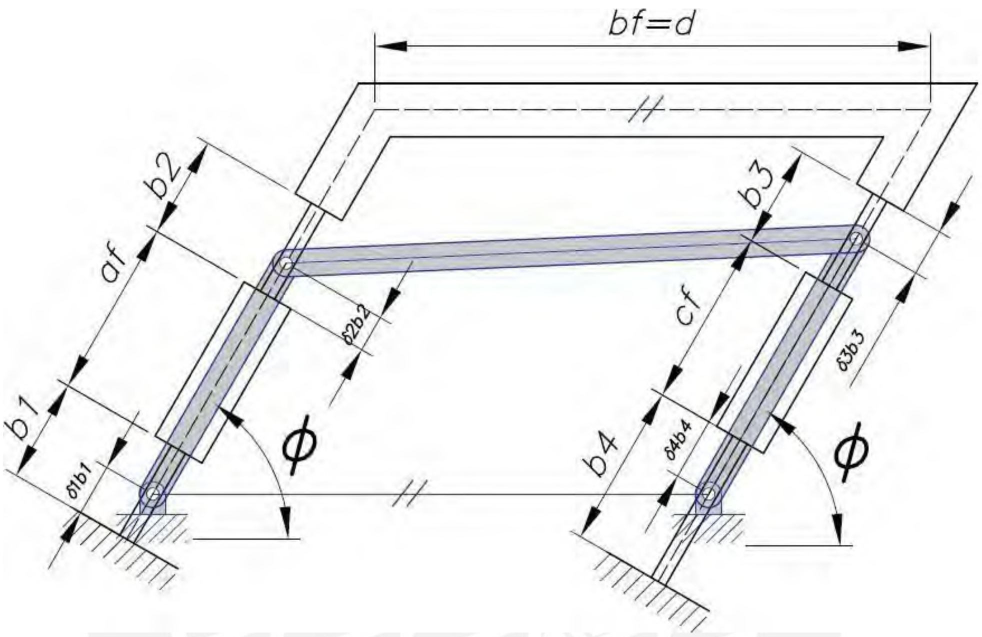



Figure 2.10: Superposed geometries of a rigid and a compliant four-bar linkage [12]

An analysis work of flexure hinges was made in [20]. In this work, the analysis calculation are based on the equation 3.8, 3.9, 3.10 and , 3.11. Here the authors make the calculations taking into account different kind of geometries of the flexure hinges, that could have circular, corner-filleted, elliptical or a variable power function contour. To facilitate the calculation procedure, a GUI in MATLAB called detasFLEX [9] was developed. In this interface, material properties, type of contour, geometrical parameters and other kind of setting can be added. A representation of the flexure hinge and some graphics can be generated through detasFLEX. With the aim of validating the analytical results, a comparison with FEM was made. The deviation of the bending stiffness depends on the type of contour. For example, in [9], it was reported that for a deflection angle of 10° in a corner-filleted contour the deviation is in the range of 0.1% and 9.4%.
Continuing with [12], a study of four-bar linkage as compliant mechanism is presented. Here, the author developed a GUI in MATLAB in order to find an analytical solution for the obtained equations based on Castigliano’s theorem. The Castigliano’s theorem is an energetic method used for the analysis of displacements and deformations of structures by using deformation energy equations. The four-bar linkage analysed in this study is depicted in Fig. 2.10. The aim of this work is to find the geometrical dimensions from a compliant mechanism by introducing

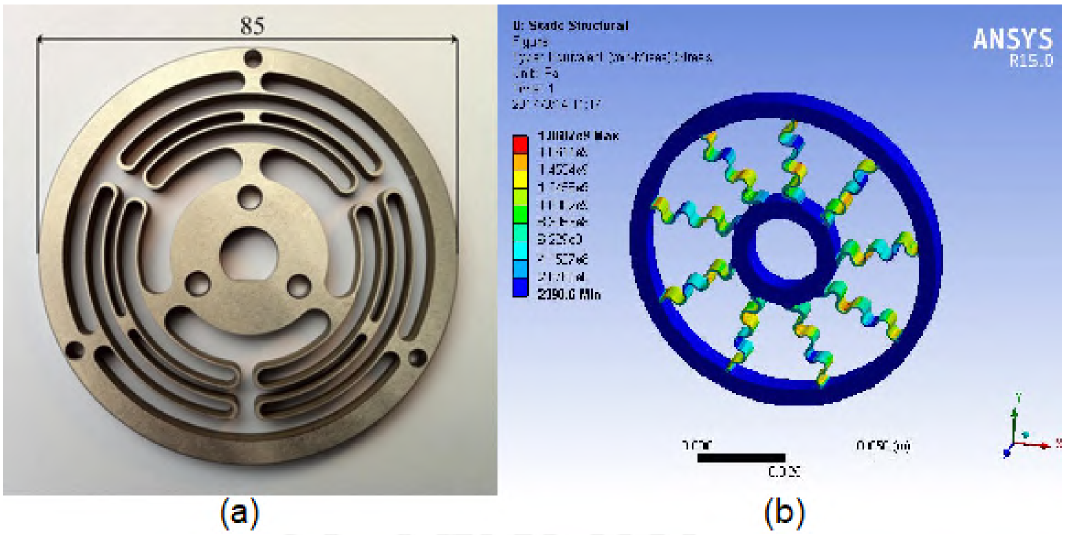
the geometrical dimensions from the original rigid four-bar linkage, the applied force and the material properties. For verification purposes, the obtained results were compared with another developed program that uses the non-linear theory [11], finding that the error between both analyses are lesser than 0.5%. However, even though the same four-bar linkage structure is being used, the developed GUI of this work only allow two types of boundaries conditions, when the applied force is at the top of the structure and when the force is applied to one side. This is because, in the Castigliano's method, when we want to analyze a structure that is under another type of boundary conditions, the new energy equations have to be calculated.


(a) Torsional spring and (b) Rotary SEA

Figure 2.11: Customize spring and SEA from [27]

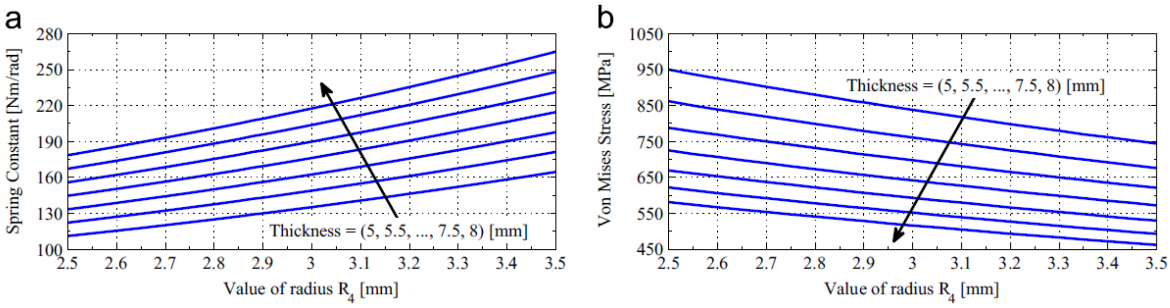
Figure 2.12: Final assembly of the SEA [5]

An adequate and well-diffused use for compliant mechanism are the SEA. The SEA consist basically in a gear-motor and an elastic element, which are connected to a load [29]. Of course the elastic element is on focus and compliant mechanisms are ideal for this purpose. The SEA have several applications in robotics [3, 4, 29], and in biomedical rehabilitation devices [5, 16, 17, 27, 28].




(a) Frontal view of torsional spring from [3] and (b) internal structure of torsional spring modeled in ANSYS from [29]

Figure 2.13: Examples of the internal geometry of the torsional spring




(a) Geometry parameters vs. Stiffness and (b) geometry variations vs. stress

Figure 2.14: Variation of stiffness and stress in the torsional spring by changing geometrical parameters [27]

The SEA offers several advantages. Firstly, they are compact, meaning that they require a minimal space for installation (Fig. 2.12 and 2.11). In addition to that, the elastic element is easily mountable and removable, facilitating maintenance work when it is necessary. Another advantage is the ability to increase the stiffness by installing an additional elastic element in parallel. For example, when it is needed to duplicate the stiffness of the system, an identical elastic element has to be installed. In Fig. 2.13 it can be seen, that the torsional springs have various structural forms, which have influence in the stiffness of the structure. This implies, that the geometric measures can be parameterized and by varying, for example, the width, the length or curvature of the torsional spring components, it becomes possible to achieve different stiffness behaviors. In Fig. 2.14 it is depicted, that the stiffness and internal stress of the torsional spring change by varying the geometrical measurements, in this case the radius of curvature an thickness of the internal element of the torsional spring from Fig. 2.11.

# 3 Development of the Analytical Model

## 3.1 Theoretical Framework

In this section we will briefly review some considerations about the CM and explain of factors have an influence in their flexibility. Then, the differential equations that describe the behavior that a beam suffers due to the application of external forces will be explained. After that, an explanation of some concepts and how the analytical model works is made. An finally, the results obtained from this model are going to be presented.
As is was mention in the chapter 2, the CM have several advantages when they are compared to rigid bodies. Some of those advantages [13] are summarized below:

- Reduction of the maintenance and lubrication requirements.
- Reduction of the number of components.
- Reduction of wear of internal components.
- Reduction or elimination of the backlash.
- Reduction of the weight of the system.
- Increase of the performance, which also means, high precision.

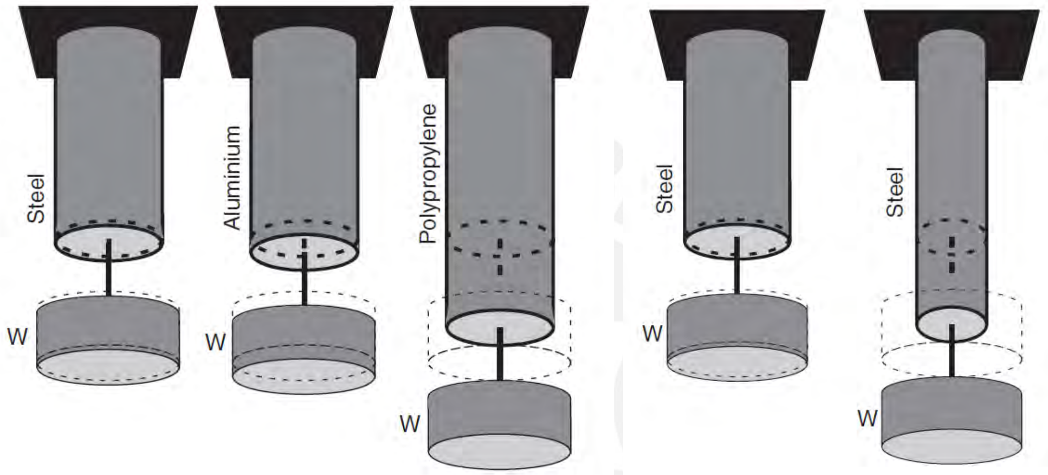
All this advantages can be traduced in reduction of cost. On the other hand, it is important to take into consideration the followings challenges on CM [13]:

- High complexity.
- Simplified linear equations are not adequated to describe the behavior of the deformation of the beam when the deflections occur in the non-linear range.
- Fatigue loads reduces the life time.
- Stress concentration may appear due to bending stresses.
- The motion is limited.

As it has been said before, a CM is an element which bends in order to fulfill a specific function. So, in this case, it is more important to take into consideration the flexibility instead of the resistance. The flexibility can be influenced by [13]:

1. The properties of the material.
2. The geometry.
3. The boundary conditions.

In the Fig. 3.1a it can be seen, that the properties of the material have an influence in the deformation of the beam. In this case, the steel has a higher $E$ than the aluminium and the polypropylene, so when the same external force is applied, the steel bar has a lower elongation than the aluminium and polypropylene bar. In the Fig. 3.1b it can be seen also, that the bar with less cross-sectional area has a higher elongation. This behaviors can be explained with the equation 3.1, which is used for axial loads, where both $E$ and $A$ are inversely proportional to $\vec{\delta_a}$. The values of the elastic modulus of some materials can be found in the table 3.1.




(a) Influence of the material properties (b) Influence of the geometry

Figure 3.1: Description of the influence in the flexibility [13]

$$\vec{\delta_a} = \frac{\vec{W}L}{EA} \tag{3.1}$$

Table 3.1: Elastic modulus of different kinds of materials [1]

| Material | Elastic modulus ($N/mm^2$) |
|---|---|
| Steel | $(1,9-2,14)\cdot 10^5$ |
| Pure Iron | $2,12\cdot 10^5$ |
| Grey Cast Iron | $(0,63-1,3\cdot 10^5$ |
| Aluminium | $(0,69-0,72)\cdot 10^5$ |
| Lead | $(0,16-0,20)\cdot 10^5$ |
| Copper | $1,24\cdot 10^5$ |
| Wolfram | $3,55\cdot 10^5$ |
| Glass | $(0,4-0,9)\cdot 10^5$ |
| Polystyrene | $4,0\cdot 10^3$ |
| Epoxy | $3,5\cdot 10^3$ |

Another example of how the geometry influences in the flexibility of a beam can be described with the Fig. 3.2 and the equation 3.2. In that equation, it can be seen the relation of the deformation ($\vec{\delta_b}$) of the beam with the external vertical force ($\vec{F}$). When the length ($L$) of the beam is doubled, then, the value of the deformation of the end-beam will be multiplied by eight. In addition to that, taking into consideration the moment of inertia ($I$) in the equation 3.3, when the width ($w$) of the cross-sectional area of the beam is doubled, assuming that the cross-sectional area is a rectangle, then the deformation of the end-beam will be divided by two and when the height ($h$) of the cross-sectional area is doubled, then, the deformation of the beam will be divided by eight.

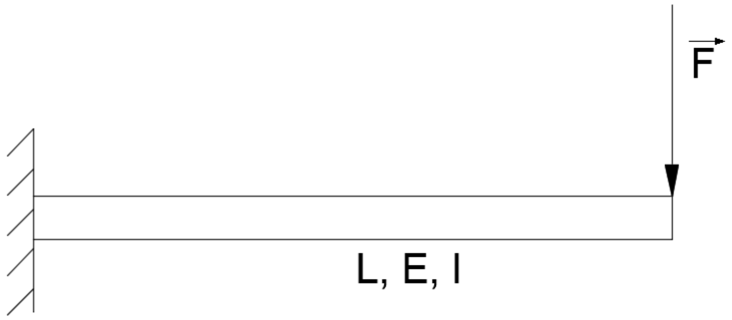



Figure 3.2: Embedded beam

$$\vec{\delta_b} = \frac{\vec{F}L^3}{3EI} \tag{3.2}$$

$$I = \frac{1}{12}wh^3 \tag{3.3}$$

In the Fig. 3.3, it can be seen that the boundary conditions have an influence in the deformation, too. Depending on how the beam is attached and how the direction of the forces are applied, the resultant deformation will vary.

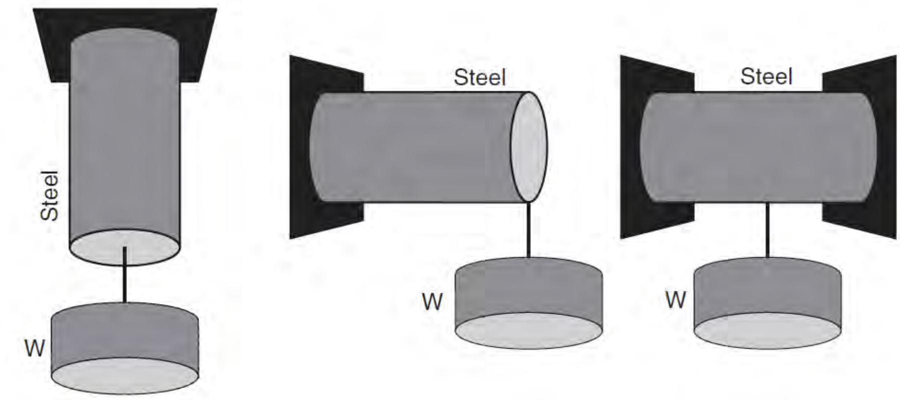


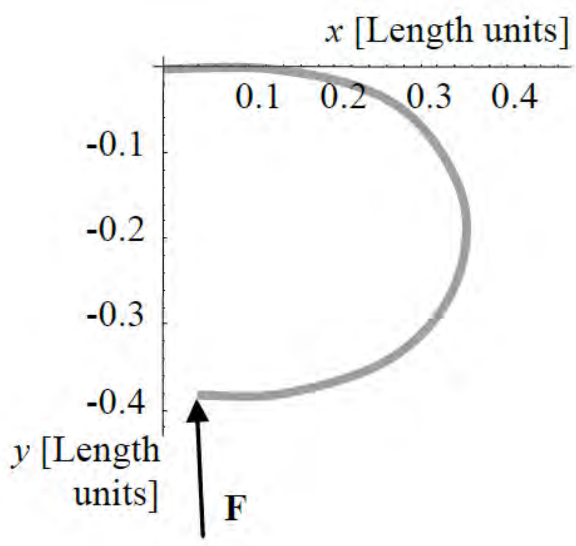

Figure 3.3: Representation of different Boundary Conditions but same force [13]

Now, in order to describe correctly the differential equation of the beam, we have to make the following assumptions [7, 30]:

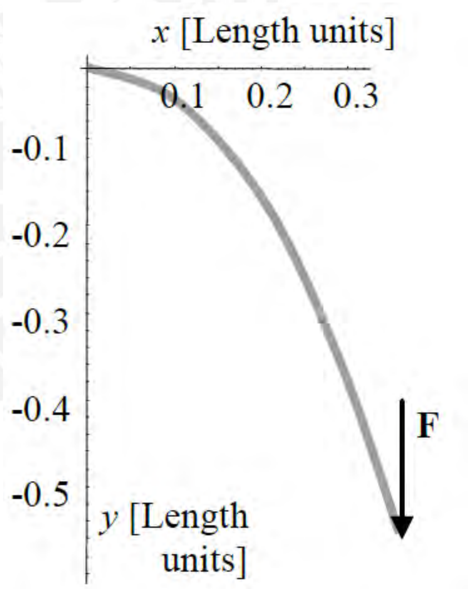
- The relation between the dimensions from the cross section must be at least ten times smaller than the length of the beam, so that the influence due to the shear forces can be neglected.
- The EBBT applies, i.e., that the cross-section of the beam remains perpendicular to the neutral axis all the time when the external forces are applied.
- The Saint-Venant's Principle (SVP) applies, that means, that it is considered that the applied force is uniformly distributed in the section of the beam.
- The deformation and the stress due to external forces will not fall into the plastic field, i.e., that the behavior of the material will always fulfills the Hooke's law (See Eq. 3.4).

$$\sigma = E \cdot \epsilon \tag{3.4}$$

- The beams that conform the compliant mechanism have a constant cross section.
- The external forces applied to the beam will maintain their directions of application. The deformation that the bar would suffer does not allow that the directions of the external forces would change. A graphic explanation can be found in the Fig. 3.4.




(a) Attached coordinate system




(b) Fixed coordinate system

Figure 3.4: Difference between attached and fixed coordinate systems and its effects [30]

In Fig. 3.5 a beam which is subjected to external forces in x and y directions and a moment can be seen. At the other end, the beam is fixed and the origin of the coordinate system is located. In this state, it is considered that the beam is static. So, when we analyze the internal forces of the beam, we obtain the following equation:

$$\sum \vec{M}_z = \vec{0} \rightarrow M_z = -F_x \cdot (y(L) - y(s)) + F_y \cdot (x(L) - x(s)) + Me \tag{3.5}$$

It is important to mention that $s$ is a variable that is going to follow the path of the neutral fiber of the beam. In addition to that, in the equation 3.5, it can be seen, that $y(L)$, $x(L)$ and $Me$ are constant, so when we derive the equation 3.5 with respect to $ds$ we obtain the equation 3.8. Moreover, we have to notice that there is a geometrical relation that can be seen in Fig. 3.6a. According to that figure, $ds$, $dx$ and $dy$ can be related and are shown in the equations 3.10 and 3.11. Finally, we have to consider the curvature of the beam which can be defined

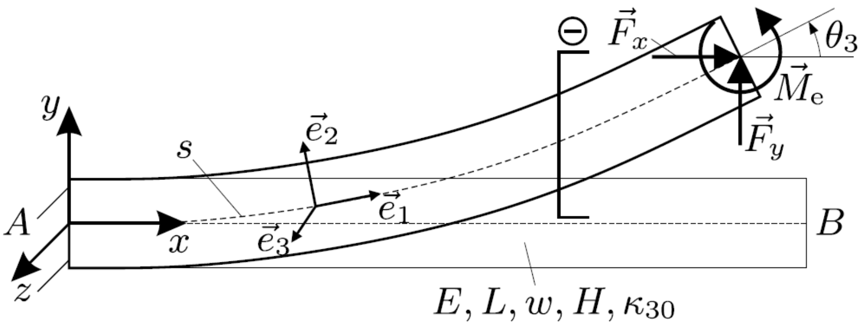
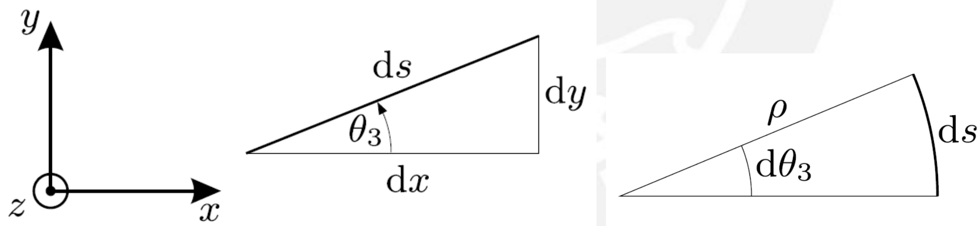



Figure 3.5: Representation of a beam under external loads [10]




(a) Relations between $ds$, $dx$ and $dy$ (b) Relation between $\rho$ and $ds$

Figure 3.6: Geometrical relations in the deformed beam [7]

as the inverse of the curvature radius in a certain point of the beam. The Fig. 3.6b and 3.7 show geometrical relations that have to be considered. According to the theory of resistance of material [1], the relation between the internal moment of the beam and the curvature can be seen in the equations 3.6 and 3.7. So, when we combine both equations, and considering a possible initial curvature, we get the equation 3.9.

$$\frac{1}{\rho} = \frac{M_z}{EI_z} \tag{3.6}$$

$$\kappa = \frac{d\theta}{ds} \tag{3.7}$$

In summary, the equations that describe the analytical behavior of the beam due to external forces are:

$$\frac{dM_z}{ds} = F_x \sin(\theta_3) - F_y \cos(\theta_3) \tag{3.8}$$

$$\frac{d\theta_3}{ds} = \kappa \text{ , with } \kappa = \frac{M_z}{EI_z} + \kappa_0 \tag{3.9}$$

$$\frac{dx}{ds} = cos(\theta_3) \tag{3.10}$$

$$\frac{dy}{ds} = sin(\theta_3) \tag{3.11}$$

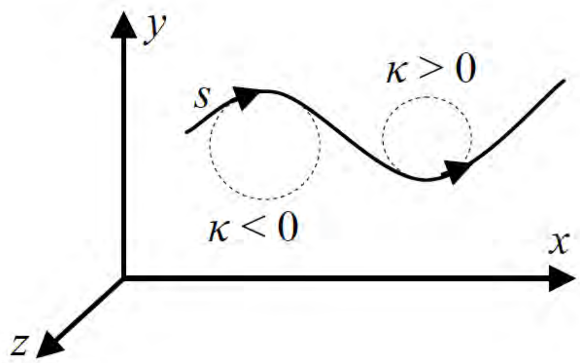



Figure 3.7: Graphical explanation of the sign of the curvature [30]

In order to solve the differential equation system above, boundary conditions are needed. Those are the conditions that the system of equation have to satisfied to get a solution. The number of boundary conditions needed is the same as the number of equations in the system. The boundary conditions define a specific problem, so they have to be consistent with the reality.

## 3.2 Development of the Analytical Model in Julia

After the review of the concerned theoretical framework of compliant mechanisms seen in 3.1, in this section, we will explain how the analytical model was developed.

In order to develop the analytical model Julia will be used. Julia is an open-source, relative new programming language dedicated to solve advanced mathematical problems. In this work, its ability to solve differential equations quickly will be leveraged. In the Fig. 3.8, it can be seen the main screen of Julia when it is opened directly. Due to the lack of functionality on the main screen, the code editor Visual Studio Code (VSC) (also called VS Code) will be used.

In order to have a good organization, the algorithms can be divided into different files. Although the algorithms are separated, it is possible to call them by the command "Include". This command will search and run the external files.

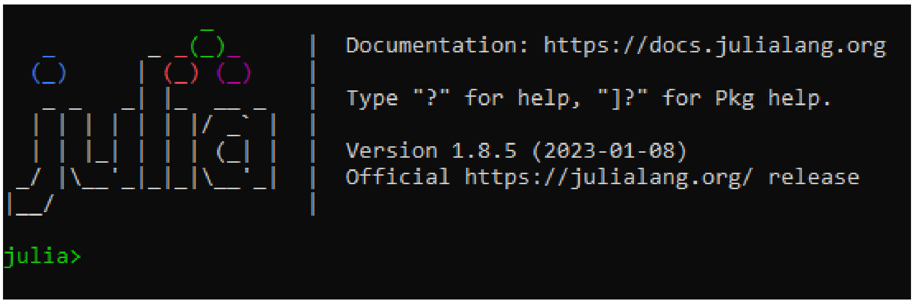
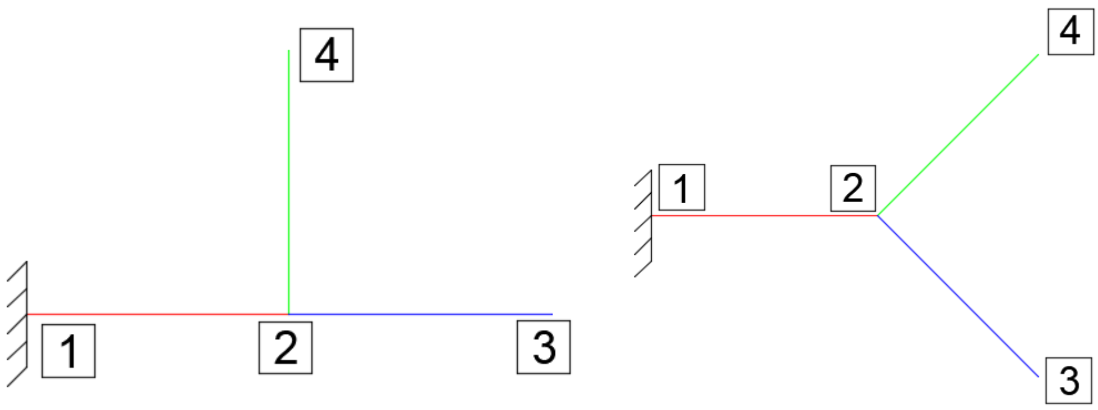



Figure 3.8: Main screen of the Julia programming language

In a first file called “Connections”, a set of structures and functions are presented. Here, a structure called “Connections” is defined and used to store information about the adjacency matrix, the nodes connected to the beams and at which end, the beams connected to the nodes and the type of node. The “adjacency matrix” is a matrix, defined by the user, which give information about how the nodes of the structure are related. The matrix 3.12 is an example of an adjacency matrix. The number of rows and columns indicate the number of nodes in the structure, so the matrix has to be quadratic and symmetric. In this case, this structure has four nodes, so, there are four rows and four columns. The position number of each column and row indicate the position numbers of the corresponding nodes. The matrix contains as elements only 1 and 0. The element 1 means that there is an adjacency connection between the nodes that correspond to that specific position in the matrix. Moreover, the element 0 means, that that connection does not exist.

$$adj = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{3.12}$$

For example, revisiting the matrix 3.12, when the first column is analysed, only an element 1 is located in the second row. That means, that the nodes 1 and 2 are connected. Then, in the second column, the elements 1 can be located in the first, third and fourth rows. That means, that the node 2 is related to the nodes 1, 3 and 4. And finally, in the third and fourth columns, only elements 1 appear in the second row. That mean, that the nodes 3 and 4 are related to the node 2, respectively. Evidently, the elements of the principal diagonal of the matrix have to be 0. As it can be depicted in the Fig. 3.9, the adjacency matrix does not define the position, but the relations of the nodes, so in that figure, two examples of structures that meet the adjacency matrix are presented.
With respect to the type of nodes, just 3 types of nodes are going to be used: “Clamp”, “Branch” ans “Free”.




(a) Example 1 (b) Example 2

Figure 3.9: Structures example

- By “Clamp”, it refers to nodes that are embedded or fixed. That means, that no displacement is going to occur at this node, even if external forces are applied.
- By “Branch”, it refers to the nodes where two beam are connected. At this type of nodes, the transition conditions of must be fulfilled.
- By “Free”, it refers to nodes that have no connection with other beam and can move freely.

For instance, in the example structures in the Fig. 3.9, the node 1 is a “Clamp” node, the nodes 2, 3 are “Branch” nodes and the node 4 is a “Free” node.
In a second file called “BeamStructure” the function “ode” is defined based on the differential equations 3.8, 3.9, 3.10 and 3.11, which are going to be solved taking into consideration the initial conditions and a defined timespan. In this case, the timespan, which correspond to the length of the beam, is defined in the range of 0 to 1. This is because, the used variables have to be normalized. The normalization of variables consist in converting the parameters into dimensionless parameters, so the process becomes independent from the physical dimensions. The normalization of the momentum, force, displacement in x, displacement in y and curvature are represented in the equations 3.13, 3.14, 3.15, 3.16 and 3.17, respectively. In case, the real dimension are required, it can be obtained by the inverse process of normalization.
Now, it is possible to complete the definition of the structures by the introduction of the physical properties of their elements. So, it is important to define the matrices“beamfeatures” and “nodefeatures”. Both can be defined in the vectors 3.18 and 3.19. In the “beamfeatures” vector, it can be found the length of each beam ($L$), the angle when the beam is going out to a node

($\theta_s$), the curvature of the beam ($\kappa$) and the angle when the beam is going in to a node ($\theta_e$).

$$\tilde{M} = \frac{ML}{EI} \tag{3.13}$$

$$\tilde{F} = \frac{ML^2}{EI} \tag{3.14}$$

$$\tilde{x} = \frac{x}{L} \tag{3.15}$$

$$\tilde{y} = \frac{y}{L} \tag{3.16}$$

$$\tilde{\kappa} = \kappa \cdot L \tag{3.17}$$

$$\text{beamfeatures} = \begin{bmatrix} L & \theta_s & \kappa & \theta_e \end{bmatrix}^T \tag{3.18}$$

$$\text{nodefeatures} = \begin{bmatrix} \triangle\theta & X_0 & Y_0 & \triangle M & \triangle F_x & \triangle F_y \end{bmatrix}^T \tag{3.19}$$

Then, in the "nodefeatures" vector, it can be found the rotation of the node ($\triangle\theta$), the initial position in the x-direction ($X_0$), the initial position in the y-direction ($Y_0$), the external moment ($\triangle M$), the external force in x-direction ($\triangle F_x$) and the external force in y-direction ($\triangle F_y$).

$$beamfeatures = \begin{bmatrix} L_1 & L_2 & L_3 \\ 0 & 0 & \pi/2 \\ 0 & 0 & 0 \\ 0 & 0 & \pi/2 \end{bmatrix} \tag{3.20}$$

$$nodefeatures = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & L_1 & (L_1 + L_2) & L_1 \\ 0 & 0 & 0 & L_3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \triangle F_x & -\triangle F_x \\ 0 & 0 & -\triangle F_y & \triangle F_y \end{bmatrix} \tag{3.21}$$

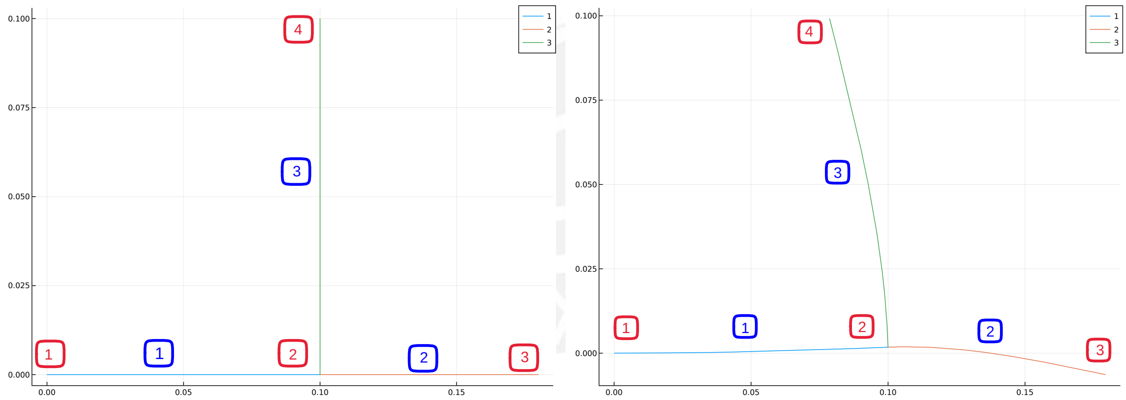
In order to introduce this parameter in the algorithms, a matrix of "beamfeatures" and a matrix of "nodefeatures" are introduced, in which the number of columns of both matrices are defined by the number of beams and nodes, respectively. An example of each matrix can be seen in 3.20 and 3.21. Now, considering these matrices in conjunction with 3.12, it is clear that they together refer to the Fig. 3.9a type structure.

## 3.3 Results of the analytical simulation

In this section, the results obtained by the simulation of different kinds of structures through Julia are going to be presented. To avoid redundant definition of the beams in the following structures, a cross-sectional area with 2 $mm$ height and 10$mm$ width will be defined. In addition to that, steel is going to be considered as the material for all the beams in the structures. Thus, according to the table 3.1, the modulus of elasticity in all the beams are going to be E = 200,000 $N/mm^2$. Evidently, this values can be changed by the user when the geometrical and material properties of the beams are different. Also, the nodes are going to be designated by numbers in red, and beams by number in blue in their respective figures. The coordinate system used in all the cases is the same as shown in the Fig. 3.6a. Also the adjacency matrices of the following structures can be found in the A.2.

### 3.3.1 Displacement determination of the nodes due to external forces

In this subsection the displacement of the nodes due to external forces are going to be obtained. The first five structures, designed by letters, have the purpose to validate the algorithms. After that, four more structures obtained from [3, 5, 27, 29], which are designed by numbers, are going to be simulated, too.



(a) Unloaded state (b) Loaded state

Figure 3.10: Schematic of the structure A in its unloaded and loaded state

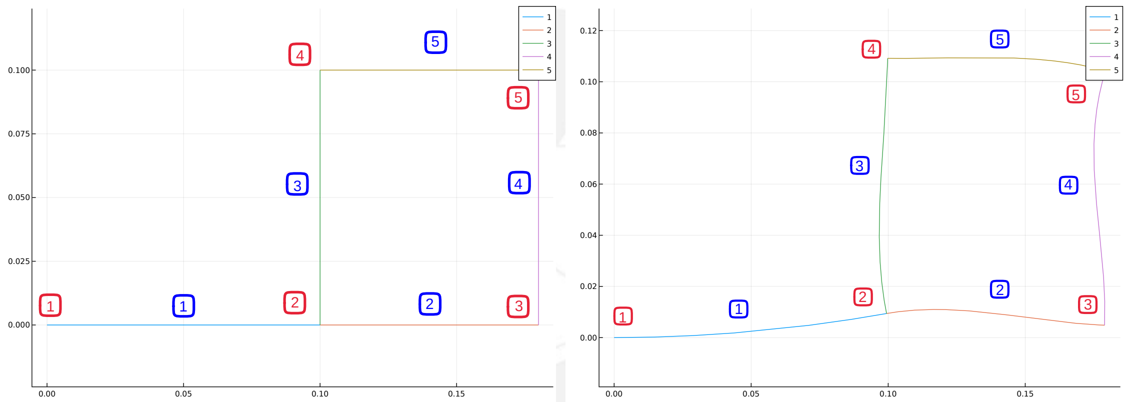
#### Structure A

The analyzed structure in its unloaded state is shown in the Fig. 3.10a. As it can be seen, it is composed by three beams and four nodes. This structure has one node of type "Clamp", which is the first node, one node of type "Branch" and two nodes of type "Free". The first and the third beams have 100$mm$ length, and the second beam has 80$mm$ length. The initial position of the nodes, the applied external forces and the displacement of the nodes are indicated in the

Table 3.2: Initial positions and displacements of the nodes in the X and Y direction in the structure A obtained by Julia

| Node | Type | Initial Position $X_0$ (mm) | $Y_0$ (mm) | External forces $M_z$ (N.mm) | $F_x$ (N) | $F_y$ (N) | New Position X (mm) | Y (mm) | Displacement $\triangle$X (mm) | $\triangle$Y (mm) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Clamp | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | Branch | 100 | 0 | 0 | 0 | 0 | 99.98 | 1.79 | -0.02 | 1.79 |
| 3 | Branch | 180 | 0 | 0 | 100 | -100 | 179.41 | -6.36 | -0.59 | -6.36 |
| 4 | Free | 100 | 100 | 0 | -100 | 100 | 78.65 | 99.17 | -21.36 | -0.83 |

table 3.2. After the application of the external forces, the deformed state is depicted in the Figure 3.10b. When the node type "Clamp" is analysed, a moment $-476.2\,\text{Nmm}$ is obtained.



(a) Unloaded state (b) Loaded state

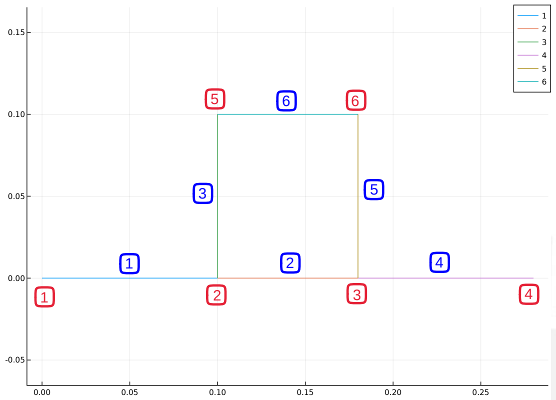
Figure 3.11: Schematic of the structure B in its unloaded and loaded state

Table 3.3: Initial positions and displacements of the nodes in the X and Y direction in the structure B obtained by Julia

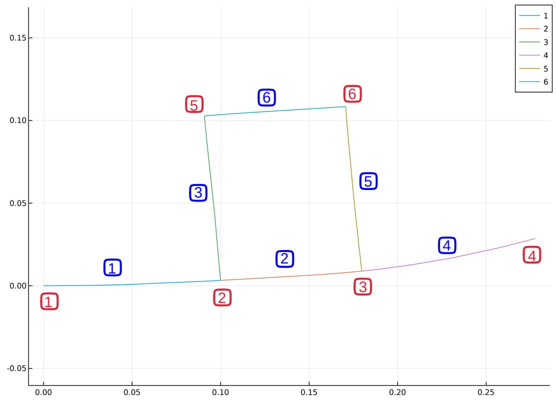
| Node | Type | Initial Position $X_0$ (mm) | $Y_0$ (mm) | External forces $M_z$ (N.mm) | $F_x$ (N) | $F_y$ (N) | New Position X (mm) | Y (mm) | Displacement $\triangle$X (mm) | $\triangle$Y (mm) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Clamp | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | Branch | 100 | 0 | 30000 | 0 | 0 | 99.41 | 9.41 | -0.59 | 9.41 |
| 3 | Branch | 180 | 0 | 0 | 100 | -100 | 178.96 | 4.84 | -1.04 | 4.84 |
| 4 | Branch | 100 | 100 | 0 | -100 | 100 | 99.80 | 109.17 | -0.20 | 9.17 |
| 5 | Branch | 180 | 100 | -30000 | 0 | 0 | 179.33 | 104.28 | -0.67 | 4.28 |

### Structure B

The unloaded state of structure B can be seen in the Fig. 3.11a. This structure has five beams and five nodes. This structure has one node of type "Clamp", which is the first node, and four nodes of type "Branch". The first, third and fourth beams have $100mm$ length and the second and fifth beams have 80$mm$ length. The initial position of the nodes, the applied external forces and the displacement of the nodes are indicated in the table 3.3. After the application of the external forces, the deformed state is depicted in the 3.11b. When the node type "Clamp" is analysed, a moment $-2516.67$ Nmm is obtained.



(a) Unloaded state



(b) Loaded state

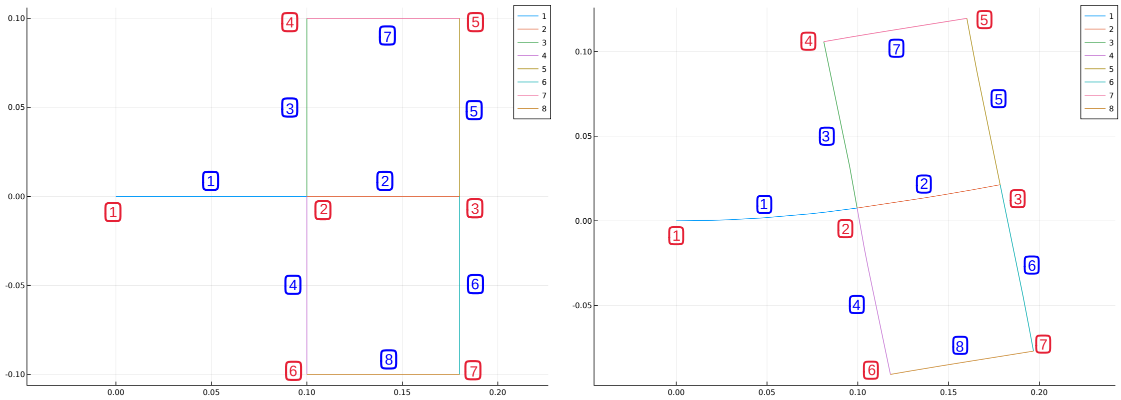
Figure 3.12: Schematic of the structure C in its unloaded and loaded state

Table 3.4: Initial positions and displacements of the nodes in the X and Y direction in the structure C obtained by Julia

| Node | Type | Initial Position $X_0$ (mm) | $Y_0$ (mm) | External forces $M_z$ (N.mm) | $F_x$ (N) | $F_y$ (N) | New Position X (mm) | Y (mm) | Displacement △X (mm) | △Y (mm) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Clamp | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | Branch | 100 | 0 | 0 | 0 | 30 | 99.93 | 3.27 | -0.07 | 3.27 |
| 3 | Branch | 180 | 0 | 0 | 30 | -30 | 179.72 | 8.88 | -0.28 | 8.88 |
| 4 | Branch | 280 | 0 | 2000 | -30 | 0 | 277.63 | 28.60 | -2.37 | 28.60 |
| 5 | Branch | 100 | 100 | 0 | -30 | 30 | 90.79 | 102.85 | -9.21 | 2.85 |
| 6 | Free | 180 | 100 | 0 | 0 | -30 | 170.59 | 108.46 | -9.41 | 8.46 |

## Structure C

The unloaded state of the structure C can be seen in the Fig 3.12a. This structure is composed by six beams and six nodes. This structure has one node of type "Clamp", which is the first node, four nodes of type "Branch" and one node of type "Free". The first, third, fourth and fifth beams have 100$mm$ length and the second and sixth beams have 80$mm$ length. The initial position of the nodes, the applied external forces and moments and the displacement of the nodes are indicated in the table 3.4. After the application of the external forces, the deformed state is depicted in the 3.12b.When the node type "Clamp" is analysed, a moment $-888.96$ Nmm is obtained.



(a) Unloaded state (b) Loaded state

Figure 3.13: Schematic of the structure D in its unloaded and loaded state

Table 3.5: Initial positions and displacements of the nodes in the X and Y direction in the structure D obtained by Julia

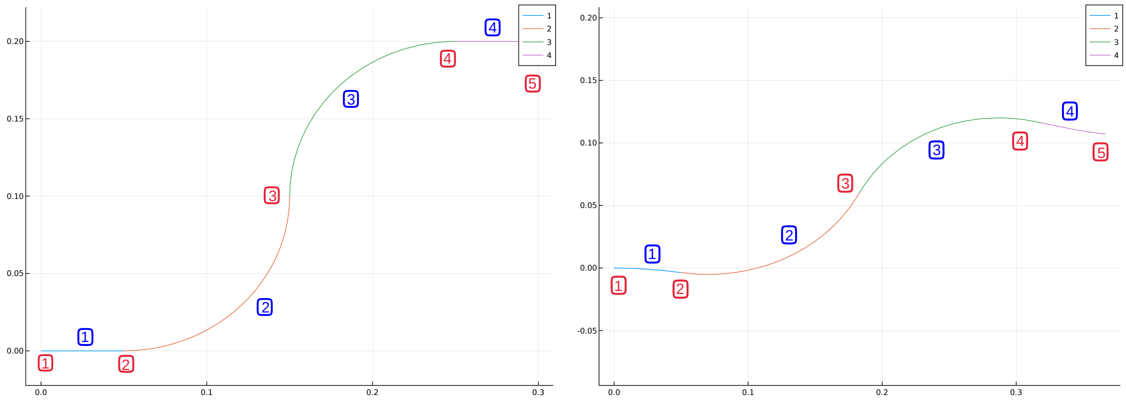
| Node | Type | Initial Position $X_0$ (mm) | Initial Position $Y_0$ (mm) | External forces $M_z$ (N.mm) | External forces $F_x$ (N) | External forces $F_y$ (N) | New Position X (mm) | New Position Y (mm) | Displacement △X (mm) | Displacement △Y (mm) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Clamp | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | Branch | 100 | 0 | -2000 | 0 | 0 | 99.62 | 7.56 | -0.38 | 7.56 |
| 3 | Branch | 180 | 0 | 2000 | 0 | 0 | 178.42 | 21.33 | -1.58 | 21.33 |
| 4 | Branch | 100 | 100 | 0 | -30 | 30 | 81.26 | 105.85 | -18.74 | 5.85 |
| 5 | Branch | 180 | 100 | 0 | 0 | 0 | 160.06 | 119.62 | -19.94 | 19.62 |
| 6 | Branch | 100 | -100 | 0 | 0 | 0 | 117.97 | -90.74 | 17.97 | 9.26 |
| 7 | Branch | 180 | -100 | 0 | 30 | -30 | 196.77 | -76.97 | 16.77 | 23.03 |

### Structure D

The unloaded state of the structure D can be seen in the Fig 3.13a. This structure is composed by eight beams and seven nodes. This structure has one node of type "Clamp", which is the first node, and the rest of the nodes are of type "Branch". All the beams have $100mm$ length except the second, seventh and eighth beams, which have $80mm$ length. The initial position of the nodes, the applied external forces and moments and the displacement of the nodes are indicated in the table 3.5. After the application of the external forces, the deformed state is depicted in the 3.13b. When the node type "Clamp" is analyzed, a moment $-2019.18$ Nmm is obtained.

### Structure E

The unloaded state of the structure E can be seen in the Fig 3.14a. This structure is composed by four beams and five nodes. This structure has one node of type "Clamp", which is the first node, three nodes are of type "Branch" and one node of type "Free".The first and the fourth beams have $50mm$ length and the second and third beam are curved beams with a radius of



(a) Unloaded state

(b) Loaded state

Figure 3.14: Schematic of the structure E in its unloaded and loaded state

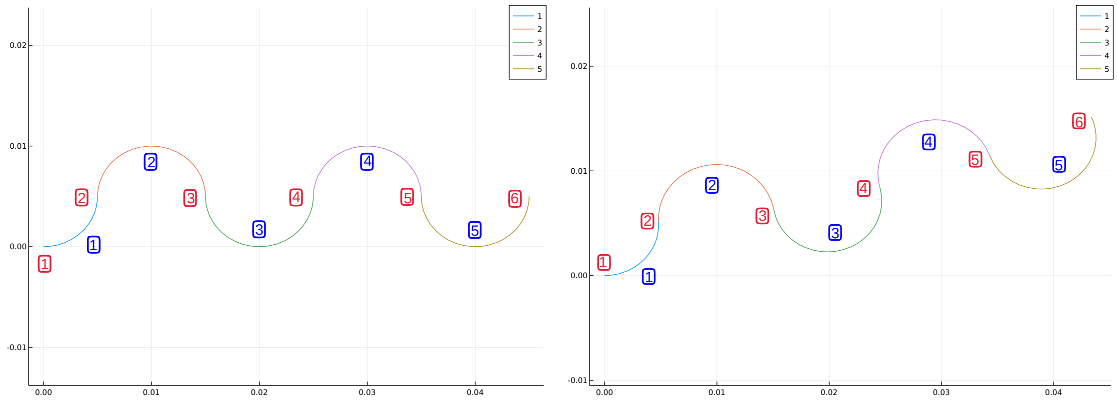
Table 3.6: Initial positions and displacements of the nodes in the X and Y direction in the structure E obtained by Julia

| Node | Type | Initial Position $X_0$ (mm) | $Y_0$ (mm) | External forces $M_z$ (N.mm) | $F_x$ (N) | $F_y$ (N) | New Position X (mm) | Y (mm) | Displacement △X (mm) | △Y (mm) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Clamp | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | Branch | 50 | 0 | 0 | -10 | 10 | 49.82 | -3.63 | -0.18 | -3.63 |
| 3 | Branch | 150 | 100 | -3000 | 0 | 0 | 182.96 | 59.89 | 32.96 | -40.11 |
| 4 | Branch | 250 | 200 | 0 | 10 | -10 | 317.60 | 116.26 | 67.60 | -83.74 |
| 5 | Free | 300 | 200 | 3000 | 0 | 0 | 366.70 | 106.96 | 66.70 | -93.04 |

curvature of $100mm$ each. The initial position of the nodes, the applied external forces and moments and the displacement of the nodes are indicated in the table 3.6. After the application of the external forces, the deformed state is depicted in the 3.14b. When the node type "Clamp" is analyzed, a moment $-3876.57\,$Nmm is obtained.

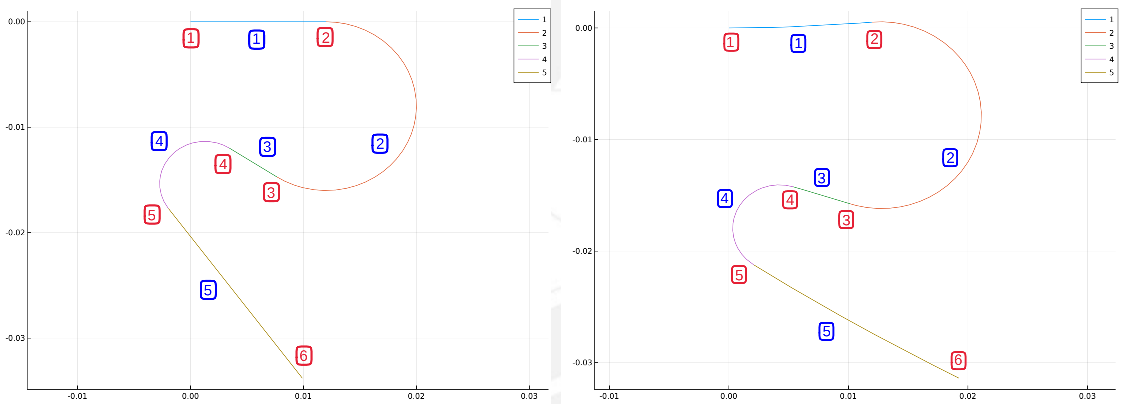
Table 3.7: Initial positions and displacements of the nodes in the X and Y direction in the structure 1 obtained by Julia

| Node | Type | Initial Position $X_0$ (mm) | $Y_0$ (mm) | External forces $M_z$ (N.mm) | $F_x$ (N) | $F_y$ (N) | New Position X (mm) | Y (mm) | Displacement △X (mm) | △Y (mm) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Clamp | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | Branch | 5 | 5 | 0 | 0 | 0 | 4.80 | 5.11 | -0.20 | 0.11 |
| 3 | Branch | 15 | 5 | 0 | 0 | 0 | 15.07 | 6.30 | 0.07 | 1.30 |
| 4 | Branch | 25 | 5 | 0 | 0 | 0 | 24.52 | 8.40 | -0.48 | 3.40 |
| 5 | Branch | 35 | 5 | 0 | 0 | 0 | 34.29 | 11.50 | -0.71 | 6.50 |
| 6 | Free | 45 | 5 | 5000 | -100 | 100 | 43.36 | 15.13 | -1.64 | 10.13 |



(a) Unloaded state (b) Loaded state

Figure 3.15: Schematic of the structure 1 in its unloaded and loaded state



(a) Unloaded state (b) Loaded state

Figure 3.16: Schematic of the structure 2 in its unloaded and loaded state

### Structure 1

This structure is obtained from [29]. The unloaded state of the structure 1 can be seen in the Fig 3.15a. This structure is composed by five curved beams and six nodes. This structure has one node of type "Clamp", which is the first node, four nodes are of type "Branch" and one node of type "Free". All the beams have the same radius of curvature, which in this case is defined by the parameter $R_1 = 5mm$. The initial position of the nodes, the applied external forces and moments and the displacement of the nodes are indicated in the table 3.7. After the application of the external forces, the deformed state is depicted in the 3.15b. When the node type "Clamp" is analysed, a moment $-10849.73\,\mathrm{Nmm}$ is obtained.

Table 3.8: Initial positions and displacements of the nodes in the X and Y direction in the structure 2 obtained by Julia

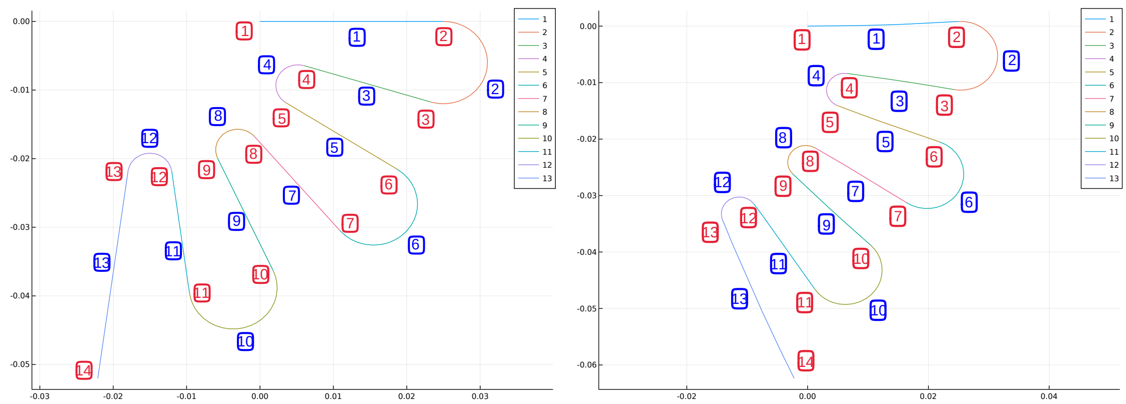
| Node | Type | Initial Position $X_0$ (mm) | $Y_0$ (mm) | External forces $M_z$ (N.mm) | $F_x$ (N) | $F_y$ (N) | New Position X (mm) | Y (mm) | Displacement $\triangle$X (mm) | $\triangle$Y (mm) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Clamp | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | Branch | 12 | 0 | 0 | 0 | 0 | 11.99 | 0.52 | -0.02 | 0.52 |
| 3 | Branch | 7.64 | -14.71 | 0 | 0 | 0 | 10.12 | -15.75 | 2.47 | -1.04 |
| 4 | Branch | 3.45 | -11.98 | 0 | 0 | 0 | 5.35 | -14.23 | 1.90 | -2.25 |
| 5 | Branch | -1.95 | -17.71 | 0 | 0 | 0 | 2.07 | -21.22 | 4.02 | -3.51 |
| 6 | Free | 9.91 | -33.81 | 5000 | 100 | 100 | 19.27 | -31.40 | 9.36 | 2.41 |

## Structure 2

This structure is obtained from [5]. The unloaded state of the structure 2 can be seen in the Fig 3.16a. This structure is composed by five beams and six nodes. This structure has one node of type "Clamp", which is the first node, four nodes are of type "Branch" and one node of type "Free". The three straight beams have different lengths and the curved beams have different radius of curvature, too. The parameters can be changed in order to obtain a different structure. In this case, the parameters are $L_1 = 12mm$, $R_1 = 8mm$ , $L_2 = 5mm$, $R_2 = 4mm$ and $L_3 = 20mm$ which correspond to the beams in ascended order respectively as depicted in the Fig 3.16a. The initial position of the nodes, the applied external forces and moments and the displacement of the nodes are indicated in the table 3.8. After the application of the external forces, which are applied only at the extreme of the structure, the deformed state is depicted in the 3.16b. When the node type "Clamp" is analyzed, a moment $-10067.35\,\text{Nmm}$ is obtained.

## Structure 3

This structure is obtained from [27]. The unloaded state of the structure 3 can be seen in the Fig 3.17a. This structure is composed by thirteen beams and fourteen nodes. This structure has one node of type "Clamp", which is the first node, twelve nodes of type "Branch" and one node of type "Free". The beams have different lengths that depend on the parameters of the structure. The parameters can be changed in order to obtain a different structure. In this case, the parameters are $L_1 = 25mm$, which correspond to the first beam, $R_1 = 6mm$ which correspond to the external curved beams, $L_2 = 12mm$, which correspond to the third, fifth, seventh, ninth and eleventh beams, $R_2 = 3mm$, which correspond to the internal curved beams, and $L_3 = 35mm$, which correspond to the thirteenth beam as depicted in the Fig 3.17a. The initial position of the nodes, the applied external forces and moments and the displacement of the nodes are indicated in the table 3.9. After the application of the external forces, which are applied only at the extreme of the structure, the deformed state is depicted in the 3.17b. When the node type "Clamp" is analyzed, a moment $-3601.43\,\text{Nmm}$ is obtained.



(a) Unloaded state (b) Loaded state

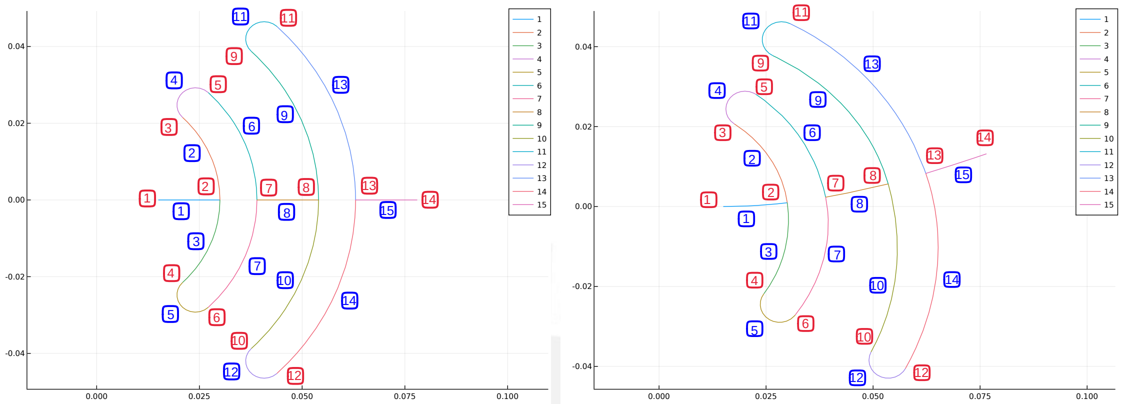
Figure 3.17: Schematic of the structure 3 in its unloaded and loaded state

Table 3.9: Initial positions and displacements of the nodes in the X and Y direction in the structure 3 obtained by Julia

| Node | Type | Initial Position $X_0$ (mm) | Initial Position $Y_0$ (mm) | External forces $M_z$ (N.mm) | External forces $F_x$ (N) | External forces $F_y$ (N) | New Position X (mm) | New Position Y (mm) | Displacement $\triangle$X (mm) | Displacement $\triangle$Y (mm) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Clamp | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 2 | Branch | 25.00 | 0.00 | 0.00 | 0.00 | 0.00 | 24.98 | 0.82 | -0.02 | 0.82 |
| 3 | Branch | 23.24 | -11.74 | 0.00 | 0.00 | 0.00 | 24.28 | -11.24 | 1.04 | 0.50 |
| 4 | Branch | 6.03 | -6.45 | 0.00 | 0.00 | 0.00 | 6.51 | -8.38 | 0.48 | -1.92 |
| 5 | Branch | 3.54 | -11.85 | 0.00 | 0.00 | 0.00 | 4.98 | -14.08 | 1.45 | -2.23 |
| 6 | Branch | 18.70 | -21.55 | 0.00 | 0.00 | 0.00 | 21.81 | -20.48 | 3.11 | 1.07 |
| 7 | Branch | 10.90 | -30.49 | 0.00 | 0.00 | 0.00 | 16.38 | -31.26 | 5.49 | -0.78 |
| 8 | Branch | -0.76 | -16.77 | 0.00 | 0.00 | 0.00 | 1.22 | -21.56 | 1.98 | -4.79 |
| 9 | Branch | -5.77 | -19.97 | 0.00 | 0.00 | 0.00 | -2.45 | -26.18 | 3.31 | -6.22 |
| 10 | Branch | 1.79 | -36.31 | 0.00 | 0.00 | 0.00 | 10.42 | -38.77 | 8.63 | -2.46 |
| 11 | Branch | -9.60 | -39.63 | 0.00 | 0.00 | 0.00 | 1.13 | -46.48 | 10.73 | -6.84 |
| 12 | Branch | -12.03 | -21.80 | 0.00 | 0.00 | 0.00 | -8.88 | -31.52 | 3.16 | -9.72 |
| 13 | Branch | -17.97 | -21.80 | 0.00 | 0.00 | 0.00 | -14.10 | -34.28 | 3.88 | -12.48 |
| 14 | Free | -22.09 | -52.02 | 3000.00 | 10.00 | 10.00 | -2.23 | -62.37 | 19.87 | -10.35 |

### Structure 4

This structure is obtained from [3]. The unloaded state of the structure 4 can be seen in the Fig 3.18a. This structure is composed by fifteen beams and fourteen nodes. This structure has one node of type "Clamp", which is the first node, twelve nodes of type "Branch" and one node of type "Free". The beams have different lengths that depend on the parameters of the structure. The parameters can be changed in order to obtain a different structure. In this case, the parameters are $R_1 = 30mm$, radius of curvature of the second and third beam, $R_2 = 2.5mm$ which correspond to the radius of curvature of the fourth, fifth, eleventh and twelfth beams and $\alpha = 1.6rad$, which correspond to angle measured between the center of curvature of the fourth, fifth, eleventh and twelfth beams. The initial position of the nodes, the applied external forces and moments and the displacement of the nodes are indicated in the table 3.10. After the application of the external forces, which are applied only at the extreme of the structure, the deformed state is depicted in the 3.18b. When the node type "Clamp" is analyzed, a moment $-11011.78\,\mathrm{Nmm}$ is obtained.



(a) Unloaded state

(b) Loaded state

Figure 3.18: Schematic of the structure 4 in its unloaded and loaded state

Table 3.10: Initial positions and displacements of the nodes in the X and Y direction in the structure 4 obtained by Julia

| Node | Type | Initial Position $X_0$ (mm) | Initial Position $Y_0$ (mm) | External forces $M_z$ (N.mm) | External forces $F_x$ (N) | External forces $F_y$ (N) | New Position X (mm) | New Position Y (mm) | Displacement $\triangle$X (mm) | Displacement $\triangle$Y (mm) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Clamp | 15.00 | 0.00 | 0.00 | 0.00 | 0.00 | 15.00 | 0.00 | 0.00 | 0.00 |
| 2 | Branch | 30.00 | 0.00 | 0.00 | 0.00 | 0.00 | 29.96 | 0.95 | -0.04 | 0.95 |
| 3 | Branch | 20.90 | 21.52 | 0.00 | -50.00 | -50.00 | 17.58 | 20.68 | -3.32 | -0.84 |
| 4 | Branch | 20.90 | -21.52 | 0.00 | 0.00 | 0.00 | 24.48 | -21.84 | 3.58 | -0.32 |
| 5 | Branch | 27.17 | 27.98 | 0.00 | 0.00 | 0.00 | 22.65 | 28.06 | -4.52 | 0.08 |
| 6 | Branch | 27.17 | -27.98 | 0.00 | 50.00 | 0.00 | 31.82 | -27.14 | 4.64 | 0.83 |
| 7 | Branch | 39.00 | 0.00 | 3000.00 | 0.00 | 0.00 | 38.91 | 2.34 | -0.09 | 2.34 |
| 8 | Branch | 54.00 | 0.00 | -3000.00 | 0.00 | 0.00 | 53.54 | 5.66 | -0.46 | 5.66 |
| 9 | Branch | 37.62 | 38.74 | 0.00 | -50.00 | -50.00 | 26.60 | 37.74 | -11.02 | -1.00 |
| 10 | Branch | 37.62 | -38.74 | 0.00 | 0.00 | 0.00 | 49.53 | -36.39 | 11.91 | 2.35 |
| 11 | Branch | 43.89 | 45.19 | 0.00 | 0.00 | 0.00 | 30.61 | 45.77 | -13.29 | 0.58 |
| 12 | Branch | 43.89 | -45.19 | 0.00 | 50.00 | 0.00 | 57.55 | -40.55 | 13.66 | 4.64 |
| 13 | Branch | 63.00 | 0.00 | 0.00 | 0.00 | 0.00 | 62.32 | 8.28 | -0.68 | 8.28 |
| 14 | Free | 78.00 | 0.00 | 3000.00 | 50.00 | 50.00 | 76.49 | 13.19 | -1.51 | 13.19 |

### 3.3.2 Force determination due to the rotation of the structure

In this subsection we will obtain the forces and moments necessary to carry out the movement of the structure. In this case, only the structures 1, 2, 3 and 4 of subsection 3.3.1 will be used because they have already engineering application. A reference point will be taken as the center of rotation and the angle to be rotated is in the range of -0.4 *rad* to 0.4 *rad* with a step of 0.1 $rad$. In other words, there will be 8 different positions for each structure. It is not considered in the following tables when the angle of rotation is $0rad$ because it is an unloaded position, therefore the external forces are zero in all cases. The complete results are shown in the Appendix A.1. In addition to that, it is also intended to find how these forces and moments vary with respect to the parameters of each structure. To do this, it is important to define the first and last node as “Clamp” type, so the rest of the structure will accommodate to the new positions. For practice, the first node is going to be fixed and only the last node is going to move and rotate. In the table 3.11 it can be seen the parameters and variations of each structure.

Table 3.11: Parameters and their variations of each structure

| Structure | Reference | Parameter | U.M. | Parameter variation | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 3.15a | R1 | mm | 5 | 6 | 7 | 8 | 9 |
| 2 | 3.16a | R1 | mm | 8 | 9 | 10 | 11 | 12 |
| | | R2 | mm | 4 | 5 | 6 | 7 | 8 |
| | | L1 | mm | 12 | 13 | 14 | 15 | 16 |
| | | L2 | mm | 5 | 6 | 7 | 8 | 9 |
| | | L3 | mm | 20 | 21 | 22 | 23 | 24 |
| 3 | 3.17a | R1 | mm | 6 | 7 | 8 | 9 | 10 |
| | | R2 | mm | 3 | 4 | 5 | 6 | 7 |
| | | L1 | mm | 25 | 26 | 27 | 28 | 29 |
| | | L2 | mm | 12 | 13 | 14 | 15 | 16 |
| | | L3 | mm | 35 | 36 | 37 | 38 | 39 |
| 4 | 3.18a | R1 | mm | 30 | 31 | 32 | 33 | 34 |
| | | $\alpha$ | rad | 0.80 | 1.00 | 1.20 | 1.40 | 1.60 |
| | | R2 | mm | 2.5 | 3 | 3.5 | 4 | 4.5 |

### Structure 1

This structure has only one parameter, which is the radius of curvature of the beams. As it can be seen in the table 3.11, the parameter $R1$ vary from $5mm$ to $9mm$ with $1mm$ steps. At the same time, the structure rotates from a node located $15mm$ to the left of the node 1 in the axis x. In the table 3.12 the angles of the resultant force, the resultant forces and the moments obtained from Julia can be seen.

### Structure 2

This structure has five parameters, which are the radius of curvature of it two curved beams and the lengths of the straight beams. As it can be seen in the table 3.11, the parameter $R1$ varies from $8mm$ to $12mm$ with $1mm$ steps, the parameter $R2$ varies from $4mm$ to $8mm$ with $1mm$ steps, $L1$ varies from $12mm$ to $16mm$ with $1mm$ steps, $L2$ varies from $5mm$ to $9mm$ with 1mm steps and $L3$ varies from $20mm$ to $24mm$ with $1mm$ steps. The structure rotates from a node located $15mm$ to the left of the node 1 in the axis x. In the tables 3.13, 3.14, 3.15, 3.16 and 3.17 the values obtained from Julia of the angles of the resultant force, the resultant forces and the moments when the parameters $R1$, $R2$, $L1$, $L2$ and $L3$ vary can be seen.

### Structure 3

This structure has five parameters, which are the radius of curvature of it the curved beams and the lengths of the straight beams. As it can be seen in the table 3.11, the parameter $R1$ varies from $6mm$ to $10mm$ with $1mm$ steps, the parameter $R2$ varies from $3mm$ to $7mm$ with $1mm$ steps, $L1$ varies from $25mm$ to $29mm$ with $1mm$ steps, $L2$ varies from $12mm$ to $16mm$ with $1mm$ steps and $L3$ varies from $35mm$ to $39mm$ with $1mm$ steps. The structure rotates from a node located $15mm$ to the left of the node 1. In the tables 3.18, 3.19, 3.20, 3.21 and 3.22 the values obtained from Julia of the angles of the resultant force, the resultant forces and the moments when the parameters $R1$, $R2$, $L1$, $L2$ and $L3$ vary can be seen.

### Structure 4

This structure has three parameters. As it can be seen in the table 3.11, the parameter $R1$ varies from $30mm$ to $34mm$ with $1mm$ steps, the parameter $\alpha$ varies from $0.8rad$ to $1.6rad$ with $0.2rad$ steps and $R2$ varies from $2.5mm$ to $4.5mm$ with $0.5mm$ steps. The structure rotates from a node located $15mm$ to the left of the node 1 in the axis x. In the tables 3.23, 3.24 and 3.25 the values obtained from Julia of the angles of the resultant force, the resultant forces and the moments when the parameters $R1$, $\alpha$ and $R2$ vary can be seen.

Table 3.12: Angle of the resultant force, resultant force and moment required to rotate with the respect to the parameter R1 of the structure 1

| R1 (mm) | Rotation (rad) | Angle (rad) | Resultant Force (N) | Moment (N.mm) |
|---|---|---|---|---|
| 5 | -0.4 | -0.81 | 7814.03 | 40802.14 |
| | -0.3 | -0.70 | 5180.78 | 29112.53 |
| | -0.2 | -0.60 | 3013.02 | 18612.32 |
| | -0.1 | -0.51 | 1285.28 | 8970.22 |
| | 0.1 | 2.74 | 866.27 | -8550.23 |
| | 0.2 | 2.71 | 1343.54 | -16680.39 |
| | 0.3 | 2.56 | 1526.40 | -24302.76 |
| | 0.4 | 2.21 | 1676.17 | -31039.31 |
| 6 | -0.4 | -0.79 | 4930.09 | 30609.73 |
| | -0.3 | -0.69 | 3310.42 | 22042.92 |
| | -0.2 | -0.59 | 1955.69 | 14198.64 |
| | -0.1 | -0.50 | 846.78 | 6893.58 |
| | 0.1 | 2.77 | 601.59 | -6617.30 |
| | 0.2 | 2.78 | 977.81 | -13008.79 |
| | 0.3 | 2.72 | 1143.43 | -19047.61 |
| | 0.4 | 2.52 | 1200.09 | -24582.90 |
| 7 | -0.4 | -0.78 | 3380.05 | 24255.78 |
| | -0.3 | -0.68 | 2286.41 | 17550.78 |
| | -0.2 | -0.58 | 1368.40 | 11378.80 |
| | -0.1 | -0.49 | 603.09 | 5549.55 |
| | 0.1 | 2.80 | 446.76 | -5371.37 |
| | 0.2 | 2.83 | 741.91 | -10601.26 |
| | 0.3 | 2.81 | 894.01 | -15603.12 |
| | 0.4 | 2.70 | 945.97 | -20259.90 |
| 8 | -0.4 | -0.77 | 2454.87 | 19956.71 |
| | -0.3 | -0.67 | 1675.39 | 14512.53 |
| | -0.2 | -0.57 | 1008.52 | 9429.62 |
| | -0.1 | -0.48 | 348.14 | 3948.93 |
| | 0.1 | 2.81 | 340.98 | -4502.37 |
| | 0.2 | 2.86 | 578.63 | -8896.47 |
| | 0.3 | 2.86 | 718.14 | -13167.85 |
| | 0.4 | 2.81 | 772.18 | -17188.89 |
| 9 | -0.4 | -0.77 | 1857.61 | 16854.63 |
| | -0.3 | -0.67 | 1275.41 | 12307.42 |
| | -0.2 | -0.57 | 771.80 | 8025.14 |
| | -0.1 | -0.48 | 348.14 | 3948.93 |
| | 0.1 | 2.82 | 271.28 | -3865.14 |
| | 0.2 | 2.88 | 465.39 | -7658.53 |
| | 0.3 | 2.90 | 587.80 | -11371.91 |
| | 0.4 | 2.88 | 643.92 | -14898.65 |

Table 3.13: Angle of the resultant force, resultant force and moment required to rotate with the respect to the parameter R1 of the structure 2

| R1 (mm) | Rotation (rad) | Angle (rad) | Resultant Force (N) | Moment (N.mm) |
|---|---|---|---|---|
| | -0.4 | -2.24 | 4521.11 | 30821.98 |
| | -0.3 | -2.18 | 3332.68 | 24092.91 |
| | -0.2 | -2.12 | 2182.31 | 16632.44 |
| 8 | -0.1 | -2.05 | 1067.50 | 8453.45 |
| | 0.1 | 1.18 | 1072.94 | -10067.96 |
| | 0.2 | 1.25 | 2110.49 | -20383.91 |
| | 0.3 | 1.31 | 3131.35 | -31353.09 |
| | 0.4 | 1.37 | 4142.14 | -42921.56 |
| | -0.4 | -2.31 | 3847.92 | 25924.70 |
| | -0.3 | -2.26 | 2855.11 | 20445.51 |
| | -0.2 | -2.20 | 1885.00 | 14373.45 |
| 9 | -0.1 | -2.15 | 938.29 | 7651.76 |
| | 0.1 | 1.13 | 901.95 | -7594.39 |
| | 0.2 | 1.18 | 1801.16 | -16086.09 |
| | 0.3 | 1.23 | 2692.74 | -25168.63 |
| | 0.4 | 1.29 | 3578.79 | -34718.81 |
| | -0.4 | -2.37 | 3251.03 | 20238.48 |
| | -0.3 | -2.32 | 2416.29 | 15929.03 |
| | -0.2 | -2.26 | 1595.00 | 11088.47 |
| 10 | -0.1 | -2.20 | 791.55 | 5762.49 |
| | 0.1 | 1.05 | 787.00 | -6412.43 |
| | 0.2 | 1.10 | 1563.05 | -13230.07 |
| | 0.3 | 1.16 | 2337.78 | -20525.40 |
| | 0.4 | 1.22 | 3110.68 | -28218.87 |
| | -0.4 | -2.43 | 2765.46 | 16025.65 |
| | -0.3 | -2.38 | 2061.69 | 12726.80 |
| | -0.2 | -2.33 | 1369.70 | 9012.22 |
| 11 | -0.1 | -2.27 | 684.73 | 4851.75 |
| | 0.1 | 1.00 | 663.90 | -4682.32 |
| | 0.2 | 1.05 | 1332.09 | -10028.17 |
| | 0.3 | 1.10 | 1998.85 | -15773.89 |
| | 0.4 | 1.16 | 2665.82 | -21846.52 |
| | -0.4 | -2.48 | 2334.00 | 11266.52 |
| | -0.3 | -2.42 | 1740.38 | 8912.24 |
| | -0.2 | -2.36 | 1152.25 | 6174.35 |
| 12 | -0.1 | -2.30 | 571.25 | 3113.56 |
| | 0.1 | 0.92 | 581.40 | -4013.63 |
| | 0.2 | 0.98 | 1156.84 | -8090.72 |
| | 0.3 | 1.04 | 1729.20 | -12437.39 |
| | 0.4 | 1.09 | 2305.26 | -17111.75 |

Table 3.14: Angle of the resultant force, resultant force and moment required to rotate with the respect to the parameter R2 of the structure 2

| R2 (mm) | Rotation (rad) | Angle (rad) | Resultant Force (N) | Moment (N.mm) |
|---|---|---|---|---|
| 4 | -0.4 | -2.24 | 4521.11 | 30821.98 |
| | -0.3 | -2.18 | 3332.68 | 24092.91 |
| | -0.2 | -2.12 | 2182.31 | 16632.44 |
| | -0.1 | -2.05 | 1067.50 | 8453.45 |
| | 0.1 | 1.18 | 1072.94 | -10067.96 |
| | 0.2 | 1.25 | 2110.49 | -20383.91 |
| | 0.3 | 1.31 | 3131.35 | -31353.09 |
| | 0.4 | 1.37 | 4142.14 | -42921.56 |
| 5 | -0.4 | -2.31 | 4196.34 | 29747.68 |
| | -0.3 | -2.25 | 3099.53 | 23449.54 |
| | -0.2 | -2.20 | 2039.61 | 16463.07 |
| | -0.1 | -2.14 | 1010.36 | 8750.60 |
| | 0.1 | 1.13 | 960.10 | -8703.80 |
| | 0.2 | 1.18 | 1907.14 | -18371.17 |
| | 0.3 | 1.24 | 2840.54 | -28637.38 |
| | 0.4 | 1.30 | 3760.79 | -39437.82 |
| 6 | -0.4 | -2.37 | 3904.19 | 27354.99 |
| | -0.3 | -2.31 | 2877.76 | 21524.81 |
| | -0.2 | -2.25 | 1881.71 | 14945.28 |
| | -0.1 | -2.19 | 925.86 | 7747.08 |
| | 0.1 | 1.05 | 906.52 | -8669.63 |
| | 0.2 | 1.11 | 1785.95 | -17751.39 |
| | 0.3 | 1.17 | 2644.27 | -27371.47 |
| | 0.4 | 1.22 | 3491.25 | -37490.86 |
| 7 | -0.4 | -2.43 | 3692.04 | 26135.16 |
| | -0.3 | -2.37 | 2721.49 | 20669.13 |
| | -0.2 | -2.31 | 1785.41 | 14531.06 |
| | -0.1 | -2.36 | 792.71 | 7705.05 |
| | 0.1 | 1.01 | 835.66 | -7791.66 |
| | 0.2 | 1.06 | 1655.72 | -16403.77 |
| | 0.3 | 1.11 | 2449.58 | -25431.52 |
| | 0.4 | 1.16 | 3233.37 | -34981.08 |
| 8 | -0.4 | -2.48 | 3512.79 | 24747.59 |
| | -0.3 | -2.42 | 2590.48 | 19677.43 |
| | -0.2 | -2.37 | 1700.00 | 13915.96 |
| | -0.1 | -2.32 | 840.96 | 7455.98 |
| | 0.1 | 0.95 | 781.99 | -7192.07 |
| | 0.2 | 1.00 | 1550.54 | -15330.74 |
| | 0.3 | 1.06 | 2298.25 | -23944.51 |
| | 0.4 | 1.11 | 3025.04 | -32921.02 |

Table 3.15: Angle of the resultant force, resultant force and moment required to rotate with the respect to the parameter L1 of the structure 2

| L1 (mm) | Rotation (rad) | Angle (rad) | Resultant Force (N) | Moment (N.mm) |
|---|---|---|---|---|
| 12 | -0.4 | -2.24 | 4521.11 | 30821.98 |
| | -0.3 | -2.18 | 3332.68 | 24092.91 |
| | -0.2 | -2.12 | 2182.31 | 16632.44 |
| | -0.1 | -2.05 | 1067.50 | 8453.45 |
| | 0.1 | 1.18 | 1072.94 | -10067.96 |
| | 0.2 | 1.25 | 2110.49 | -20383.91 |
| | 0.3 | 1.31 | 3131.35 | -31353.09 |
| | 0.4 | 1.37 | 4142.14 | -42921.56 |
| 13 | -0.4 | -2.21 | 4422.36 | 30927.86 |
| | -0.3 | -2.15 | 3269.73 | 24300.17 |
| | -0.2 | -2.09 | 2151.11 | 16935.20 |
| | -0.1 | -2.03 | 1065.53 | 8883.08 |
| | 0.1 | 1.23 | 1033.27 | -9375.11 |
| | 0.2 | 1.30 | 2055.59 | -19563.27 |
| | 0.3 | 1.36 | 3061.92 | -30431.83 |
| | 0.4 | 1.42 | 4057.44 | -41893.50 |
| 14 | -0.4 | -2.18 | 4297.81 | 29965.23 |
| | -0.3 | -2.12 | 3178.44 | 23484.26 |
| | -0.2 | -2.06 | 2091.65 | 16302.51 |
| | -0.1 | -1.99 | 1031.72 | 8443.62 |
| | 0.1 | 1.26 | 1020.58 | -9423.18 |
| | 0.2 | 1.33 | 2021.82 | -19419.33 |
| | 0.3 | 1.39 | 3013.02 | -30098.74 |
| | 0.4 | 1.46 | 3993.31 | -41406.72 |
| 15 | -0.4 | -2.15 | 4171.59 | 29152.92 |
| | -0.3 | -2.09 | 3089.41 | 22860.44 |
| | -0.2 | -2.03 | 2033.30 | 15856.22 |
| | -0.1 | -1.96 | 1005.38 | 8222.41 |
| | 0.1 | 1.30 | 999.49 | -9238.51 |
| | 0.2 | 1.37 | 1977.55 | -18959.60 |
| | 0.3 | 1.43 | 2946.23 | -29411.99 |
| | 0.4 | 1.50 | 3906.43 | -40510.19 |
| 16 | -0.4 | -2.13 | 4048.77 | 28528.64 |
| | -0.3 | -2.07 | 3003.10 | 22386.62 |
| | -0.2 | -2.00 | 1981.21 | 15608.02 |
| | -0.1 | -1.94 | 983.24 | 8160.97 |
| | 0.1 | 1.34 | 965.57 | -8763.58 |
| | 0.2 | 1.41 | 1920.89 | -18269.64 |
| | 0.3 | 1.47 | 2868.81 | -28508.05 |
| | 0.4 | 1.54 | 3807.22 | -39379.30 |

Table 3.16: Angle of the resultant force, resultant force and moment required to rotate with the respect to the parameter L2 of the structure 2

| L2 (mm) | Rotation (rad) | Angle (rad) | Resultant Force (N) | Moment (N.mm) |
|---|---|---|---|---|
| 5 | -0.4 | -2.24 | 4521.11 | 30821.98 |
| | -0.3 | -2.18 | 3332.68 | 24092.91 |
| | -0.2 | -2.12 | 2182.31 | 16632.44 |
| | -0.1 | -2.05 | 1067.50 | 8453.45 |
| | 0.1 | 1.18 | 1072.94 | -10067.96 |
| | 0.2 | 1.25 | 2110.49 | -20383.91 |
| | 0.3 | 1.31 | 3131.35 | -31353.09 |
| | 0.4 | 1.37 | 4142.14 | -42921.56 |
| 6 | -0.4 | -2.28 | 4267.67 | 30013.66 |
| | -0.3 | -2.22 | 3155.80 | 23612.90 |
| | -0.2 | -2.17 | 2080.88 | 16536.63 |
| | -0.1 | -2.11 | 1032.53 | 8741.29 |
| | 0.1 | 1.16 | 989.63 | -8814.59 |
| | 0.2 | 1.21 | 1972.47 | -18550.96 |
| | 0.3 | 1.26 | 2939.60 | -28833.95 |
| | 0.4 | 1.32 | 3904.53 | -39661.99 |
| 7 | -0.4 | -2.31 | 4004.66 | 28155.42 |
| | -0.3 | -2.26 | 2961.76 | 22079.28 |
| | -0.2 | -2.20 | 1951.60 | 15404.47 |
| | -0.1 | -2.15 | 968.68 | 8078.84 |
| | 0.1 | 1.10 | 940.08 | -8455.30 |
| | 0.2 | 1.16 | 1870.37 | -17534.66 |
| | 0.3 | 1.21 | 2790.08 | -27130.34 |
| | 0.4 | 1.26 | 3708.22 | -37191.66 |
| 8 | -0.4 | -2.35 | 3766.00 | 26505.02 |
| | -0.3 | -2.30 | 2788.11 | 20787.40 |
| | -0.2 | -2.24 | 1840.17 | 14515.12 |
| | -0.1 | -2.19 | 912.21 | 7587.10 |
| | 0.1 | 1.06 | 887.07 | -7861.45 |
| | 0.2 | 1.11 | 1767.30 | -16363.73 |
| | 0.3 | 1.15 | 2637.61 | -25250.75 |
| | 0.4 | 1.20 | 3511.32 | -34532.94 |
| 9 | -0.4 | -2.39 | 3553.08 | 25154.52 |
| | -0.3 | -2.34 | 2636.61 | 19789.16 |
| | -0.2 | -2.29 | 1742.79 | 13860.91 |
| | -0.1 | -2.24 | 867.61 | 7365.61 |
| | 0.1 | 1.02 | 827.89 | -7086.21 |
| | 0.2 | 1.06 | 1659.11 | -14981.12 |
| | 0.3 | 1.11 | 2481.69 | -23174.57 |
| | 0.4 | 1.15 | 3306.77 | -31681.67 |

Table 3.17: Angle of the resultant force, resultant force and moment required to rotate with the respect to the parameter L3 of the structure 2

| L3 (mm) | Rotation (rad) | Angle (rad) | Resultant Force (N) | Moment (N.mm) |
|---|---|---|---|---|
| 20 | -0.4 | -2.24 | 4521.11 | 30821.98 |
| | -0.3 | -2.18 | 3332.68 | 24092.91 |
| | -0.2 | -2.12 | 2182.31 | 16632.44 |
| | -0.1 | -2.05 | 1067.50 | 8453.45 |
| | 0.1 | 1.18 | 1072.94 | -10067.96 |
| | 0.2 | 1.25 | 2110.49 | -20383.91 |
| | 0.3 | 1.31 | 3131.35 | -31353.09 |
| | 0.4 | 1.37 | 4142.14 | -42921.56 |
| 21 | -0.4 | -2.22 | 4438.77 | 31290.65 |
| | -0.3 | -2.16 | 3264.45 | 24441.72 |
| | -0.2 | -2.10 | 2136.33 | 16893.05 |
| | -0.1 | -2.03 | 1041.95 | 8580.13 |
| | 0.1 | 1.20 | 1044.06 | -10209.85 |
| | 0.2 | 1.27 | 2048.80 | -20707.85 |
| | 0.3 | 1.33 | 3032.48 | -31828.04 |
| | 0.4 | 1.39 | 3999.74 | -43530.83 |
| 22 | -0.4 | -2.21 | 4356.69 | 31625.70 |
| | -0.3 | -2.15 | 3198.72 | 24725.95 |
| | -0.2 | -2.08 | 2086.69 | 17060.11 |
| | -0.1 | -2.02 | 1016.48 | 8666.34 |
| | 0.1 | 1.22 | 1016.19 | -10343.58 |
| | 0.2 | 1.29 | 1986.11 | -20889.97 |
| | 0.3 | 1.35 | 2929.49 | -32070.70 |
| | 0.4 | 1.41 | 3857.44 | -43917.65 |
| 23 | -0.4 | -2.19 | 4272.84 | 31802.41 |
| | -0.3 | -2.13 | 3135.01 | 24887.29 |
| | -0.2 | -2.07 | 2041.17 | 17177.03 |
| | -0.1 | -2.00 | 991.25 | 8716.91 |
| | 0.1 | 1.25 | 987.36 | -10394.05 |
| | 0.2 | 1.31 | 1923.96 | -20982.54 |
| | 0.3 | 1.38 | 2831.33 | -32241.12 |
| | 0.4 | 1.44 | 3716.28 | -44103.19 |
| 24 | -0.4 | -2.18 | 4196.53 | 31931.85 |
| | -0.3 | -2.11 | 3068.46 | 24953.13 |
| | -0.2 | -2.05 | 1996.23 | 17235.57 |
| | -0.1 | -1.98 | 966.39 | 8736.37 |
| | 0.1 | 1.27 | 958.92 | -10407.31 |
| | 0.2 | 1.33 | 1862.75 | -20995.98 |
| | 0.3 | 1.40 | 2730.97 | -32214.75 |
| | 0.4 | 1.46 | 3574.03 | -44069.58 |

Table 3.18: Angle of the resultant force, resultant force and moment required to rotate with the respect to the parameter R1 of the structure 3

| R1 (mm) | Rotation (rad) | Angle (rad) | Resultant Force (N) | Moment (N.mm) |
|---|---|---|---|---|
| 6 | -0.4 | -2.53 | 1209.82 | 7138.24 |
| | -0.3 | -2.47 | 916.99 | 6053.36 |
| | -0.2 | -2.41 | 618.24 | 4551.15 |
| | -0.1 | -2.34 | 313.32 | 2604.97 |
| | 0.1 | 0.94 | 309.82 | -2812.13 |
| | 0.2 | 1.00 | 624.67 | -6376.39 |
| | 0.3 | 1.06 | 940.01 | -10582.92 |
| | 0.4 | 1.13 | 1254.34 | -15460.44 |
| 7 | -0.4 | -2.63 | 990.00 | 3774.52 |
| | -0.3 | -2.57 | 751.90 | 3342.36 |
| | -0.2 | -2.50 | 506.86 | 2585.90 |
| | -0.1 | -2.44 | 256.78 | 1486.20 |
| | 0.1 | 0.83 | 261.51 | -1912.46 |
| | 0.2 | 0.89 | 526.96 | -4271.57 |
| | 0.3 | 0.96 | 794.74 | -7116.43 |
| | 0.4 | 1.02 | 1065.32 | -10493.38 |
| 8 | -0.4 | -2.73 | 803.29 | 1401.72 |
| | -0.3 | -2.66 | 612.23 | 1490.35 |
| | -0.2 | -2.60 | 415.01 | 1330.81 |
| | -0.1 | -2.55 | 211.18 | 895.49 |
| | 0.1 | 0.75 | 212.53 | -883.92 |
| | 0.2 | 0.80 | 430.39 | -2263.45 |
| | 0.3 | 0.86 | 652.35 | -4003.67 |
| | 0.4 | 0.92 | 876.04 | -6115.09 |
| 9 | -0.4 | -2.81 | 641.50 | -953.30 |
| | -0.3 | -2.75 | 490.52 | -414.98 |
| | -0.2 | -2.68 | 332.06 | -78.59 |
| | -0.1 | -2.62 | 168.73 | 48.70 |
| | 0.1 | 0.64 | 173.91 | -382.21 |
| | 0.2 | 0.70 | 351.30 | -965.53 |
| | 0.3 | 0.76 | 532.04 | -1803.53 |
| | 0.4 | 0.83 | 714.58 | -2908.11 |
| 10 | -0.4 | -2.89 | 510.43 | -2360.74 |
| | -0.3 | -2.83 | 391.06 | -1529.47 |
| | -0.2 | -2.77 | 266.26 | -854.31 |
| | -0.1 | -2.71 | 135.53 | -336.10 |
| | 0.1 | 0.56 | 139.54 | 175.81 |
| | 0.2 | 0.62 | 282.96 | 153.31 |
| | 0.3 | 0.68 | 429.08 | -64.02 |
| | 0.4 | 0.74 | 576.79 | -476.92 |

Table 3.19: Angle of the resultant force, resultant force and moment required to rotate with the respect to the parameter R2 of the structure 3

| R2 (mm) | Rotation (rad) | Angle (rad) | Resultant Force (N) | Moment (N.mm) |
|---|---|---|---|---|
| 3 | -0.4 | -2.53 | 1209.82 | 7138.24 |
| | -0.3 | -2.47 | 916.99 | 6053.36 |
| | -0.2 | -2.41 | 618.24 | 4551.15 |
| | -0.1 | -2.34 | 313.32 | 2604.97 |
| | 0.1 | 0.94 | 309.82 | -2812.13 |
| | 0.2 | 1.00 | 624.67 | -6376.39 |
| | 0.3 | 1.06 | 940.01 | -10582.92 |
| | 0.4 | 1.13 | 1254.34 | -15460.44 |
| 4 | -0.4 | -2.66 | 1003.26 | 3603.41 |
| | -0.3 | -2.60 | 764.64 | 3272.92 |
| | -0.2 | -2.53 | 517.92 | 2612.82 |
| | -0.1 | -2.47 | 263.66 | 1597.05 |
| | 0.1 | 0.82 | 263.54 | -1649.71 |
| | 0.2 | 0.88 | 534.89 | -3962.49 |
| | 0.3 | 0.94 | 810.45 | -6811.40 |
| | 0.4 | 1.00 | 1087.89 | -10236.71 |
| 5 | -0.4 | -2.78 | 803.08 | 488.36 |
| | -0.3 | -2.72 | 615.57 | 794.70 |
| | -0.2 | -2.65 | 418.62 | 852.00 |
| | -0.1 | -2.59 | 214.09 | 640.04 |
| | 0.1 | 0.69 | 217.98 | -694.82 |
| | 0.2 | 0.75 | 444.03 | -1873.30 |
| | 0.3 | 0.81 | 675.29 | -3436.31 |
| | 0.4 | 0.87 | 910.31 | -5424.60 |
| 6 | -0.4 | -2.89 | 619.87 | -2022.28 |
| | -0.3 | -2.83 | 478.42 | -1233.60 |
| | -0.2 | -2.76 | 327.20 | -621.61 |
| | -0.1 | -2.70 | 167.71 | -205.23 |
| | 0.1 | 0.57 | 175.27 | -23.23 |
| | 0.2 | 0.63 | 357.03 | -289.00 |
| | 0.3 | 0.69 | 544.62 | -820.06 |
| | 0.4 | 0.75 | 737.40 | -1637.94 |
| 7 | -0.4 | -3.00 | 463.75 | -3411.21 |
| | -0.3 | -2.94 | 361.55 | -2338.67 |
| | -0.2 | -2.88 | 249.31 | -1387.40 |
| | -0.1 | -2.83 | 128.60 | -565.68 |
| | 0.1 | 0.46 | 136.56 | 626.86 |
| | 0.2 | 0.52 | 279.79 | 979.09 |
| | 0.3 | 0.58 | 429.17 | 1143.43 |
| | 0.4 | 0.64 | 583.93 | 1118.40 |

Table 3.20: Angle of the resultant force, resultant force and moment required to rotate with the respect to the parameter L1 of the structure 3

| L1 (mm) | Rotation (rad) | Angle (rad) | Resultant Force (N) | Moment (N.mm) |
|---|---|---|---|---|
| 25 | -0.4 | -2.53 | 1209.82 | 7138.24 |
| | -0.3 | -2.47 | 916.99 | 6053.36 |
| | -0.2 | -2.41 | 618.24 | 4551.15 |
| | -0.1 | -2.34 | 313.32 | 2604.97 |
| | 0.1 | 0.94 | 309.82 | -2812.13 |
| | 0.2 | 1.00 | 624.67 | -6376.39 |
| | 0.3 | 1.06 | 940.01 | -10582.92 |
| | 0.4 | 1.13 | 1254.34 | -15460.44 |
| 26 | -0.4 | -2.51 | 1209.97 | 7405.26 |
| | -0.3 | -2.44 | 915.37 | 6202.62 |
| | -0.2 | -2.37 | 614.98 | 4593.43 |
| | -0.1 | -2.31 | 310.13 | 2548.92 |
| | 0.1 | 0.97 | 312.85 | -3057.30 |
| | 0.2 | 1.03 | 626.98 | -6697.00 |
| | 0.3 | 1.09 | 942.34 | -10978.11 |
| | 0.4 | 1.16 | 1254.98 | -15882.22 |
| 27 | -0.4 | -2.48 | 1207.81 | 7759.47 |
| | -0.3 | -2.41 | 913.59 | 6451.04 |
| | -0.2 | -2.35 | 612.90 | 4737.58 |
| | -0.1 | -2.28 | 308.63 | 2598.94 |
| | 0.1 | 0.99 | 312.25 | -3173.41 |
| | 0.2 | 1.06 | 625.18 | -6888.46 |
| | 0.3 | 1.12 | 938.44 | -11212.07 |
| | 0.4 | 1.19 | 1249.41 | -16145.21 |
| 28 | -0.4 | -2.46 | 1203.86 | 8185.96 |
| | -0.3 | -2.39 | 910.16 | 6776.79 |
| | -0.2 | -2.32 | 611.06 | 4973.53 |
| | -0.1 | -2.26 | 307.27 | 2733.20 |
| | 0.1 | 1.02 | 308.58 | -3184.06 |
| | 0.2 | 1.09 | 619.27 | -6960.54 |
| | 0.3 | 1.15 | 929.59 | -11311.36 |
| | 0.4 | 1.22 | 1238.67 | -16258.26 |
| 29 | -0.4 | -2.44 | 1198.34 | 8686.19 |
| | -0.3 | -2.37 | 905.76 | 7180.37 |
| | -0.2 | -2.30 | 609.37 | 5292.28 |
| | -0.1 | -2.24 | 308.46 | 2970.75 |
| | 0.1 | 1.06 | 302.82 | -3099.46 |
| | 0.2 | 1.12 | 609.97 | -6915.08 |
| | 0.3 | 1.18 | 917.74 | -11298.48 |
| | 0.4 | 1.25 | 1223.11 | -16242.48 |

Table 3.21: Angle of the resultant force, resultant force and moment required to rotate with the respect to the parameter L2 of the structure 3

| L2 (mm) | Rotation (rad) | Angle (rad) | Resultant Force (N) | Moment (N.mm) |
|---|---|---|---|---|
| 12 | -0.4 | -2.53 | 1209.82 | 7138.24 |
| | -0.3 | -2.47 | 916.99 | 6053.36 |
| | -0.2 | -2.41 | 618.24 | 4551.15 |
| | -0.1 | -2.34 | 313.32 | 2604.97 |
| | 0.1 | 0.94 | 309.82 | -2812.13 |
| | 0.2 | 1.00 | 624.67 | -6376.39 |
| | 0.3 | 1.06 | 940.01 | -10582.92 |
| | 0.4 | 1.13 | 1254.34 | -15460.44 |
| 13 | -0.4 | -2.55 | 1119.40 | 5762.64 |
| | -0.3 | -2.48 | 848.72 | 4916.83 |
| | -0.2 | -2.42 | 572.08 | 3685.19 |
| | -0.1 | -2.35 | 294.37 | 2130.74 |
| | 0.1 | 0.91 | 296.86 | -2776.93 |
| | 0.2 | 0.97 | 595.17 | -6016.82 |
| | 0.3 | 1.04 | 894.36 | -9863.35 |
| | 0.4 | 1.10 | 1194.30 | -14368.05 |
| 14 | -0.4 | -2.57 | 1041.04 | 5117.77 |
| | -0.3 | -2.51 | 791.67 | 4456.70 |
| | -0.2 | -2.44 | 535.33 | 3430.57 |
| | -0.1 | -2.38 | 272.26 | 2003.77 |
| | 0.1 | 0.90 | 273.24 | -2275.60 |
| | 0.2 | 0.96 | 552.95 | -5216.61 |
| | 0.3 | 1.02 | 835.78 | -8758.37 |
| | 0.4 | 1.08 | 1119.03 | -12916.82 |
| 15 | -0.4 | -2.59 | 961.16 | 4033.94 |
| | -0.3 | -2.52 | 731.90 | 3594.20 |
| | -0.2 | -2.46 | 495.01 | 2797.77 |
| | -0.1 | -2.40 | 250.98 | 1612.98 |
| | 0.1 | 0.87 | 257.17 | -2097.61 |
| | 0.2 | 0.93 | 519.25 | -4717.44 |
| | 0.3 | 0.99 | 786.00 | -7919.08 |
| | 0.4 | 1.05 | 1054.01 | -11716.52 |
| 16 | -0.4 | -2.60 | 886.61 | 3117.32 |
| | -0.3 | -2.54 | 676.54 | 2870.48 |
| | -0.2 | -2.48 | 458.13 | 2292.10 |
| | -0.1 | -2.42 | 232.21 | 1337.19 |
| | 0.1 | 0.84 | 240.11 | -1840.57 |
| | 0.2 | 0.90 | 485.56 | -4162.10 |
| | 0.3 | 0.96 | 735.13 | -7025.80 |
| | 0.4 | 1.02 | 987.97 | -10468.91 |

Table 3.22: Angle of the resultant force, resultant force and moment required to rotate with the respect to the parameter L3 of the structure 3

| L3 (mm) | Rotation (rad) | Angle (rad) | Resultant Force (N) | Moment (N.mm) |
|---|---|---|---|---|
| 35 | -0.4 | -2.53 | 1209.82 | 7138.24 |
| | -0.3 | -2.47 | 916.99 | 6053.36 |
| | -0.2 | -2.41 | 618.24 | 4551.15 |
| | -0.1 | -2.34 | 313.32 | 2604.97 |
| | 0.1 | 0.94 | 309.82 | -2812.13 |
| | 0.2 | 1.00 | 624.67 | -6376.39 |
| | 0.3 | 1.06 | 940.01 | -10582.92 |
| | 0.4 | 1.13 | 1254.34 | -15460.44 |
| 36 | -0.4 | -2.53 | 1205.17 | 7486.38 |
| | -0.3 | -2.47 | 913.79 | 6333.86 |
| | -0.2 | -2.40 | 614.89 | 4748.95 |
| | -0.1 | -2.34 | 311.64 | 2720.36 |
| | 0.1 | 0.95 | 307.00 | -2923.55 |
| | 0.2 | 1.01 | 619.25 | -6640.20 |
| | 0.3 | 1.07 | 929.88 | -10995.55 |
| | 0.4 | 1.13 | 1238.28 | -16037.41 |
| 37 | -0.4 | -2.53 | 1201.69 | 7819.45 |
| | -0.3 | -2.46 | 910.35 | 6595.69 |
| | -0.2 | -2.40 | 612.75 | 4944.88 |
| | -0.1 | -2.34 | 309.89 | 2818.37 |
| | 0.1 | 0.95 | 304.58 | -3037.91 |
| | 0.2 | 1.01 | 612.93 | -6873.86 |
| | 0.3 | 1.07 | 919.44 | -11385.91 |
| | 0.4 | 1.14 | 1220.24 | -16551.24 |
| 38 | -0.4 | -2.52 | 1196.53 | 8120.90 |
| | -0.3 | -2.46 | 906.05 | 6840.24 |
| | -0.2 | -2.39 | 609.72 | 5116.41 |
| | -0.1 | -2.33 | 308.03 | 2909.34 |
| | 0.1 | 0.96 | 302.01 | -3143.95 |
| | 0.2 | 1.02 | 606.72 | -7099.57 |
| | 0.3 | 1.08 | 907.07 | -11722.58 |
| | 0.4 | 1.14 | 1201.52 | -17021.52 |
| 39 | -0.4 | -2.52 | 1192.56 | 8409.52 |
| | -0.3 | -2.45 | 902.18 | 7066.42 |
| | -0.2 | -2.39 | 605.82 | 5274.60 |
| | -0.1 | -2.33 | 306.72 | 3005.16 |
| | 0.1 | 0.96 | 298.76 | -3230.15 |
| | 0.2 | 1.02 | 598.88 | -7287.04 |
| | 0.3 | 1.08 | 894.89 | -12034.78 |
| | 0.4 | 1.14 | 1182.32 | -17439.68 |

Table 3.23: Angle of the resultant force, resultant force and moment required to rotate with the respect to the parameter R1 of the structure 4

| R1 (mm) | Rotation (rad) | Angle (rad) | Resultant Force (N) | Moment (N.mm) |
|---|---|---|---|---|
| 30 | -0.4 | -1.49 | 1744.64 | 29630.91 |
| | -0.3 | -1.44 | 1266.34 | 22530.04 |
| | -0.2 | -1.44 | 800.50 | 15147.22 |
| | -0.1 | -1.49 | 380.81 | 7605.02 |
| | 0.1 | 1.49 | 380.81 | -7605.02 |
| | 0.2 | 1.44 | 800.50 | -15147.22 |
| | 0.3 | 1.44 | 1266.34 | -22530.04 |
| | 0.4 | 1.49 | 1744.64 | -29630.91 |
| 31 | -0.4 | -1.45 | 1590.70 | 27659.84 |
| | -0.3 | -1.41 | 1145.54 | 21016.88 |
| | -0.2 | -1.42 | 717.31 | 14129.64 |
| | -0.1 | -1.47 | 339.46 | 7100.51 |
| | 0.1 | 1.47 | 339.46 | -7100.51 |
| | 0.2 | 1.42 | 717.31 | -14129.64 |
| | 0.3 | 1.41 | 1145.54 | -21016.88 |
| | 0.4 | 1.45 | 1590.70 | -27659.84 |
| 32 | -0.4 | -1.41 | 1458.40 | 25916.71 |
| | -0.3 | -1.37 | 1044.54 | 19689.90 |
| | -0.2 | -1.38 | 649.77 | 13239.22 |
| | -0.1 | -1.46 | 303.57 | 6654.66 |
| | 0.1 | 1.46 | 303.57 | -6654.66 |
| | 0.2 | 1.38 | 649.77 | -13239.22 |
| | 0.3 | 1.37 | 1044.54 | -19689.90 |
| | 0.4 | 1.41 | 1458.40 | -25916.71 |
| 33 | -0.4 | -1.38 | 1343.34 | 24343.36 |
| | -0.3 | -1.35 | 956.50 | 18510.98 |
| | -0.2 | -1.36 | 589.78 | 12431.94 |
| | -0.1 | -1.43 | 274.03 | 6254.26 |
| | 0.1 | 1.43 | 274.03 | -6254.26 |
| | 0.2 | 1.36 | 589.78 | -12431.94 |
| | 0.3 | 1.35 | 956.50 | -18510.98 |
| | 0.4 | 1.38 | 1343.34 | -24343.36 |
| 34 | -0.4 | -1.35 | 1246.18 | 22935.71 |
| | -0.3 | -1.31 | 881.03 | 17431.50 |
| | -0.2 | -1.33 | 539.17 | 11723.18 |
| | -0.1 | -1.43 | 247.69 | 5897.12 |
| | 0.1 | 1.43 | 247.69 | -5897.12 |
| | 0.2 | 1.33 | 539.17 | -11723.18 |
| | 0.3 | 1.31 | 881.03 | -17431.50 |
| | 0.4 | 1.35 | 1246.18 | -22935.71 |

Table 3.24: Angle of the resultant force, resultant force and moment required to rotate with the respect to the parameter $\alpha$ of the structure 4

| $\alpha$ (rad) | Rotation (rad) | Angle (rad) | Resultant Force (N) | Moment (N.mm) |
|---|---|---|---|---|
| 0.8 | -0.4 | -1.15 | 2693.29 | 32280.23 |
| | -0.3 | -1.14 | 1766.45 | 24783.79 |
| | -0.2 | -1.19 | 1005.09 | 16810.10 |
| | -0.1 | -1.33 | 433.45 | 8492.23 |
| | 0.1 | 1.33 | 433.45 | -8492.23 |
| | 0.2 | 1.19 | 1005.09 | -16810.10 |
| | 0.3 | 1.14 | 1766.45 | -24783.79 |
| | 0.4 | 1.15 | 2693.29 | -32280.23 |
| 1.0 | -0.4 | -1.32 | 2134.38 | 31721.89 |
| | -0.3 | -1.30 | 1480.09 | 24327.20 |
| | -0.2 | -1.34 | 897.66 | 16477.01 |
| | -0.1 | -1.43 | 412.44 | 8314.56 |
| | 0.1 | 1.43 | 412.44 | -8314.56 |
| | 0.2 | 1.34 | 897.66 | -16477.01 |
| | 0.3 | 1.30 | 1480.09 | -24327.20 |
| | 0.4 | 1.32 | 2134.38 | -31721.89 |
| 1.2 | -0.4 | -1.42 | 1906.32 | 31112.47 |
| | -0.3 | -1.39 | 1363.16 | 23810.53 |
| | -0.2 | -1.40 | 850.80 | 16096.66 |
| | -0.1 | -1.47 | 400.33 | 8111.94 |
| | 0.1 | 1.47 | 400.33 | -8111.94 |
| | 0.2 | 1.40 | 850.80 | -16096.66 |
| | 0.3 | 1.39 | 1363.16 | -23810.53 |
| | 0.4 | 1.42 | 1906.32 | -31112.47 |
| 1.4 | -0.4 | -1.47 | 1801.28 | 30425.80 |
| | -0.3 | -1.43 | 1304.51 | 23218.95 |
| | -0.2 | -1.43 | 822.87 | 15658.41 |
| | -0.1 | -1.48 | 390.50 | 7877.98 |
| | 0.1 | 1.48 | 390.50 | -7877.98 |
| | 0.2 | 1.43 | 822.87 | -15658.41 |
| | 0.3 | 1.43 | 1304.51 | -23218.95 |
| | 0.4 | 1.47 | 1801.28 | -30425.80 |
| 1.6 | -0.4 | -1.49 | 1744.64 | 29630.91 |
| | -0.3 | -1.44 | 1266.34 | 22530.04 |
| | -0.2 | -1.44 | 800.50 | 15147.22 |
| | -0.1 | -1.49 | 380.81 | 7605.02 |
| | 0.1 | 1.49 | 380.81 | -7605.02 |
| | 0.2 | 1.44 | 800.50 | -15147.22 |
| | 0.3 | 1.44 | 1266.34 | -22530.04 |
| | 0.4 | 1.49 | 1744.64 | -29630.91 |

Table 3.25: Angle of the resultant force, resultant force and moment required to rotate with the respect to the parameter R2 of the structure 4

| R2 (mm) | Rotation (rad) | Angle (rad) | Resultant Force (N) | Moment (N.mm) |
|---|---|---|---|---|
| 2.5 | -0.4 | -1.49 | 1744.64 | 29630.91 |
| | -0.3 | -1.44 | 1266.34 | 22530.04 |
| | -0.2 | -1.44 | 800.50 | 15147.22 |
| | -0.1 | -1.49 | 380.81 | 7605.02 |
| | 0.1 | 1.49 | 380.81 | -7605.02 |
| | 0.2 | 1.44 | 800.50 | -15147.22 |
| | 0.3 | 1.44 | 1266.34 | -22530.04 |
| | 0.4 | 1.49 | 1744.64 | -29630.91 |
| 3.0 | -0.4 | -1.56 | 1522.36 | 27831.42 |
| | -0.3 | -1.51 | 1114.53 | 21157.82 |
| | -0.2 | -1.51 | 716.00 | 14210.38 |
| | -0.1 | -1.53 | 346.98 | 7143.45 |
| | 0.1 | 1.53 | 346.98 | -7143.45 |
| | 0.2 | 1.51 | 716.00 | -14210.38 |
| | 0.3 | 1.51 | 1114.53 | -21157.82 |
| | 0.4 | 1.56 | 1522.36 | -27831.42 |
| 3.5 | -0.4 | -1.61 | 1355.05 | 26193.27 |
| | -0.3 | -1.57 | 999.16 | 19899.23 |
| | -0.2 | -1.55 | 649.51 | 13369.00 |
| | -0.1 | -1.55 | 318.00 | 6718.46 |
| | 0.1 | 1.55 | 318.00 | -6718.46 |
| | 0.2 | 1.55 | 649.51 | -13369.00 |
| | 0.3 | 1.57 | 999.16 | -19899.23 |
| | 0.4 | 1.61 | 1355.05 | -26193.27 |
| 4.0 | -0.4 | -1.66 | 1222.55 | 24688.02 |
| | -0.3 | -1.61 | 906.72 | 18735.58 |
| | -0.2 | -1.58 | 594.31 | 12590.00 |
| | -0.1 | -1.57 | 292.66 | 6327.05 |
| | 0.1 | 1.57 | 292.66 | -6327.05 |
| | 0.2 | 1.58 | 594.31 | -12590.00 |
| | 0.3 | 1.61 | 906.72 | -18735.58 |
| | 0.4 | 1.66 | 1222.55 | -24688.02 |
| 4.5 | -0.4 | -1.69 | 1112.73 | 23287.75 |
| | -0.3 | -1.64 | 828.93 | 17671.47 |
| | -0.2 | -1.60 | 546.39 | 11876.92 |
| | -0.1 | -1.58 | 270.26 | 5966.34 |
| | 0.1 | 1.58 | 270.26 | -5966.34 |
| | 0.2 | 1.60 | 546.39 | -11876.92 |
| | 0.3 | 1.64 | 828.93 | -17671.47 |
| | 0.4 | 1.69 | 1112.73 | -23287.75 |

# 4 Development of the FEM Models

Because a verification of the analytical results of the model in Julia is needed, the FEM models were developed in Ansys. A description of how the models are developed will be explained in the sections 4.1. After that, the structures of the section 3.3 will proved and the results will be presented. Finally, the results obtained in this chapter will be compared with the analytical results in the chapter 5.

## 4.1 Description of the FEM models

### 4.1.1 Procedure to obtain the displacements of the nodes due to external forces

In this subsection, the required steps to develop the FEM models, which can determine the displacement of the nodes due to external forces, are going to be described:

1. Start the program and select the “Static-Analysis” application.
2. Import the structure modeled in a Computer-Aided-Design (CAD) program. Because of compatibility, the model must be saved as STEP format.
3. The material must be assigned. As it was mentioned in the section 3.3, in this work only steel is going to be considered, which has a modulus of elasticity E = 200,000 $N/mm^2$.
4. In order to obtain the deformation values at the desired points, it is useful to define auxiliary coordinate systems at the nodes. After that, paths can be defined between the nodes. They are going to be useful to obtain the displacement at the nodes.
5. After that, the boundary conditions have to be defined. In Ansys Mechanical the geometrical conditions and the external loads can be defined . In this part, it is important to activate the “Large Deformations” field, which is located in “Analyses Settings”.
6. Finally, the mesh and the type of results to be obtained must be defined and the simulation executed.

### 4.1.2 Procedure to obtain the external forces due to the rotation of the structure

In this subsection, the required steps to develop the FEM models, which can determine the external forces due to the rotation of the structures, are going to be described:

1. Follow the steps 1 - 3 from 4.1.1.

2. The boundary conditions have to be defined. In this case, the first node of all structures are fixed and an external displacement has to be defined at the last node. It is important to consider not only the displacement but also the rotation of the node. In this part, it is important to activate the “Large Deformations” field, which is located in “Analyses Settings”.
3. Finally, the mesh and the type of results to be obtained must be defined and the simulation executed.

## 4.2 Results of the FEM simulation

### 4.2.1 Displacement determination of the nodes due to external forces

In this subsection, the results obtained in Ansys are presented. The geometrical measurements and boundary conditions are the same described in the subsection 3.3.1.

#### Structure A

This structure is conform by four nodes and three straight beams. Just the node 1 is fixed and the external forces are applied in the nodes three and four. The loaded and unloaded states can be seen in the Fig. 4.1 and the results of the displacements of the nodes can be seen in the table 4.1. The designation of the nodes and beam are the same as in the Fig. 3.10a. The moment obtained in the fixed node is $-486.88\,\mathrm{Nmm}$.

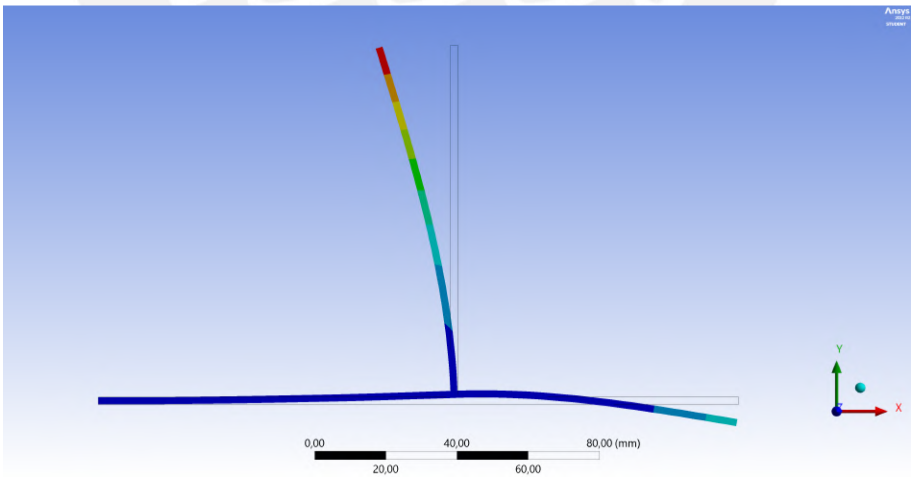



Figure 4.1: Loaded and unloaded states of the structure A

Table 4.1: Initial positions and displacements of the nodes in the X and Y direction in the structure A obtained by Ansys

| Node | Type | Initial Position $X_0$ (mm) | $Y_0$ (mm) | External forces $M_z$ (N.mm) | $F_x$ (N) | $F_y$ (N) | Displacement $\triangle$X (mm) | $\triangle$Y (mm) |
|---|---|---|---|---|---|---|---|---|
| 1 | Clamp | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | Branch | 100.00 | 0.00 | 0 | 0 | 0 | -0.02 | 1.81 |
| 3 | Branch | 180.00 | 0.00 | 0 | 100 | -100 | -0.56 | -6.13 |
| 4 | Free | 100.00 | 100.00 | 0 | -100 | 100 | -21.07 | -0.74 |

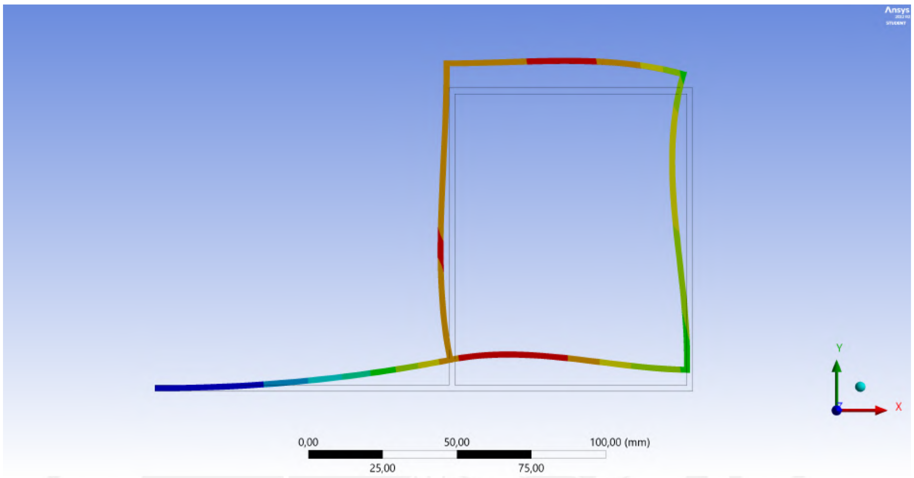



Figure 4.2: Loaded and unloaded states of the structure B

### Structure B

This structure is conform by five nodes and five straight beams. Just the node 1 is fixed and the external forces are applied at the rest of the nodes. The loaded and unloaded states can be seen in the Fig. 4.2 and the results of the displacements of the nodes can be seen in the table 4.2. The designation of the nodes and beams are the same as in the Fig. 3.11a. The moment in the fixed node is $-2451.9\,\mathrm{Nmm}$.

Table 4.2: Initial positions and displacements of the nodes in the X and Y direction in the structure B obtained by Ansys

| Node | Type | Initial Position $X_0$ (mm) | $Y_0$ (mm) | External forces $M_z$ (N.mm) | $F_x$ (N) | $F_y$ (N) | Displacement $\triangle$X (mm) | $\triangle$Y (mm) |
|---|---|---|---|---|---|---|---|---|
| 1 | Clamp | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | Branch | 100.00 | 0.00 | 30000.00 | 0.00 | 0.00 | -0.55 | 9.09 |
| 3 | Branch | 180.00 | 0.00 | 0.00 | 100.00 | -100.00 | -0.97 | 4.89 |
| 4 | Branch | 100.00 | 100.00 | 0.00 | -100.00 | 100.00 | -0.67 | 8.86 |
| 5 | Branch | 180.00 | 100.00 | -30000.00 | 0.00 | 0.00 | -1.10 | 4.34 |

### Structure C

This structure is conform by six nodes and six straight beams. Just the node 1 is fixed and the external forces are applied in the rest of the nodes. The loaded and unloaded states can be seen in the Fig. 4.3 and the results of the displacements of the nodes can be seen in the table 4.3. The designation of the nodes and beams are the same as in the Fig. 3.12a. The moment in the fixed node is $-821.79\,\mathrm{Nmm}$.

Table 4.3: Initial positions and displacements of the nodes in the X and Y direction in the structure C obtained by Ansys

| Node | Type | Initial Position $X_0$ (mm) | $Y_0$ (mm) | External forces $M_z$ (N.mm) | $F_x$ (N) | $F_y$ (N) | Displacement $\triangle$X (mm) | $\triangle$Y (mm) |
|---|---|---|---|---|---|---|---|---|
| 1 | Clamp | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | Branch | 100.00 | 0.00 | 0.00 | 0.00 | 30.00 | -0.06 | 3.00 |
| 3 | Branch | 180.00 | 0.00 | 0.00 | 30.00 | -30.00 | -0.23 | 8.12 |
| 4 | Branch | 280.00 | 0.00 | 2000.00 | -30.00 | 0.00 | -2.13 | 26.86 |
| 5 | Branch | 100.00 | 100.00 | 0.00 | -30.00 | 30.00 | -8.50 | 2.64 |
| 6 | Free | 180.00 | 100.00 | 0.00 | 0.00 | -30.00 | -8.66 | 7.76 |

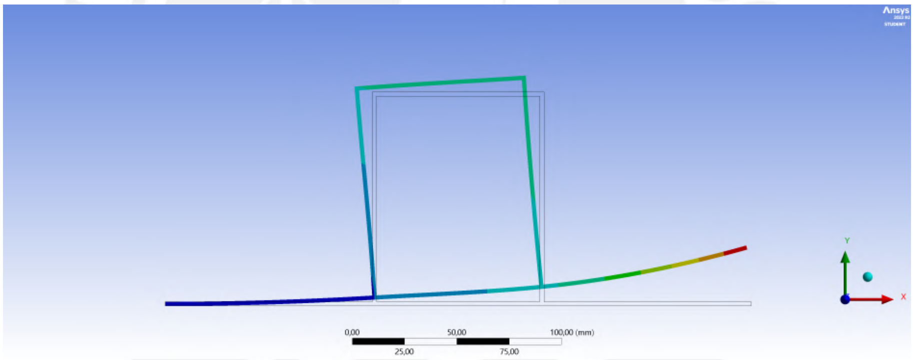


Figure 4.3: Loaded and unloaded states of the structure C

Table 4.4: Initial positions and displacements of the nodes in the X and Y direction in the structure D obtained by Ansys

| Node | Type | Initial Position $X_0$ (mm) | $Y_0$ (mm) | External forces $M_z$ (N.mm) | $F_x$ (N) | $F_y$ (N) | Displacement $\triangle$X (mm) | $\triangle$Y (mm) |
|---|---|---|---|---|---|---|---|---|
| 1 | Clamp | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | Branch | 100.00 | 0.00 | -2000.00 | 0.00 | 0.00 | -0.38 | 7.56 |
| 3 | Branch | 180.00 | 0.00 | 2000.00 | 0.00 | 0.00 | -1.57 | 21.24 |
| 4 | Branch | 100.00 | 100.00 | 0.00 | -30.00 | 30.00 | -18.57 | 5.88 |
| 5 | Branch | 180.00 | 100.00 | 0.00 | 0.00 | 0.00 | -19.75 | 19.57 |
| 6 | Branch | 100.00 | -100.00 | 0.00 | 0.00 | 0.00 | 17.79 | 9.23 |
| 7 | Branch | 180.00 | -100.00 | 0.00 | 30.00 | -30.00 | 16.61 | 22.91 |

### Structure D

This structure is conform by seven nodes and eight straight beams. Just the node 1 is fixed and the external forces are applied in the nodes two, three, four and seven. The loaded and unloaded states can be seen in the Fig. 4.4 and the results of the displacements of the nodes can be seen in the table 4.4. The designation of the nodes and beams can be seen in the Fig. 3.13a. The moment in the fixed node is $-2034.2\,\text{Nmm}$.

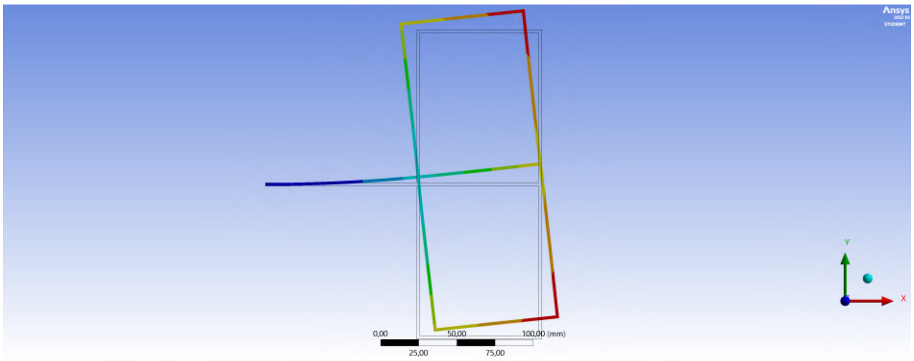


Figure 4.4: Loaded and unloaded states of the structure D

Table 4.5: Initial positions and displacements of the nodes in the X and Y direction in the structure E obtained by Ansys

| Node | Type | Initial Position $X_0$ (mm) | $Y_0$ (mm) | External forces $M_z$ (N.mm) | $F_x$ (N) | $F_y$ (N) | Displacement $\triangle$X (mm) | $\triangle$Y (mm) |
|---|---|---|---|---|---|---|---|---|
| 1 | Clamp | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | Branch | 50.00 | 0.00 | 0.00 | -10.00 | 10.00 | -0.17 | -3.58 |
| 3 | Branch | 150.00 | 100.00 | -3000.00 | 0.00 | 0.00 | 32.91 | -39.95 |
| 4 | Branch | 250.00 | 200.00 | 0.00 | 10.00 | -10.00 | 67.50 | -83.50 |
| 5 | Free | 300.00 | 200.00 | 3000.00 | 0.00 | 0.00 | 66.60 | -92.79 |

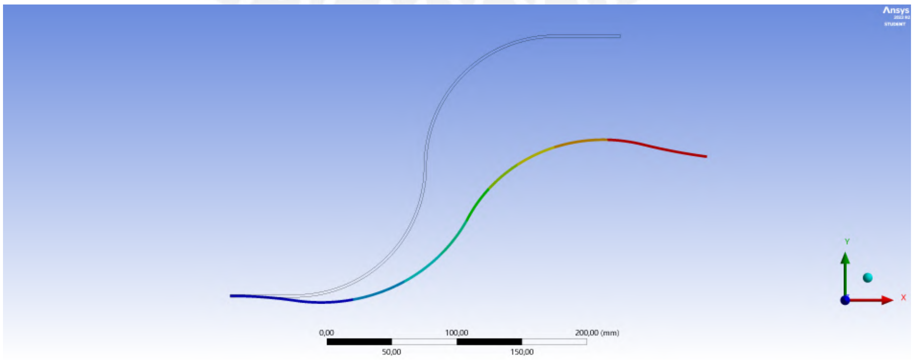


Figure 4.5: Loaded and unloaded states of the structure E

### Structure E

This structure is conform by five nodes and four beams, of which two beam are curved. Just the node 1 is fixed and the external forces are applied in the nodes two, three and four. The loaded and unloaded states can be seen in the Fig. 4.5 and the results of the displacements of the nodes can be seen in the table 4.5. The designation of the nodes and beam are the same as in the Fig. 3.14a. The moment in the fixed node is $-3880.6\,\text{Nmm}$.

Table 4.6: Initial positions and displacements of the nodes in the X and Y direction in the structure 1 obtained by Ansys

| Node | Type | Initial Position $X_0$ (mm) | $Y_0$ (mm) | External forces $M_z$ (N.mm) | $F_x$ (N) | $F_y$ (N) | Displacement $\triangle$X (mm) | $\triangle$Y (mm) |
|---|---|---|---|---|---|---|---|---|
| 1 | Clamp | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | Branch | 5.00 | 5.00 | 0.00 | 0.00 | 0.00 | -0.19 | 0.11 |
| 3 | Branch | 15.00 | 5.00 | 0.00 | 0.00 | 0.00 | 0.07 | 1.24 |
| 4 | Branch | 25.00 | 5.00 | 0.00 | 0.00 | 0.00 | -0.45 | 3.26 |
| 5 | Branch | 35.00 | 5.00 | 0.00 | 0.00 | 0.00 | -0.65 | 6.22 |
| 6 | Free | 45.00 | 5.00 | 5000.00 | -100.00 | 100.00 | -1.50 | 9.70 |

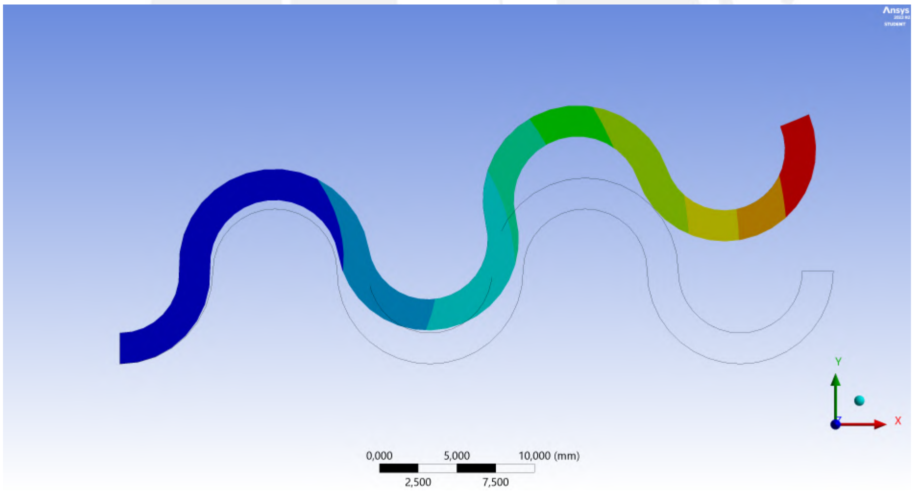



Figure 4.6: Loaded and unloaded states of the structure 1

### Structure 1

This structure is obtained from [29]. Just the node 1 is fixed and the external forces are applied in the sixth node. The loaded and unloaded states can be seen in the Fig. 4.6 and the results of the displacements of the nodes can be seen in the table 4.6. The designation of the nodes and beams can be seen in the Fig. 3.15a. The moment obtained in the fixed node is $-10787\,\text{Nmm}$.

### Structure 2

This structure is obtained from [5]. The first node is fixed and the external forces are applied in the sixth node. The loaded and unloaded states can be seen in the Fig. 4.7 and the results of the displacements of the nodes can be seen in the table 4.7. The designation of the nodes and beam are the same as in the Fig. 3.16a. The moment obtained in the fixed node is $-9997.3\,\mathrm{Nmm}$.

Table 4.7: Initial positions and displacements of the nodes in the X and Y direction in the structure 2 obtained by Ansys

| Node | Type | Initial Position | | External forces | | | Displacement | |
|---|---|---|---|---|---|---|---|---|
| | | $X_0$ (mm) | $Y_0$ (mm) | $M_z$ (N.mm) | $F_x$ (N) | $F_y$ (N) | $\triangle$X (mm) | $\triangle$Y (mm) |
| 1 | Clamp | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 2 | Branch | 12.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.50 |
| 3 | Branch | 7.64 | -14.71 | 0.00 | 0.00 | 0.00 | 2.38 | -1.01 |
| 4 | Branch | 3.45 | -11.98 | 0.00 | 0.00 | 0.00 | 1.82 | -2.16 |
| 5 | Branch | -1.95 | -17.71 | 0.00 | 0.00 | 0.00 | 3.84 | -3.39 |
| 6 | Free | 9.91 | -33.81 | 5000.00 | 100.00 | 100.00 | 9.01 | 2.24 |

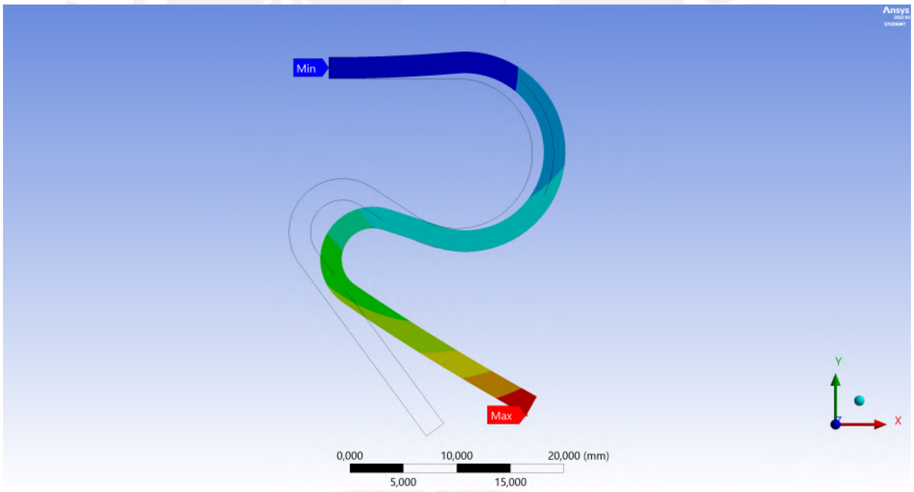



Figure 4.7: Loaded and unloaded states of the structure 2

### Structure 3

This structure is obtained from [27]. The node 1 is fixed and the external forces are applied in the fourteenth node. The loaded and unloaded states can be seen in the Fig. 4.8 and the results of the displacements of the nodes can be seen in the table 4.8. The designation of the nodes and beams are the same as in the Fig. 3.17a. The moment obtained in the fixed node is $-3595.8\,\mathrm{Nmm}$.

Table 4.8: Initial positions and displacements of the nodes in the X and Y direction in the structure 3 obtained by Ansys

| Node | Type | Initial Position $X_0$ (mm) | $Y_0$ (mm) | External forces $M_z$ (N.mm) | $F_x$ (N) | $F_y$ (N) | Displacement $\triangle$X (mm) | $\triangle$Y (mm) |
|---|---|---|---|---|---|---|---|---|
| 1 | Clamp | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 2 | Branch | 25.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | 0.81 |
| 3 | Branch | 23.24 | -11.74 | 0.00 | 0.00 | 0.00 | 1.02 | 0.49 |
| 4 | Branch | 6.03 | -6.45 | 0.00 | 0.00 | 0.00 | 0.47 | -1.88 |
| 5 | Branch | 3.54 | -11.85 | 0.00 | 0.00 | 0.00 | 1.41 | -2.19 |
| 6 | Branch | 18.70 | -21.55 | 0.00 | 0.00 | 0.00 | 3.05 | 1.04 |
| 7 | Branch | 10.90 | -30.49 | 0.00 | 0.00 | 0.00 | 5.37 | -0.77 |
| 8 | Branch | -0.76 | -16.77 | 0.00 | 0.00 | 0.00 | 1.93 | -4.69 |
| 9 | Branch | -5.77 | -19.97 | 0.00 | 0.00 | 0.00 | 3.23 | -6.08 |
| 10 | Branch | 1.79 | -36.31 | 0.00 | 0.00 | 0.00 | 8.44 | -2.43 |
| 11 | Branch | -9.60 | -39.63 | 0.00 | 0.00 | 0.00 | 10.48 | -6.72 |
| 12 | Branch | -12.03 | -21.80 | 0.00 | 0.00 | 0.00 | 3.06 | -9.50 |
| 13 | Branch | -17.97 | -21.80 | 0.00 | 0.00 | 0.00 | 3.75 | -12.21 |
| 14 | Free | -22.09 | -52.02 | 3000.00 | 10.00 | 10.00 | 19.41 | -10.21 |

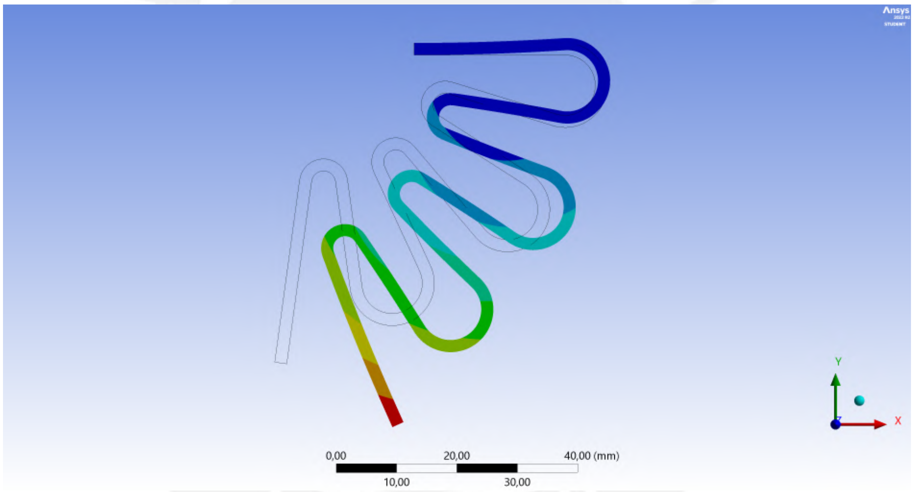



Figure 4.8: Loaded and unloaded states of the structure 3

### Structure 4

This structure is obtained from [3]. Just the node 1 is fixed and the external forces are applied in the third, sixth, seventh, eighth, ninth, twelfth, and fourteenth nodes. The loaded and unloaded states can be seen in the Fig. 4.9 and the results of the displacements of the nodes can be seen in the table 4.9. The designation of the nodes and beams are the same as in the Fig. 3.18a. The moment obtained in the fixed node is $-11054\,$Nmm.

Table 4.9: Initial positions and displacements of the nodes in the X and Y direction in the structure 4 obtained by Ansys

| Node | Type | Initial Position $X_0$ (mm) | $Y_0$ (mm) | External forces $M_z$ (N.mm) | $F_x$ (N) | $F_y$ (N) | Displacement $\triangle$X (mm) | $\triangle$Y (mm) |
|---|---|---|---|---|---|---|---|---|
| 1 | Clamp | 15.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 2 | Branch | 30.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 | 0.92 |
| 3 | Branch | 20.90 | 21.52 | 0.00 | -50.00 | -50.00 | -3.11 | -0.75 |
| 4 | Branch | 20.90 | -21.52 | 0.00 | 0.00 | 0.00 | 3.33 | -0.29 |
| 5 | Branch | 27.17 | 27.98 | 0.00 | 0.00 | 0.00 | -4.23 | 0.13 |
| 6 | Branch | 27.17 | -27.98 | 0.00 | 50.00 | 0.00 | 4.34 | 0.79 |
| 7 | Branch | 39.00 | 0.00 | 3000.00 | 0.00 | 0.00 | -0.07 | 2.21 |
| 8 | Branch | 54.00 | 0.00 | -3000.00 | 0.00 | 0.00 | -0.40 | 5.32 |
| 9 | Branch | 37.62 | 38.74 | 0.00 | -50.00 | -50.00 | -10.37 | -0.84 |
| 10 | Branch | 37.62 | -38.74 | 0.00 | 0.00 | 0.00 | 11.19 | 2.10 |
| 11 | Branch | 43.89 | 45.19 | 0.00 | 0.00 | 0.00 | -12.49 | 0.66 |
| 12 | Branch | 43.89 | -45.19 | 0.00 | 50.00 | 0.00 | 12.85 | 4.24 |
| 13 | Branch | 63.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.58 | 7.79 |
| 14 | Free | 78.00 | 0.00 | 3000.00 | 50.00 | 50.00 | -1.31 | 12.40 |

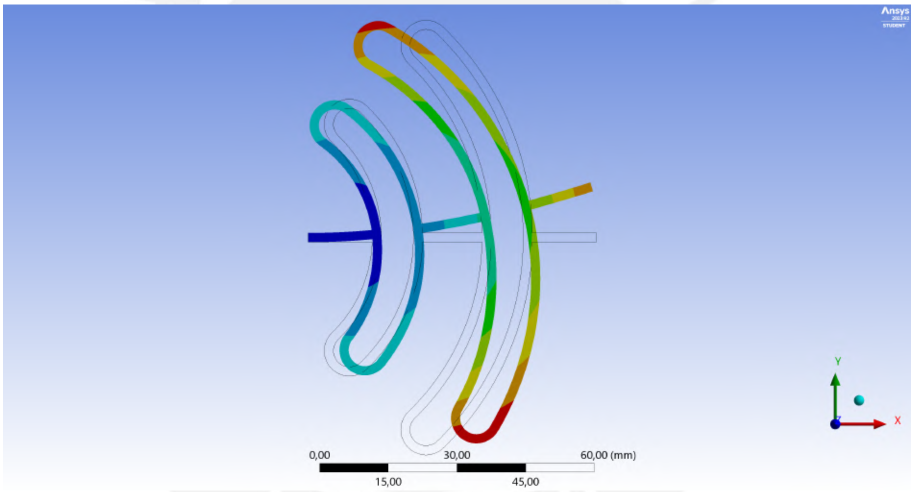



Figure 4.9: Loaded and unloaded states of the structure 4

### 4.2.2 Force determination due to rotation of the structures

In this subsection we will obtain the forces and moments necessary to carry out the movement of the structure. In this case, structures 1, 2, 3 and 4 of subsection 3.3.2 will be used. In all the cases, the first node is going to be fixed and the last nodes are going to move and rotate. At the same time, the parameter are going to variate. In the table 3.11 it can be seen the parameters and variations of each structure.

#### Structure 1

This structure has only one parameter, which is the radius of curvature of the beams. As it can be seen in the table 3.11, the parameter $R1$ varies from $5mm$ to $9mm$ with $1mm$ steps. At the same time, the structure rotates from a node located 15 $mm$ to the left of the node 1. The element size of the net used is $1mm$ in order to obtain better results. In the table 4.10 can be seen the angles of the resultant force, the resultant forces and moments obtained from the simulation in Ansys when the parameter $R1$ varies.

#### Structure 2

This structure has five parameters, which are the radius of curvature of it two curved beams and the lengths of the straight beams. As it can be seen in the table 3.11, the parameter $R1$ varies from $8mm$ to $12mm$ with $1mm$ steps, the parameter $R2$ varies from $4mm$ to $8mm$ with $1mm$ steps, $L1$ varies from $12mm$ to $16mm$ with $1mm$ steps, $L2$ varies from $5mm$ to $9mm$ with 1mm steps and $L3$ varies from $20mm$ to $24mm$ with $1mm$ steps. In the tables 4.11, 4.12, 4.13, 4.14 and 4.15 can be seen the angles of the resultant force, the resultant forces and moments obtained from the simulation in Ansys when the parameters $R1$, $R2$, $L1$, $L2$ and $L3$ vary, respectively.

#### Structure 3

This structure has five parameters, which are the radius of curvature of the curved beams and the lengths of the straight beams. As it can be seen in the table 3.11, the parameter $R1$ varies from $6mm$ to $10mm$ with $1mm$ steps, the parameter $R2$ varies from $3mm$ to $7mm$ with $1mm$ steps, $L1$ varies from $25mm$ to $29mm$ with $1mm$ steps, $L2$ varies from $12mm$ to $16mm$ with $1mm$ steps and $L3$ varies from $35mm$ to $39mm$ with $1mm$ steps. In the tables 4.16, 4.17, 4.18, 4.19 and 4.20 can be seen the angles of the resultant force, the resultant forces and moments obtained from the simulation in Ansys when the parameters $R1$, $R2$, $L1$, $L2$ and $L3$ vary, respectively.

#### Structure 4

This structure has three parameters. As it can be seen in the table 3.11, the parameter $R1$ varies from $30mm$ to $34mm$ with $1mm$ steps, the parameter $\alpha$ varies from $0.8rad$ to $1.6rad$ with $0.2rad$ steps and $R2$ varies from $2.5mm$ to $4.5mm$ with $0.5mm$ steps. In the tables 4.21, 4.22 can be seen the angles of the resultant force, the resultant forces and moments obtained from the simulation in Ansys when the parameter $R1$, $\alpha$ and $R2$ vary, respectively.

Table 4.10: Angle of the resultant force, resultant force and moment required to rotate with the respect to the parameter R1 of the structure 1

| R1 (mm) | Rotation (rad) | Angle (rad) | Resultant Force (N) | Moment (N.mm) |
|---|---|---|---|---|
| 5 | -0.4 | -0.81 | 8438.58 | 43278.00 |
| | -0.3 | -0.70 | 5477.39 | 30300.00 |
| | -0.2 | -0.60 | 3128.77 | 19104.00 |
| | -0.1 | -0.51 | 1320.74 | 9138.40 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 2.74 | 890.20 | -8734.50 |
| | 0.2 | 2.71 | 1398.26 | -17206.00 |
| | 0.3 | 2.57 | 1617.91 | -25461.00 |
| | 0.4 | 2.22 | 1807.52 | -33159.00 |
| 6 | -0.4 | -0.79 | 5321.46 | 32541.00 |
| | -0.3 | -0.69 | 3505.84 | 22994.00 |
| | -0.2 | -0.59 | 2035.56 | 14611.00 |
| | -0.1 | -0.50 | 873.23 | 7041.30 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 2.77 | 620.29 | -6770.40 |
| | 0.2 | 2.79 | 1021.44 | -13438.00 |
| | 0.3 | 2.73 | 1216.91 | -19953.00 |
| | 0.4 | 2.54 | 1296.77 | -26234.00 |
| 7 | -0.4 | -0.78 | 3483.16 | 24693.00 |
| | -0.3 | -0.68 | 2418.55 | 18298.00 |
| | -0.2 | -0.58 | 1421.74 | 11701.00 |
| | -0.1 | -0.49 | 622.17 | 5668.60 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 2.80 | 461.46 | -5494.10 |
| | 0.2 | 2.83 | 775.45 | -10940.00 |
| | 0.3 | 2.82 | 952.29 | -16320.00 |
| | 0.4 | 2.72 | 1027.78 | -21569.00 |
| 8 | -0.4 | -0.77 | 2527.54 | 20319.00 |
| | -0.3 | -0.67 | 1769.22 | 15118.00 |
| | -0.2 | -0.57 | 1049.49 | 9698.10 |
| | -0.1 | -0.48 | 462.24 | 4723.40 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 2.81 | 352.00 | -4602.20 |
| | 0.2 | 2.86 | 604.61 | -9170.50 |
| | 0.3 | 2.88 | 764.59 | -13755.00 |
| | 0.4 | 2.83 | 839.86 | -18269.00 |
| 9 | -0.4 | -0.76 | 1993.80 | 17845.00 |
| | -0.3 | -0.66 | 1343.67 | 12803.00 |
| | -0.2 | -0.57 | 801.97 | 8246.80 |
| | -0.1 | -0.47 | 358.48 | 4029.90 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 2.83 | 279.63 | -3948.90 |
| | 0.2 | 2.88 | 485.53 | -7889.50 |
| | 0.3 | 2.92 | 624.89 | -11858.00 |
| | 0.4 | 2.91 | 700.42 | -15806.00 |

Table 4.11: Angle of the resultant force, resultant force and moment required to rotate with the respect to the parameter R1 of the structure 2

| R1 (mm) | Rotation (rad) | Angle (rad) | Resultant Force (N) | Moment (N.mm) |
|---|---|---|---|---|
| 8 | -0.4 | -2.24 | 4694.63 | 31824.00 |
| | -0.3 | -2.18 | 3451.56 | 24844.00 |
| | -0.2 | -2.12 | 2255.30 | 17184.00 |
| | -0.1 | -2.06 | 1108.44 | 8939.40 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.20 | 1081.27 | -9662.40 |
| | 0.2 | 1.26 | 2143.65 | -20067.00 |
| | 0.3 | 1.32 | 3210.67 | -31287.00 |
| | 0.4 | 1.38 | 4285.20 | -43481.00 |
| 9 | -0.4 | -2.31 | 3977.92 | 25940.00 |
| | -0.3 | -2.25 | 2932.33 | 20267.00 |
| | -0.2 | -2.19 | 1924.49 | 14106.00 |
| | -0.1 | -2.13 | 950.69 | 7343.30 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.12 | 932.02 | -7933.10 |
| | 0.2 | 1.18 | 1858.12 | -16549.00 |
| | 0.3 | 1.24 | 2787.62 | -25776.00 |
| | 0.4 | 1.30 | 3729.14 | -35789.00 |
| 10 | -0.4 | -2.37 | 3371.35 | 20632.00 |
| | -0.3 | -2.31 | 2490.21 | 16141.00 |
| | -0.2 | -2.26 | 1640.21 | 11257.00 |
| | -0.1 | -2.20 | 811.68 | 5875.80 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.05 | 802.41 | -6391.30 |
| | 0.2 | 1.11 | 1601.43 | -13271.00 |
| | 0.3 | 1.17 | 2405.28 | -20703.00 |
| | 0.4 | 1.22 | 3221.60 | -28744.00 |
| 11 | -0.4 | -2.43 | 2855.69 | 15883.00 |
| | -0.3 | -2.37 | 2118.07 | 12507.00 |
| | -0.2 | -2.32 | 1396.75 | 8723.80 |
| | -0.1 | -2.26 | 694.24 | 4577.80 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 0.99 | 688.13 | -4986.20 |
| | 0.2 | 1.05 | 1372.33 | -10330.00 |
| | 0.3 | 1.10 | 2068.42 | -16226.00 |
| | 0.4 | 1.16 | 2770.89 | -22502.00 |
| 12 | -0.4 | -2.48 | 2422.05 | 11740.00 |
| | -0.3 | -2.43 | 1800.89 | 9322.00 |
| | -0.2 | -2.37 | 1188.71 | 6540.20 |
| | -0.1 | -2.31 | 591.65 | 3440.40 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 0.94 | 588.84 | -3786.10 |
| | 0.2 | 0.99 | 1176.03 | -7855.80 |
| | 0.3 | 1.05 | 1772.72 | -12326.00 |
| | 0.4 | 1.10 | 2378.16 | -17152.00 |

Table 4.12: Angle of the resultant force, resultant force and moment required to rotate with the respect to the parameter R2 of the structure 2

| R2 (mm) | Rotation (rad) | Angle (rad) | Resultant Force (N) | Moment (N.mm) |
|---|---|---|---|---|
| 4 | -0.4 | -2.24 | 4694.63 | 31824.00 |
| | -0.3 | -2.18 | 3451.56 | 24844.00 |
| | -0.2 | -2.12 | 2255.30 | 17184.00 |
| | -0.1 | -2.06 | 1108.44 | 8939.40 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.20 | 1081.27 | -9662.40 |
| | 0.2 | 1.26 | 2143.65 | -20067.00 |
| | 0.3 | 1.32 | 3210.67 | -31287.00 |
| | 0.4 | 1.38 | 4285.20 | -43481.00 |
| 5 | -0.4 | -2.30 | 4332.98 | 29812.00 |
| | -0.3 | -2.25 | 3179.23 | 23264.00 |
| | -0.2 | -2.19 | 2076.84 | 16142.00 |
| | -0.1 | -2.13 | 1018.51 | 8388.70 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.13 | 991.21 | -9052.30 |
| | 0.2 | 1.19 | 1967.82 | -18831.00 |
| | 0.3 | 1.25 | 2940.77 | -29289.00 |
| | 0.4 | 1.30 | 3923.26 | -40663.00 |
| 6 | -0.4 | -2.36 | 4048.97 | 28031.00 |
| | -0.3 | -2.31 | 2966.07 | 21875.00 |
| | -0.2 | -2.25 | 1932.25 | 15157.00 |
| | -0.1 | -2.19 | 948.31 | 7902.60 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.06 | 920.59 | -8526.60 |
| | 0.2 | 1.12 | 1821.36 | -17702.00 |
| | 0.3 | 1.18 | 2715.63 | -27482.00 |
| | 0.4 | 1.23 | 3615.37 | -38087.00 |
| 7 | -0.4 | -2.42 | 3807.18 | 26298.00 |
| | -0.3 | -2.36 | 2788.07 | 20592.00 |
| | -0.2 | -2.31 | 1817.09 | 13490.00 |
| | -0.1 | -2.25 | 887.64 | 7423.70 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.00 | 859.20 | -8038.20 |
| | 0.2 | 1.06 | 1698.52 | -16665.00 |
| | 0.3 | 1.12 | 2530.70 | -25934.00 |
| | 0.4 | 1.17 | 3361.94 | -35880.00 |
| 8 | -0.4 | -2.47 | 3616.08 | 24750.00 |
| | -0.3 | -2.42 | 2646.41 | 19438.00 |
| | -0.2 | -2.36 | 1719.77 | 13490.00 |
| | -0.1 | -2.30 | 842.76 | 7045.90 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 0.95 | 812.19 | -7629.60 |
| | 0.2 | 1.00 | 1598.19 | -15781.00 |
| | 0.3 | 1.06 | 2377.10 | -24563.00 |
| | 0.4 | 1.11 | 3149.77 | -33942.00 |

Table 4.13: Angle of the resultant force, resultant force and moment required to rotate with the respect to the parameter L1 of the structure 2

| L1 (mm) | Rotation (rad) | Angle (rad) | Resultant Force (N) | Moment (N.mm) |
|---|---|---|---|---|
| 12 | -0.4 | -2.24 | 4694.63 | 31824.00 |
| | -0.3 | -2.18 | 3451.56 | 24844.00 |
| | -0.2 | -2.12 | 2255.30 | 17184.00 |
| | -0.1 | -2.06 | 1108.44 | 8939.40 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.20 | 1081.27 | -9662.40 |
| | 0.2 | 1.26 | 2143.65 | -20067.00 |
| | 0.3 | 1.32 | 3210.67 | -31287.00 |
| | 0.4 | 1.38 | 4285.20 | -43481.00 |
| 13 | -0.4 | -2.21 | 4583.60 | 31312.00 |
| | -0.3 | -2.15 | 3369.05 | 24391.00 |
| | -0.2 | -2.09 | 2205.44 | 16916.00 |
| | -0.1 | -2.02 | 1086.98 | 8804.90 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.24 | 1059.92 | -9497.50 |
| | 0.2 | 1.30 | 2112.75 | -19798.00 |
| | 0.3 | 1.36 | 3162.50 | -30896.00 |
| | 0.4 | 1.42 | 4228.52 | -43004.00 |
| 14 | -0.4 | -2.18 | 4458.30 | 30604.00 |
| | -0.3 | -2.12 | 3285.40 | 23878.00 |
| | -0.2 | -2.05 | 2153.73 | 16542.00 |
| | -0.1 | -1.99 | 1061.47 | 8619.00 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.28 | 1037.87 | -9300.10 |
| | 0.2 | 1.34 | 2073.28 | -19441.00 |
| | 0.3 | 1.40 | 3107.82 | -30346.00 |
| | 0.4 | 1.47 | 4154.71 | -42272.00 |
| 15 | -0.4 | -2.15 | 4337.08 | 29899.00 |
| | -0.3 | -2.09 | 3191.92 | 23245.00 |
| | -0.2 | -2.02 | 2094.79 | 16144.00 |
| | -0.1 | -1.96 | 1035.32 | 8394.30 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.31 | 1016.02 | -9101.90 |
| | 0.2 | 1.38 | 2025.87 | -18984.00 |
| | 0.3 | 1.44 | 3044.57 | -29725.00 |
| | 0.4 | 1.51 | 4072.72 | -41452.00 |
| 16 | -0.4 | -2.13 | 4210.38 | 29031.00 |
| | -0.3 | -2.06 | 3107.03 | 22631.00 |
| | -0.2 | -2.00 | 2033.74 | 15653.00 |
| | -0.1 | -1.93 | 1007.11 | 8171.90 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.34 | 990.23 | -8857.80 |
| | 0.2 | 1.41 | 1977.39 | -18486.00 |
| | 0.3 | 1.48 | 2971.44 | -28939.00 |
| | 0.4 | 1.55 | 3979.18 | -40452.00 |

Table 4.14: Angle of the resultant force, resultant force and moment required to rotate with the respect to the parameter L2 of the structure 2

| L2 (mm) | Rotation (rad) | Angle (rad) | Resultant Force (N) | Moment (N.mm) |
|---|---|---|---|---|
| | -0.4 | -2.24 | 4694.63 | 31824.00 |
| | -0.3 | -2.18 | 3451.56 | 24844.00 |
| | -0.2 | -2.12 | 2255.30 | 17184.00 |
| | -0.1 | -2.06 | 1108.44 | 8939.40 |
| 5 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.20 | 1081.27 | -9662.40 |
| | 0.2 | 1.26 | 2143.65 | -20067.00 |
| | 0.3 | 1.32 | 3210.67 | -31287.00 |
| | 0.4 | 1.38 | 4285.20 | -43481.00 |
| | -0.4 | -2.27 | 4415.25 | 30147.00 |
| | -0.3 | -2.22 | 3243.36 | 23469.00 |
| | -0.2 | -2.16 | 2124.14 | 16270.00 |
| | -0.1 | -2.10 | 1045.26 | 8438.90 |
| 6 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.15 | 1021.59 | -9125.80 |
| | 0.2 | 1.21 | 2030.63 | -18903.00 |
| | 0.3 | 1.27 | 3040.35 | -29396.00 |
| | 0.4 | 1.32 | 4064.58 | -40743.00 |
| | -0.4 | -2.31 | 4145.67 | 28417.00 |
| | -0.3 | -2.25 | 3053.37 | 22181.00 |
| | -0.2 | -2.20 | 1999.67 | 15346.00 |
| | -0.1 | -2.14 | 985.07 | 7967.40 |
| 7 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.11 | 964.01 | -8547.00 |
| | 0.2 | 1.16 | 1918.09 | -17729.00 |
| | 0.3 | 1.21 | 2879.24 | -27545.00 |
| | 0.4 | 1.27 | 3848.13 | -38016.00 |
| | -0.4 | -2.35 | 3898.54 | 26689.00 |
| | -0.3 | -2.29 | 2873.26 | 20846.00 |
| | -0.2 | -2.24 | 1881.85 | 14418.00 |
| | -0.1 | -2.18 | 927.33 | 7455.30 |
| 8 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.06 | 911.61 | -8017.30 |
| | 0.2 | 1.11 | 1812.34 | -16570.00 |
| | 0.3 | 1.16 | 2720.05 | -25652.00 |
| | 0.4 | 1.21 | 3636.73 | -35292.00 |
| | -0.4 | -2.39 | 3656.96 | 24919.00 |
| | -0.3 | -2.33 | 2702.70 | 19511.00 |
| | -0.2 | -2.28 | 1773.65 | 13502.00 |
| | -0.1 | -2.23 | 876.15 | 6995.40 |
| 9 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.01 | 859.95 | -7477.00 |
| | 0.2 | 1.06 | 1713.13 | -15430.00 |
| | 0.3 | 1.11 | 2567.19 | -23768.00 |
| | 0.4 | 1.15 | 3433.68 | -32591.00 |

Table 4.15: Angle of the resultant force, resultant force and moment required to rotate with the respect to the parameter L3 of the structure 2

| L3 (mm) | Rotation (rad) | Angle (rad) | Resultant Force (N) | Moment (N.mm) |
|---|---|---|---|---|
| 20 | -0.4 | -2.24 | 4694.63 | 31824.00 |
| | -0.3 | -2.18 | 3451.56 | 24844.00 |
| | -0.2 | -2.12 | 2255.30 | 17184.00 |
| | -0.1 | -2.06 | 1108.44 | 8939.40 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.20 | 1081.27 | -9662.40 |
| | 0.2 | 1.26 | 2143.65 | -20067.00 |
| | 0.3 | 1.32 | 3210.67 | -31287.00 |
| | 0.4 | 1.38 | 4285.20 | -43481.00 |
| 21 | -0.4 | -2.22 | 4610.77 | 32316.00 |
| | -0.3 | -2.16 | 3381.89 | 25199.00 |
| | -0.2 | -2.10 | 2206.95 | 17472.00 |
| | -0.1 | -2.04 | 1079.38 | 9044.20 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.22 | 1050.04 | -9801.90 |
| | 0.2 | 1.28 | 2081.68 | -20381.00 |
| | 0.3 | 1.34 | 3107.84 | -31725.00 |
| | 0.4 | 1.40 | 4145.76 | -44174.00 |
| 22 | -0.4 | -2.21 | 4527.91 | 32674.00 |
| | -0.3 | -2.15 | 3312.58 | 25446.00 |
| | -0.2 | -2.08 | 2156.68 | 17638.00 |
| | -0.1 | -2.02 | 1055.89 | 9170.20 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.24 | 1018.84 | -9892.80 |
| | 0.2 | 1.30 | 2016.02 | -20562.00 |
| | 0.3 | 1.36 | 2983.49 | -31788.00 |
| | 0.4 | 1.43 | 3999.52 | -44584.00 |
| 23 | -0.4 | -2.19 | 4444.78 | 32945.00 |
| | -0.3 | -2.13 | 3246.36 | 25640.00 |
| | -0.2 | -2.07 | 2106.80 | 17736.00 |
| | -0.1 | -2.00 | 1027.24 | 9197.00 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.26 | 990.61 | -9931.90 |
| | 0.2 | 1.33 | 1954.50 | -20684.00 |
| | 0.3 | 1.39 | 2899.65 | -32267.00 |
| | 0.4 | 1.45 | 3854.36 | -44833.00 |
| 24 | -0.4 | -2.18 | 4364.68 | 33079.00 |
| | -0.3 | -2.11 | 3178.97 | 25708.00 |
| | -0.2 | -2.05 | 2057.77 | 17775.00 |
| | -0.1 | -1.98 | 1002.78 | 9214.70 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.28 | 960.19 | -9943.70 |
| | 0.2 | 1.35 | 1889.69 | -20659.00 |
| | 0.3 | 1.41 | 2799.95 | -32321.00 |
| | 0.4 | 1.47 | 3709.31 | -44957.00 |

Table 4.16: Angle of the resultant force, resultant force and moment required to rotate with the respect to the parameter R1 of the structure 3

| R1 (mm) | Rotation (rad) | Angle (rad) | Resultant Force (N) | Moment (N.mm) |
|---|---|---|---|---|
| 6 | -0.4 | -2.53 | 1254.90 | 7084.60 |
| | -0.3 | -2.46 | 946.24 | 5975.20 |
| | -0.2 | -2.40 | 632.81 | 4430.60 |
| | -0.1 | -2.33 | 318.45 | 2468.50 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 0.94 | 321.50 | -3014.30 |
| | 0.2 | 1.00 | 646.65 | -6653.90 |
| | 0.3 | 1.07 | 975.30 | -10971.00 |
| | 0.4 | 1.13 | 1308.16 | -15911.00 |
| 7 | -0.4 | -2.63 | 1027.65 | 3863.90 |
| | -0.3 | -2.57 | 776.85 | 3428.60 |
| | -0.2 | -2.50 | 522.19 | 2653.60 |
| | -0.1 | -2.44 | 263.06 | 1520.50 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 0.83 | 268.18 | -1946.10 |
| | 0.2 | 0.90 | 541.33 | -4356.60 |
| | 0.3 | 0.96 | 819.60 | -7277.30 |
| | 0.4 | 1.02 | 1104.20 | -10758.00 |
| 8 | -0.4 | -2.71 | 830.04 | 1083.50 |
| | -0.3 | -2.66 | 628.97 | 1303.90 |
| | -0.2 | -2.60 | 424.14 | 1160.20 |
| | -0.1 | -2.53 | 214.63 | 728.71 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 0.73 | 219.26 | -1048.60 |
| | 0.2 | 0.80 | 443.86 | -2432.80 |
| | 0.3 | 0.86 | 673.95 | -4185.20 |
| | 0.4 | 0.92 | 909.36 | -6329.70 |
| 9 | -0.4 | -2.81 | 663.45 | -922.46 |
| | -0.3 | -2.75 | 504.49 | -353.08 |
| | -0.2 | -2.68 | 340.71 | -11.64 |
| | -0.1 | -2.62 | 172.97 | 110.07 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 0.64 | 177.46 | -346.03 |
| | 0.2 | 0.70 | 359.46 | -944.37 |
| | 0.3 | 0.77 | 546.20 | -1785.60 |
| | 0.4 | 0.83 | 737.16 | -2909.60 |
| 10 | -0.4 | -2.89 | 526.26 | -2464.30 |
| | -0.3 | -2.83 | 401.41 | -1572.10 |
| | -0.2 | -2.77 | 271.85 | -869.53 |
| | -0.1 | -2.71 | 138.00 | -347.22 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 0.56 | 142.58 | 166.00 |
| | 0.2 | 0.62 | 289.08 | 151.29 |
| | 0.3 | 0.68 | 439.88 | -48.19 |
| | 0.4 | 0.74 | 594.35 | -445.64 |

Table 4.17: Angle of the resultant force, resultant force and moment required to rotate with the respect to the parameter R2 of the structure 3

| R2 (mm) | Rotation (rad) | Angle (rad) | Resultant Force (N) | Moment (N.mm) |
|---|---|---|---|---|
| | -0.4 | -2.53 | 1254.90 | 7084.60 |
| | -0.3 | -2.46 | 946.24 | 5975.20 |
| | -0.2 | -2.40 | 632.81 | 4430.60 |
| | -0.1 | -2.33 | 318.45 | 2468.50 |
| 3 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 0.94 | 321.50 | -3014.30 |
| | 0.2 | 1.00 | 646.65 | -6653.90 |
| | 0.3 | 1.07 | 975.30 | -10971.00 |
| | 0.4 | 1.13 | 1308.16 | -15911.00 |
| | -0.4 | -2.66 | 1039.96 | 3422.90 |
| | -0.3 | -2.59 | 787.50 | 3103.50 |
| | -0.2 | -2.53 | 530.47 | 2441.70 |
| | -0.1 | -2.46 | 267.73 | 1418.10 |
| 4 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 0.81 | 273.72 | -1855.50 |
| | 0.2 | 0.87 | 553.55 | -4196.30 |
| | 0.3 | 0.94 | 840.40 | -7096.00 |
| | 0.4 | 1.00 | 1133.72 | -10603.00 |
| | -0.4 | -2.77 | 831.85 | 309.46 |
| | -0.3 | -2.71 | 634.15 | 653.53 |
| | -0.2 | -2.65 | 429.30 | 725.09 |
| | -0.1 | -2.58 | 218.32 | 513.95 |
| 5 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 0.68 | 224.88 | -832.18 |
| | 0.2 | 0.75 | 457.74 | -2018.00 |
| | 0.3 | 0.81 | 697.62 | -3578.60 |
| | 0.4 | 0.88 | 945.71 | -5605.50 |
| | -0.4 | -2.89 | 641.42 | -2075.70 |
| | -0.3 | -2.83 | 493.18 | -1233.70 |
| | -0.2 | -2.76 | 336.23 | -604.13 |
| | -0.1 | -2.70 | 171.41 | -193.22 |
| 6 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 0.57 | 179.21 | -30.29 |
| | 0.2 | 0.63 | 366.04 | -303.36 |
| | 0.3 | 0.69 | 560.76 | -830.09 |
| | 0.4 | 0.75 | 763.26 | -1649.00 |
| | -0.4 | -3.00 | 479.45 | -3647.70 |
| | -0.3 | -2.94 | 371.95 | -2502.50 |
| | -0.2 | -2.88 | 255.69 | -1515.50 |
| | -0.1 | -2.82 | 131.66 | -679.40 |
| 7 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 0.45 | 139.36 | 523.76 |
| | 0.2 | 0.51 | 286.37 | 891.80 |
| | 0.3 | 0.58 | 441.30 | 1092.20 |
| | 0.4 | 0.64 | 603.44 | 1114.10 |

Table 4.18: Angle of the resultant force, resultant force and moment required to rotate with the respect to the parameter L1 of the structure 3

| L1 (mm) | Rotation (rad) | Angle (rad) | Resultant Force (N) | Moment (N.mm) |
|---|---|---|---|---|
| | -0.4 | -2.53 | 1254.90 | 7084.60 |
| | -0.3 | -2.46 | 946.24 | 5975.20 |
| | -0.2 | -2.40 | 632.81 | 4430.60 |
| | -0.1 | -2.33 | 318.45 | 2468.50 |
| 25 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 0.94 | 321.50 | -3014.30 |
| | 0.2 | 1.00 | 646.65 | -6653.90 |
| | 0.3 | 1.07 | 975.30 | -10971.00 |
| | 0.4 | 1.13 | 1308.16 | -15911.00 |
| | -0.4 | -2.50 | 1258.39 | 7571.30 |
| | -0.3 | -2.44 | 946.91 | 6322.40 |
| | -0.2 | -2.37 | 633.54 | 4669.40 |
| | -0.1 | -2.31 | 317.31 | 2570.20 |
| 26 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 0.97 | 320.49 | -3115.30 |
| | 0.2 | 1.03 | 644.55 | -6843.80 |
| | 0.3 | 1.10 | 972.61 | -11238.00 |
| | 0.4 | 1.16 | 1305.43 | -16354.00 |
| | -0.4 | -2.48 | 1257.21 | 7999.60 |
| | -0.3 | -2.40 | 914.74 | 6157.50 |
| | -0.2 | -2.35 | 632.17 | 4869.00 |
| | -0.1 | -2.28 | 316.61 | 2669.50 |
| 27 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.00 | 319.44 | -3203.00 |
| | 0.2 | 1.06 | 641.72 | -7001.80 |
| | 0.3 | 1.13 | 967.85 | -11460.00 |
| | 0.4 | 1.19 | 1298.62 | -16587.00 |
| | -0.4 | -2.46 | 1251.79 | 8357.10 |
| | -0.3 | -2.39 | 940.75 | 6886.30 |
| | -0.2 | -2.32 | 627.96 | 5023.90 |
| | -0.1 | -2.25 | 314.82 | 2750.60 |
| 28 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.02 | 317.43 | -3270.10 |
| | 0.2 | 1.09 | 637.11 | -7116.20 |
| | 0.3 | 1.16 | 952.71 | -11527.00 |
| | 0.4 | 1.22 | 1289.31 | -16747.00 |
| | -0.4 | -2.43 | 1244.17 | 8669.10 |
| | -0.3 | -2.36 | 934.63 | 7107.90 |
| | -0.2 | -2.30 | 623.83 | 5167.10 |
| | -0.1 | -2.23 | 312.79 | 2817.30 |
| 29 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.05 | 314.58 | -3318.70 |
| | 0.2 | 1.12 | 631.33 | -7201.80 |
| | 0.3 | 1.18 | 943.08 | -11625.00 |
| | 0.4 | 1.25 | 1278.48 | -16845.00 |

Table 4.19: Angle of the resultant force, resultant force and moment required to rotate with the respect to the parameter L2 of the structure 3

| L2 (mm) | Rotation (rad) | Angle (rad) | Resultant Force (N) | Moment (N.mm) |
|---|---|---|---|---|
| 12 | -0.4 | -2.53 | 1254.90 | 7084.60 |
| | -0.3 | -2.46 | 946.24 | 5975.20 |
| | -0.2 | -2.40 | 632.81 | 4430.60 |
| | -0.1 | -2.33 | 318.45 | 2468.50 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 0.94 | 321.50 | -3014.30 |
| | 0.2 | 1.00 | 646.65 | -6653.90 |
| | 0.3 | 1.07 | 975.30 | -10971.00 |
| | 0.4 | 1.13 | 1308.16 | -15911.00 |
| 13 | -0.4 | -2.55 | 1165.85 | 6081.90 |
| | -0.3 | -2.48 | 880.13 | 5188.10 |
| | -0.2 | -2.42 | 590.25 | 3898.30 |
| | -0.1 | -2.35 | 296.91 | 2184.80 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 0.92 | 301.35 | -2713.50 |
| | 0.2 | 0.98 | 608.38 | -6036.10 |
| | 0.3 | 1.04 | 920.24 | -9997.50 |
| | 0.4 | 1.10 | 1235.94 | -14559.00 |
| 14 | -0.4 | -2.57 | 1080.50 | 5100.70 |
| | -0.3 | -2.50 | 816.89 | 4425.80 |
| | -0.2 | -2.44 | 549.15 | 3374.40 |
| | -0.1 | -2.38 | 277.36 | 1921.90 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 0.89 | 282.29 | -2422.50 |
| | 0.2 | 0.95 | 570.69 | -5420.70 |
| | 0.3 | 1.02 | 864.95 | -9040.10 |
| | 0.4 | 1.08 | 1165.37 | -13361.00 |
| 15 | -0.4 | -2.58 | 998.92 | 4144.00 |
| | -0.3 | -2.52 | 757.07 | 3686.70 |
| | -0.2 | -2.46 | 509.84 | 2864.70 |
| | -0.1 | -2.40 | 257.93 | 1649.30 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 0.87 | 264.43 | -2142.90 |
| | 0.2 | 0.93 | 534.50 | -4825.30 |
| | 0.3 | 0.99 | 810.43 | -8086.90 |
| | 0.4 | 1.05 | 1094.57 | -12031.00 |
| 16 | -0.4 | -2.60 | 922.15 | 3227.30 |
| | -0.3 | -2.54 | 699.24 | 2976.30 |
| | -0.2 | -2.48 | 472.09 | 2375.10 |
| | -0.1 | -2.42 | 239.50 | 1401.20 |
| | 0 | - | 0.00 | 0.00 |
| | 0.1 | 0.84 | 246.06 | -1862.50 |
| | 0.2 | 0.91 | 531.70 | -4526.60 |
| | 0.3 | 0.97 | 806.86 | -7657.80 |
| | 0.4 | 1.03 | 1091.51 | -11528.00 |

Table 4.20: Angle of the resultant force, resultant force and moment required to rotate with the respect to the parameter L3 of the structure 3

| L3 (mm) | Rotation (rad) | Angle (rad) | Resultant Force (N) | Moment (N.mm) |
|---|---|---|---|---|
| | -0.4 | -2.53 | 1254.90 | 7084.60 |
| | -0.3 | -2.46 | 946.24 | 5975.20 |
| | -0.2 | -2.40 | 632.81 | 4430.60 |
| | -0.1 | -2.33 | 318.45 | 2468.50 |
| 35 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 0.94 | 321.50 | -3014.30 |
| | 0.2 | 1.00 | 646.65 | -6653.90 |
| | 0.3 | 1.07 | 975.30 | -10971.00 |
| | 0.4 | 1.13 | 1308.16 | -15911.00 |
| | -0.4 | -2.53 | 1250.13 | 7444.70 |
| | -0.3 | -2.46 | 942.29 | 6263.90 |
| | -0.2 | -2.39 | 630.06 | 4631.90 |
| | -0.1 | -2.33 | 316.79 | 2575.60 |
| 36 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 0.94 | 319.10 | -3138.10 |
| | 0.2 | 1.01 | 640.97 | -6922.20 |
| | 0.3 | 1.07 | 964.78 | -11395.00 |
| | 0.4 | 1.13 | 1291.17 | -16472.00 |
| | -0.4 | -2.52 | 1245.68 | 7780.20 |
| | -0.3 | -2.46 | 938.05 | 6523.30 |
| | -0.2 | -2.39 | 626.43 | 4820.60 |
| | -0.1 | -2.32 | 315.01 | 2676.10 |
| 37 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 0.95 | 315.74 | -3248.50 |
| | 0.2 | 1.01 | 634.07 | -7164.60 |
| | 0.3 | 1.07 | 953.28 | -11770.00 |
| | 0.4 | 1.14 | 1274.44 | -17160.00 |
| | -0.4 | -2.52 | 1241.90 | 8101.90 |
| | -0.3 | -2.45 | 934.37 | 6774.50 |
| | -0.2 | -2.39 | 623.31 | 4995.20 |
| | -0.1 | -2.32 | 312.37 | 2766.80 |
| 38 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 0.95 | 313.02 | -3354.60 |
| | 0.2 | 1.02 | 627.08 | -7380.10 |
| | 0.3 | 1.08 | 941.17 | -12146.00 |
| | 0.4 | 1.14 | 1255.45 | -17677.00 |
| | -0.4 | -2.51 | 1236.35 | 8392.10 |
| | -0.3 | -2.45 | 929.96 | 7010.20 |
| | -0.2 | -2.38 | 620.03 | 5157.30 |
| | -0.1 | -2.32 | 310.37 | 2852.60 |
| 39 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 0.96 | 310.13 | -3452.20 |
| | 0.2 | 1.02 | 620.22 | -7587.90 |
| | 0.3 | 1.08 | 920.25 | -12381.00 |
| | 0.4 | 1.15 | 1235.54 | -18114.00 |

Table 4.21: Angle of the resultant force, resultant force and moment required to rotate with the respect to the parameter R1 of the structure 4

| R1 (mm) | Rotation (rad) | Angle (rad) | Resultant Force (N) | Moment (N.mm) |
|---|---|---|---|---|
| | -0.4 | -1.50 | 1798.06 | 30162.00 |
| | -0.3 | -1.44 | 1341.02 | 23419.00 |
| | -0.2 | -1.45 | 834.47 | 15536.00 |
| | -0.1 | -1.50 | 394.73 | 7737.20 |
| 30 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.49 | 394.90 | -7736.00 |
| | 0.2 | 1.45 | 834.71 | -15532.00 |
| | 0.3 | 1.44 | 1341.34 | -23413.00 |
| | 0.4 | 1.47 | 1902.93 | -31360.00 |
| | -0.4 | -1.46 | 1635.28 | 28123.00 |
| | -0.3 | -1.41 | 1212.37 | 21806.00 |
| | -0.2 | -1.43 | 746.13 | 14474.00 |
| | -0.1 | -1.48 | 351.85 | 7214.70 |
| 31 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.47 | 351.96 | -7215.30 |
| | 0.2 | 1.43 | 746.52 | -14476.00 |
| | 0.3 | 1.41 | 1212.92 | -21811.00 |
| | 0.4 | 1.46 | 1636.28 | -28130.00 |
| | -0.4 | -1.40 | 1604.71 | 27307.00 |
| | -0.3 | -1.37 | 1118.26 | 20378.00 |
| | -0.2 | -1.39 | 675.01 | 13552.00 |
| | -0.1 | -1.46 | 313.81 | 6760.30 |
| 32 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.47 | 313.82 | -6758.60 |
| | 0.2 | 1.40 | 674.23 | -13550.00 |
| | 0.3 | 1.38 | 1100.19 | -20413.00 |
| | 0.4 | 1.43 | 1495.73 | -26331.00 |
| | -0.4 | -1.37 | 1477.04 | 25604.00 |
| | -0.3 | -1.34 | 1006.91 | 19192.00 |
| | -0.2 | -1.37 | 616.04 | 12706.00 |
| | -0.1 | -1.44 | 282.48 | 6350.50 |
| 33 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.44 | 282.37 | -6350.70 |
| | 0.2 | 1.37 | 616.33 | -12704.00 |
| | 0.3 | 1.35 | 1007.50 | -19171.00 |
| | 0.4 | 1.40 | 1374.96 | -24721.00 |
| | -0.4 | -1.36 | 1273.51 | 23279.00 |
| | -0.3 | -1.32 | 924.90 | 18042.00 |
| | -0.2 | -1.34 | 557.33 | 11992.00 |
| | -0.1 | -1.44 | 255.23 | 5984.20 |
| 34 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.44 | 255.29 | -5984.80 |
| | 0.2 | 1.34 | 558.04 | -11986.00 |
| | 0.3 | 1.31 | 925.44 | -18049.00 |
| | 0.4 | 1.36 | 1274.00 | -23286.00 |

Table 4.22: Angle of the resultant force, resultant force and moment required to rotate with the respect to the parameter $\alpha$ of the structure 4

| R2 (mm) | Rotation (rad) | Angle (rad) | Resultant Force (N) | Moment (N.mm) |
|---|---|---|---|---|
| | -0.4 | -1.14 | 2821.61 | 32764.00 |
| | -0.3 | -1.12 | 1908.83 | 25702.00 |
| | -0.2 | -1.18 | 1058.36 | 17227.00 |
| | -0.1 | -1.33 | 451.30 | 8634.30 |
| 0.8 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.34 | 450.64 | -8633.90 |
| | 0.2 | 1.18 | 1056.25 | -17225.00 |
| | 0.3 | 1.12 | 1905.34 | -25700.00 |
| | 0.4 | 1.14 | 2817.35 | -32759.00 |
| | -0.4 | -1.29 | 2366.68 | 33529.00 |
| | -0.3 | -1.29 | 1583.90 | 25264.00 |
| | -0.2 | -1.33 | 940.00 | 16898.00 |
| | -0.1 | -1.43 | 429.11 | 8456.00 |
| 1.0 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.43 | 429.01 | -8455.60 |
| | 0.2 | 1.33 | 939.63 | -16896.00 |
| | 0.3 | 1.29 | 1583.16 | -25260.00 |
| | 0.4 | 1.29 | 2367.31 | -33528.00 |
| | -0.4 | -1.43 | 1979.20 | 31648.00 |
| | -0.3 | -1.38 | 1453.71 | 24723.00 |
| | -0.2 | -1.41 | 889.00 | 16506.00 |
| | -0.1 | -1.47 | 415.89 | 8251.50 |
| 1.2 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.47 | 415.78 | -8251.60 |
| | 0.2 | 1.41 | 888.61 | -16507.00 |
| | 0.3 | 1.38 | 1453.38 | -24720.00 |
| | 0.4 | 1.43 | 1978.34 | -31649.00 |
| | -0.4 | -1.48 | 1861.71 | 30966.00 |
| | -0.3 | -1.42 | 1383.23 | 24133.00 |
| | -0.2 | -1.44 | 858.69 | 16058.00 |
| | -0.1 | -1.49 | 405.48 | 8013.60 |
| 1.4 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.49 | 405.32 | -8013.60 |
| | 0.2 | 1.44 | 858.11 | -16058.00 |
| | 0.3 | 1.42 | 1386.49 | -24134.00 |
| | 0.4 | 1.48 | 1860.37 | -30965.00 |
| | -0.4 | -1.50 | 1798.06 | 30162.00 |
| | -0.3 | -1.44 | 1341.02 | 23419.00 |
| | -0.2 | -1.45 | 834.47 | 15536.00 |
| | -0.1 | -1.50 | 394.73 | 7737.20 |
| 1.6 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.49 | 394.90 | -7736.00 |
| | 0.2 | 1.45 | 834.71 | -15532.00 |
| | 0.3 | 1.44 | 1341.34 | -23413.00 |
| | 0.4 | 1.47 | 1902.93 | -31360.00 |

Table 4.23: Angle of the resultant force, resultant force and moment required to rotate with the respect to the parameter R2 of the structure 4

| L1 (mm) | Rotation (rad) | Angle (rad) | Resultant Force (N) | Moment (N.mm) |
|---|---|---|---|---|
| | -0.4 | -1.50 | 1798.06 | 30162.00 |
| | -0.3 | -1.44 | 1341.02 | 23419.00 |
| | -0.2 | -1.45 | 834.47 | 15536.00 |
| | -0.1 | -1.50 | 394.73 | 7737.20 |
| 2.5 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.49 | 394.90 | -7736.00 |
| | 0.2 | 1.45 | 834.71 | -15532.00 |
| | 0.3 | 1.44 | 1341.34 | -23413.00 |
| | 0.4 | 1.47 | 1902.93 | -31360.00 |
| | -0.4 | -1.57 | 1576.61 | 28364.00 |
| | -0.3 | -1.51 | 1183.55 | 22010.00 |
| | -0.2 | -1.51 | 748.82 | 14585.00 |
| | -0.1 | -1.53 | 360.27 | 7254.20 |
| 3.0 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.53 | 360.50 | -7271.80 |
| | 0.2 | 1.51 | 748.77 | -14584.00 |
| | 0.3 | 1.51 | 1183.44 | -22009.00 |
| | 0.4 | 1.54 | 1688.39 | -29413.00 |
| | -0.4 | -1.62 | 1408.57 | 26738.00 |
| | -0.3 | -1.57 | 1063.11 | 20712.00 |
| | -0.2 | -1.55 | 680.36 | 13732.00 |
| | -0.1 | -1.56 | 330.35 | 6824.20 |
| 3.5 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.56 | 330.35 | -6823.70 |
| | 0.2 | 1.55 | 680.35 | -13730.00 |
| | 0.3 | 1.57 | 1063.11 | -20708.00 |
| | 0.4 | 1.62 | 1408.56 | -26731.00 |
| | -0.4 | -1.64 | 1330.68 | 26184.00 |
| | -0.3 | -1.60 | 963.91 | 19508.00 |
| | -0.2 | -1.58 | 623.27 | 12930.00 |
| | -0.1 | -1.57 | 304.83 | 6443.80 |
| 4.0 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.57 | 304.84 | -6443.70 |
| | 0.2 | 1.58 | 623.29 | -12930.00 |
| | 0.3 | 1.60 | 964.00 | -19506.00 |
| | 0.4 | 1.64 | 1343.29 | -26138.00 |
| | -0.4 | -1.68 | 1213.48 | 24689.00 |
| | -0.3 | -1.63 | 887.68 | 18378.00 |
| | -0.2 | -1.59 | 570.66 | 12222.00 |
| | -0.1 | -1.59 | 281.71 | 6077.50 |
| 4.5 | 0 | - | 0.00 | 0.00 |
| | 0.1 | 1.59 | 281.71 | -6077.40 |
| | 0.2 | 1.59 | 570.61 | -12222.00 |
| | 0.3 | 1.63 | 887.22 | -18378.00 |
| | 0.4 | 1.67 | 1223.40 | -24645.00 |

# 5 Comparison of Results

In this chapter the comparison of the results obtained from the chapters 3 and 4 are going to be presented. The results will be divided into the sections 5.1, 5.2 and 5.3. Finally, a discussion of the results and conclusions are going to be presented.

## 5.1 Comparison of displacement of the nodes due to external forces

In this Section, the comparison of the results obtained in the subsections 3.3.1 and 4.2.1 are presented. Evidently, because the first nodes of all the structures are type "Clamp", there are no displacement, so those nodes are not going to be taken into consideration. In its place, the moments generated at these nodes are shown in the Table 5.10. In the tables 5.1, 5.2, 5.3, 5.4, 5.5, 5.6, 5.7, 5.8 and 5.9 the relative and absolute errors are shown.

Table 5.1: Relative and absolute errors between the analytical and FEM results in the structure A

| Node | analytical Results | | FEM Results | | Rel. Error | | Abs. Error | |
|---|---|---|---|---|---|---|---|---|
| | △ X (mm) | △ Y (mm) | △ X (mm) | △ Y (mm) | X (%) | Y (%) | X (mm) | Y (mm) |
| 2 | -0.02 | 1.79 | -0.02 | 1.81 | -2.74 | -1.45 | 0.00 | -0.03 |
| 3 | -0.59 | -6.36 | -0.56 | -6.22 | 3.85 | 3.83 | -0.02 | -0.23 |
| 4 | -21.35 | -0.83 | -21.07 | -0.74 | 1.36 | 12.16 | -0.29 | -0.09 |

Table 5.2: Relative and absolute errors between the analytical and FEM results in the structure B

| Node | analytical Results | | FEM Results | | Rel. Error | | Abs. Error | |
|---|---|---|---|---|---|---|---|---|
| | △ X (mm) | △ Y (mm) | △ X (mm) | △ Y (mm) | X (%) | Y (%) | X (mm) | Y (mm) |
| 2 | -0.58 | 9.41 | -0.55 | 9.09 | -5.18 | 3.50 | -0.03 | 0.32 |
| 3 | -1.04 | 4.84 | -0.97 | 4.89 | 7.54 | -0.91 | -0.07 | -0.04 |
| 4 | -0.20 | 9.18 | -0.67 | 8.86 | -70.36 | 3.70 | 0.47 | 0.33 |
| 5 | -0.65 | 4.27 | -1.10 | 4.34 | -40.68 | -1.51 | 0.45 | -0.07 |

Table 5.3: Relative and absolute errors between the analytical and FEM results in the structure C

| Node | analytical Results | | FEM Results | | Rel. Error | | Abs. Error | |
|---|---|---|---|---|---|---|---|---|
| | △ X (mm) | △ Y (mm) | △ X (mm) | △ Y (mm) | X (%) | Y (%) | X (mm) | Y (mm) |
| 2 | -0.07 | 3.27 | -0.06 | 3.00 | 17.02 | 8.96 | -0.01 | 0.27 |
| 3 | -0.28 | 8.88 | -0.23 | 8.12 | 18.82 | 9.31 | -0.04 | 0.76 |
| 4 | -2.37 | 28.60 | -2.13 | 26.86 | 11.11 | 6.45 | -0.24 | 1.73 |
| 5 | -9.21 | 2.85 | -8.50 | 2.64 | 8.40 | 7.82 | -0.71 | 0.21 |
| 6 | -9.41 | 8.46 | -8.66 | 7.76 | 8.62 | 8.95 | -0.75 | 0.69 |

Table 5.4: Relative and absolute errors between the analytical and FEM results in the structure D

| Node | analytical Results | | FEM Results | | Rel. Error | | Abs. Error | |
|---|---|---|---|---|---|---|---|---|
| | △ X (mm) | △ Y (mm) | △ X (mm) | △ Y (mm) | X (%) | Y (%) | X (mm) | Y (mm) |
| 2 | -0.38 | 7.56 | -0.38 | 7.56 | -0.29 | 0.01 | 0.00 | 0.00 |
| 3 | -1.58 | 21.33 | -1.57 | 21.24 | 0.85 | 0.39 | -0.01 | 0.08 |
| 4 | -18.74 | 5.85 | -18.57 | 5.88 | 0.93 | -0.55 | -0.17 | -0.03 |
| 5 | -19.94 | 19.62 | -19.75 | 19.57 | 0.95 | 0.26 | -0.19 | 0.05 |
| 6 | 17.97 | 9.26 | 17.79 | 9.23 | 0.99 | 0.36 | 0.18 | 0.03 |
| 7 | 16.77 | 23.03 | 16.61 | 22.91 | 0.97 | 0.51 | 0.16 | 0.12 |

Table 5.5: Relative and absolute errors between the analytical and FEM results in the structure E

| Node | analytical Results | | FEM Results | | Rel. Error | | Abs. Error | |
|---|---|---|---|---|---|---|---|---|
| | △ X (mm) | △ Y (mm) | △ X (mm) | △ Y (mm) | X (%) | Y (%) | X (mm) | Y (mm) |
| 2 | -0.18 | -3.63 | -0.18 | -3.63 | 0.00 | 0.00 | 0.00 | -0.00 |
| 3 | 32.96 | -40.11 | 32.96 | -40.11 | 0.03 | 0.00 | 0.01 | -0.00 |
| 4 | 67.60 | -83.74 | 67.58 | -83.74 | 0.02 | 0.01 | 0.02 | -0.01 |
| 5 | 66.70 | -93.04 | 66.68 | -93.04 | 0.02 | 0.01 | 0.02 | -0.01 |

Table 5.6: Relative and absolute errors between the analytical and FEM results in the structure 1

| Node | analytical Results | | FEM Results | | Rel. Error | | Abs. Error | |
|---|---|---|---|---|---|---|---|---|
| | △ X (mm) | △ Y (mm) | △ X (mm) | △ Y (mm) | X (%) | Y (%) | X (mm) | Y (mm) |
| 2 | -0.20 | 0.11 | -0.19 | 0.11 | 6.89 | 2.73 | -0.01 | 0.00 |
| 3 | 0.07 | 1.30 | 0.07 | 1.24 | -7.38 | 4.56 | -0.01 | 0.06 |
| 4 | -0.48 | 3.40 | -0.45 | 3.26 | 8.46 | 4.30 | -0.04 | 0.14 |
| 5 | -0.71 | 6.50 | -0.65 | 6.22 | 9.94 | 4.42 | -0.06 | 0.28 |
| 6 | -1.64 | 10.13 | -1.50 | 9.70 | 9.24 | 4.43 | -0.14 | 0.43 |

Table 5.7: Relative and absolute errors between the analytical and FEM results in the structure 2

| Node | analytical Results | | FEM Results | | Rel. Error | | Abs. Error | |
|---|---|---|---|---|---|---|---|---|
| | △ X (mm) | △ Y (mm) | △ X (mm) | △ Y (mm) | X (%) | Y (%) | X (mm) | Y (mm) |
| 2 | -0.02 | 0.52 | -0.01 | 0.50 | 18.11 | 3.65 | -0.00 | 0.02 |
| 3 | 2.47 | -1.04 | 2.38 | -1.01 | 4.00 | 3.13 | 0.10 | -0.03 |
| 4 | 1.90 | -2.25 | 1.83 | -2.16 | 4.32 | 3.79 | 0.08 | -0.08 |
| 5 | 4.02 | -3.51 | 3.84 | -3.39 | 4.50 | 3.46 | 0.17 | -0.12 |
| 6 | 9.36 | 2.41 | 9.01 | 2.24 | 3.89 | 7.88 | 0.35 | 0.18 |

Table 5.8: Relative and absolute errors between the analytical and FEM results in the structure 3

| Node | analytical Results $\triangle$ X (mm) | $\triangle$ Y (mm) | FEM Results $\triangle$ X (mm) | $\triangle$ Y (mm) | Rel. Error X (%) | Y (%) | Abs. Error X (mm) | Y (mm) |
|---|---|---|---|---|---|---|---|---|
| 2 | -0.02 | 0.82 | -0.02 | 0.81 | 6.51 | 2.14 | -0.00 | 0.02 |
| 3 | 1.04 | 0.50 | 1.02 | 0.49 | 2.07 | 2.08 | 0.02 | 0.01 |
| 4 | 0.48 | -1.92 | 0.47 | -1.88 | 2.77 | 2.26 | 0.01 | -0.04 |
| 5 | 1.45 | -2.23 | 1.41 | -2.19 | 2.40 | 2.13 | 0.03 | -0.05 |
| 6 | 3.11 | 1.07 | 3.05 | 1.04 | 2.03 | 2.48 | 0.06 | 0.03 |
| 7 | 5.49 | -0.78 | 5.37 | -0.77 | 2.18 | 1.16 | 0.12 | -0.01 |
| 8 | 1.98 | -4.79 | 1.93 | -4.69 | 2.81 | 2.27 | 0.05 | -0.11 |
| 9 | 3.31 | -6.22 | 3.23 | -6.08 | 2.73 | 2.17 | 0.09 | -0.13 |
| 10 | 8.63 | -2.46 | 8.44 | -2.43 | 2.23 | 1.31 | 0.19 | -0.03 |
| 11 | 10.73 | -6.84 | 10.48 | -6.72 | 2.41 | 1.78 | 0.25 | -0.12 |
| 12 | 3.16 | -9.72 | 3.06 | -9.50 | 3.26 | 2.32 | 0.10 | -0.22 |
| 13 | 3.88 | -12.48 | 3.75 | -12.21 | 3.37 | 2.26 | 0.13 | -0.28 |
| 14 | 19.87 | -10.35 | 19.41 | -10.21 | 2.34 | 1.39 | 0.45 | -0.14 |

Table 5.9: Relative and absolute errors between the analytical and FEM results in the structure 4

| Node | analytical Results $\triangle$ X (mm) | $\triangle$ Y (mm) | FEM Results $\triangle$ X (mm) | $\triangle$ Y (mm) | Rel. Error X (%) | Y (%) | Abs. Error X (mm) | Y (mm) |
|---|---|---|---|---|---|---|---|---|
| 2 | -0,04 | 0,95 | -0,04 | 0,92 | 0,08 | 0,04 | 0,00 | 0,03 |
| 3 | -3,32 | -0,84 | -3,11 | -0,75 | 0,07 | 0,13 | -0,21 | -0,10 |
| 4 | 3,58 | -0,32 | 3,33 | -0,29 | 0,07 | 0,10 | 0,24 | -0,03 |
| 5 | -4,52 | 0,08 | -4,23 | 0,13 | 0,07 | -0,36 | -0,29 | -0,04 |
| 6 | 4,64 | 0,83 | 4,34 | 0,79 | 0,07 | 0,06 | 0,30 | 0,05 |
| 7 | -0,09 | 2,34 | -0,07 | 2,21 | 0,18 | 0,06 | -0,01 | 0,13 |
| 8 | -0,46 | 5,66 | -0,40 | 5,32 | 0,15 | 0,06 | -0,06 | 0,34 |
| 9 | -11,02 | -1,00 | -10,37 | -0,84 | 0,06 | 0,18 | -0,65 | -0,15 |
| 10 | 11,91 | 2,35 | 11,19 | 2,10 | 0,06 | 0,12 | 0,72 | 0,25 |
| 11 | -13,29 | 0,58 | -12,49 | 0,66 | 0,06 | -0,12 | -0,80 | -0,08 |
| 12 | 13,66 | 4,64 | 12,85 | 4,24 | 0,06 | 0,09 | 0,80 | 0,40 |

Table 5.10: Relative and absolute errors in the moments generated at the nodes 1 in each structure

| Structure | Results in Julia (N.mm) | Results in Ansys (N.mm) | Rel. Error (%) | Abs. Error (N.mm) |
|---|---|---|---|---|
| A | -476,20 | -486,88 | -2,19 | 10,68 |
| B | -2516,67 | -2451,90 | 2,64 | -64,77 |
| C | -888,96 | -821,79 | 8,17 | -67,17 |
| D | -2019,18 | -2034,20 | -0,74 | 15,02 |
| E | -3876,57 | -3880,60 | -0,10 | 4,03 |
| 1 | -10849,73 | -10787,00 | 0,58 | -62,73 |
| 2 | -10067,35 | -9997,30 | 0,70 | -70,05 |
| 3 | -3601,43 | -3595,80 | 0,16 | -5,63 |
| 4 | -11011,79 | -11054,00 | -0,38 | 42,21 |

## 5.2 Comparison of forces due to rotation of the structure

In this section, the comparison of the results obtained from the subsections 3.3.2 and 4.2.2 are presented. As in the previous section, the case when the rotation is zero will be ignored.

### 5.2.1 Structure 1

The relative error of the angles of the resultant force, the result forces and the moments of the structure 1 are shown in the table 5.11. In the first column it can be seen the angle of rotation of the structure and in the other columns the relative error are presented for every value of the parameter $R1$.

Table 5.11: Relative errors between the analytical and FEM models of structure 1 with respect to the variation of the parameter R1

| Ang | R1 = 5mm | | | R1 = 6mm | | | R1 = 7mm | | | R1 = 8mm | | | R1 = 9mm | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (Rad) | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ |
| -0,4 | 0,3 | -7,4 | -5,7 | 0,2 | -7,4 | -5,9 | 0,1 | -3,0 | -1,8 | 0,1 | -2,9 | -1,8 | 0,6 | -6,8 | -5,5 |
| -0,3 | 0,3 | -5,4 | -3,9 | 0,4 | -5,6 | -4,1 | 0,4 | -5,5 | -4,1 | 0,5 | -5,3 | -4,0 | 0,5 | -5,1 | -3,9 |
| -0,2 | 0,2 | -3,7 | -2,6 | 0,3 | -3,9 | -2,8 | 0,4 | -3,8 | -2,8 | 0,4 | -3,9 | -2,8 | 0,4 | -3,8 | -2,7 |
| -0,1 | 0,0 | -2,7 | -1,8 | 0,2 | -3,0 | -2,1 | 0,4 | -3,1 | -2,1 | 0,4 | -3,0 | -2,1 | 0,4 | -2,9 | -2,0 |
| 0,1 | -0,1 | -2,7 | -2,1 | -0,2 | -3,0 | -2,3 | -0,1 | -3,2 | -2,2 | -0,1 | -3,1 | -2,2 | -0,1 | -3,0 | -2,1 |
| 0,2 | -0,1 | -3,9 | -3,1 | -0,2 | -4,3 | -3,2 | -0,2 | -4,3 | -3,1 | -0,2 | -4,3 | -3,0 | -0,2 | -4,1 | -2,9 |
| 0,3 | -0,3 | -5,7 | -4,5 | -0,5 | -6,0 | -4,5 | -0,5 | -6,1 | -4,4 | -0,5 | -6,1 | -4,3 | -0,4 | -5,9 | -4,1 |
| 0,4 | -0,2 | -7,3 | -6,4 | -0,8 | -7,5 | -6,3 | -0,9 | -8,0 | -6,1 | -1,0 | -8,1 | -5,9 | -0,9 | -8,1 | -5,7 |

### 5.2.2 Structure 2

The relative error of the angles of the resultant force, the result forces and the moments of the structure 1 are shown in the tables 5.12, 5.13, 5.14, 5.15 and 5.16. In the first column it can be seen the angle of rotation of the structure and in the other columns the relative error are presented for every value of the parameter $R1$, $R2$, $L1$, $L2$ and $L3$.

Table 5.12: Relative errors between the analytical and FEM models of structure 2 with respect to the variation of the parameter R1

| Ang | R1 = 8mm | | | R1 = 9mm | | | R1 = 10mm | | | R1 = 11mm | | | R1 = 12mm | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (Rad) | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ |
| -0,4 | 0,0 | -3,7 | -3,1 | 0,2 | -3,3 | -0,1 | 0,1 | -3,6 | -1,9 | 0,2 | -3,2 | 0,9 | -0,1 | -3,6 | -4,0 |
| -0,3 | 0,0 | -3,4 | -3,0 | 0,3 | -2,6 | 0,9 | 0,1 | -3,0 | -1,3 | 0,3 | -2,7 | 1,8 | -0,1 | -3,4 | -4,4 |
| -0,2 | 0,0 | -3,2 | -3,2 | 0,5 | -2,1 | 1,9 | 0,1 | -2,8 | -1,5 | 0,4 | -1,9 | 3,3 | -0,2 | -3,1 | -5,6 |
| -0,1 | -0,3 | -3,7 | -5,4 | 0,7 | -1,3 | 4,2 | 0,0 | -2,5 | -1,9 | 0,7 | -1,4 | 6,0 | -0,5 | -3,4 | -9,5 |
| 0,1 | -1,5 | -0,8 | 4,2 | 0,4 | -3,2 | -4,3 | -0,6 | -1,9 | 0,3 | 0,8 | -3,5 | -6,1 | -1,6 | -1,3 | 6,0 |
| 0,2 | -1,1 | -1,5 | 1,6 | -0,1 | -3,1 | -2,8 | -0,6 | -2,4 | -0,3 | 0,3 | -2,9 | -2,9 | -0,8 | -1,6 | 3,0 |
| 0,3 | -0,9 | -2,5 | 0,2 | -0,3 | -3,4 | -2,4 | -0,6 | -2,8 | -0,9 | -0,1 | -3,4 | -2,8 | -0,8 | -2,5 | 0,9 |
| 0,4 | -0,8 | -3,3 | -1,3 | -0,4 | -4,0 | -3,0 | -0,6 | -3,4 | -1,8 | -0,3 | -3,8 | -2,9 | -0,8 | -3,1 | -0,2 |

Table 5.13: Relative errors between the analytical and FEM models of structure 2 with respect to the variation of the parameter R2

| Ang | R2 = 4mm | | | R2 = 5mm | | | R2 = 6mm | | | R2 = 7mm | | | R2 = 8mm | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (Rad) | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ |
| -0,4 | 0,0 | -3,7 | -3,1 | 0,2 | -3,2 | -0,2 | 0,2 | -3,6 | -2,4 | 0,3 | -3,0 | -0,6 | 0,3 | -2,9 | 0,0 |
| -0,3 | 0,0 | -3,4 | -3,0 | 0,3 | -2,5 | 0,8 | 0,1 | -3,0 | -1,6 | 0,2 | -2,4 | 0,4 | 0,2 | -2,1 | 1,2 |
| -0,2 | 0,0 | -3,2 | -3,2 | 0,4 | -1,8 | 2,0 | 0,1 | -2,6 | -1,4 | 0,3 | -1,7 | 1,7 | 0,4 | -1,1 | 3,2 |
| -0,1 | -0,3 | -3,7 | -5,4 | 0,6 | -0,8 | 4,3 | 0,0 | -2,4 | -2,0 | 4,7 | -10,7 | 3,8 | 0,6 | -0,2 | 5,8 |
| 0,1 | -1,5 | -0,8 | 4,2 | 0,3 | -3,1 | -3,8 | -1,0 | -1,5 | 1,7 | 0,1 | -2,7 | -3,1 | 0,6 | -3,7 | -5,7 |
| 0,2 | -1,1 | -1,5 | 1,6 | -0,2 | -3,1 | -2,4 | -0,7 | -1,9 | 0,3 | -0,3 | -2,5 | -1,6 | 0,0 | -3,0 | -2,9 |
| 0,3 | -0,9 | -2,5 | 0,2 | -0,4 | -3,4 | -2,2 | -0,8 | -2,6 | -0,4 | -0,5 | -3,2 | -1,9 | -0,3 | -3,3 | -2,5 |
| 0,4 | -0,8 | -3,3 | -1,3 | -0,5 | -4,1 | -3,0 | -0,7 | -3,4 | -1,6 | -0,5 | -3,8 | -2,5 | -0,4 | -4,0 | -3,0 |

Table 5.14: Relative errors between the analytical and FEM models of structure 2 with respect to the variation of the parameter L1

| Ang | L1 = 12mm | | | L1 = 13mm | | | L1 = 14mm | | | L1 = 15mm | | | L1 = 16mm | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (Rad) | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ |
| -0,4 | 0,0 | -3,7 | -3,1 | 0,2 | -3,5 | -1,2 | 0,1 | -3,6 | -2,1 | 0,1 | -3,8 | -2,5 | 0,2 | -3,8 | -1,7 |
| -0,3 | 0,0 | -3,4 | -3,0 | 0,2 | -2,9 | -0,4 | 0,0 | -3,3 | -1,6 | -0,1 | -3,2 | -1,7 | 0,2 | -3,3 | -1,1 |
| -0,2 | 0,0 | -3,2 | -3,2 | 0,3 | -2,5 | 0,1 | 0,2 | -2,9 | -1,4 | 0,2 | -2,9 | -1,8 | 0,3 | -2,6 | -0,3 |
| -0,1 | -0,3 | -3,7 | -5,4 | 0,4 | -2,0 | 0,9 | 0,1 | -2,8 | -2,0 | 0,1 | -2,9 | -2,0 | 0,3 | -2,4 | -0,1 |
| 0,1 | -1,5 | -0,8 | 4,2 | -0,4 | -2,5 | -1,3 | -0,9 | -1,7 | 1,3 | -0,9 | -1,6 | 1,5 | -0,4 | -2,5 | -1,1 |
| 0,2 | -1,1 | -1,5 | 1,6 | -0,5 | -2,7 | -1,2 | -0,8 | -2,5 | -0,1 | -0,8 | -2,4 | -0,1 | -0,6 | -2,9 | -1,2 |
| 0,3 | -0,9 | -2,5 | 0,2 | -0,5 | -3,2 | -1,5 | -0,7 | -3,1 | -0,8 | -0,7 | -3,2 | -1,1 | -0,5 | -3,5 | -1,5 |
| 0,4 | -0,8 | -3,3 | -1,3 | -0,6 | -4,0 | -2,6 | -0,7 | -3,9 | -2,0 | -0,7 | -4,1 | -2,3 | -0,5 | -4,3 | -2,7 |

Table 5.15: Relative errors between the analytical and FEM models of structure 2 with respect to the variation of the parameter L2

| Ang | L2 = 5mm | | | L2 = 6mm | | | L2 = 7mm | | | L2 = 8mm | | | L2 = 9mm | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (Rad) | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ |
| -0,4 | 0,0 | -3,7 | -3,1 | 0,2 | -3,3 | -0,4 | 0,2 | -3,4 | -0,9 | 0,2 | -3,4 | -0,7 | 0,1 | -2,8 | 0,9 |
| -0,3 | 0,0 | -3,4 | -3,0 | 0,3 | -2,7 | 0,6 | 0,2 | -3,0 | -0,5 | 0,3 | -3,0 | -0,3 | 0,4 | -2,4 | 1,4 |
| -0,2 | 0,0 | -3,2 | -3,2 | 0,4 | -2,0 | 1,6 | 0,3 | -2,4 | 0,4 | 0,3 | -2,2 | 0,7 | 0,5 | -1,7 | 2,7 |
| -0,1 | -0,3 | -3,7 | -5,4 | 0,7 | -1,2 | 3,6 | 0,4 | -1,7 | 1,4 | 0,5 | -1,6 | 1,8 | 0,8 | -1,0 | 5,3 |
| 0,1 | -1,5 | -0,8 | 4,2 | 0,1 | -3,1 | -3,4 | -0,5 | -2,5 | -1,1 | -0,3 | -2,7 | -1,9 | 0,6 | -3,7 | -5,2 |
| 0,2 | -1,1 | -1,5 | 1,6 | -0,4 | -2,9 | -1,9 | -0,5 | -2,5 | -1,1 | -0,5 | -2,5 | -1,2 | -0,1 | -3,2 | -2,9 |
| 0,3 | -0,9 | -2,5 | 0,2 | -0,4 | -3,3 | -1,9 | -0,6 | -3,1 | -1,5 | -0,6 | -3,0 | -1,6 | -0,4 | -3,3 | -2,5 |
| 0,4 | -0,8 | -3,3 | -1,3 | -0,6 | -3,9 | -2,7 | -0,7 | -3,6 | -2,2 | -0,6 | -3,4 | -2,2 | -0,5 | -3,7 | -2,8 |

Table 5.16: Relative errors between the analytical and FEM models of structure 2 with respect to the variation of the parameter L3

| Ang | L3 = 20mm | | | L3 = 21mm | | | L3 = 22mm | | | L3 = 23mm | | | L3 = 24mm | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (Rad) | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ |
| -0,4 | 0,0 | -3,7 | -3,1 | 0,0 | -3,7 | -3,2 | 0,1 | -3,8 | -3,2 | 0,1 | -3,9 | -3,5 | 0,1 | -3,9 | -3,5 |
| -0,3 | 0,0 | -3,4 | -3,0 | 0,0 | -3,5 | -3,0 | 0,0 | -3,4 | -2,8 | 0,0 | -3,4 | -2,9 | 0,0 | -3,5 | -2,9 |
| -0,2 | 0,0 | -3,2 | -3,2 | 0,0 | -3,2 | -3,3 | 0,0 | -3,2 | -3,3 | 0,0 | -3,1 | -3,2 | 0,0 | -3,0 | -3,0 |
| -0,1 | -0,3 | -3,7 | -5,4 | -0,3 | -3,5 | -5,1 | -0,3 | -3,7 | -5,5 | -0,3 | -3,5 | -5,2 | -0,2 | -3,6 | -5,2 |
| 0,1 | -1,5 | -0,8 | 4,2 | -1,4 | -0,6 | 4,2 | -1,4 | -0,3 | 4,6 | -1,5 | -0,3 | 4,7 | -1,4 | -0,1 | 4,7 |
| 0,2 | -1,1 | -1,5 | 1,6 | -1,1 | -1,6 | 1,6 | -1,0 | -1,5 | 1,6 | -1,0 | -1,6 | 1,4 | -1,0 | -1,4 | 1,6 |
| 0,3 | -0,9 | -2,5 | 0,2 | -0,8 | -2,4 | 0,3 | -0,7 | -1,8 | 0,9 | -0,8 | -2,4 | -0,1 | -0,8 | -2,5 | -0,3 |
| 0,4 | -0,8 | -3,3 | -1,3 | -0,9 | -3,5 | -1,5 | -0,9 | -3,6 | -1,5 | -0,9 | -3,6 | -1,6 | -0,7 | -3,6 | -2,0 |

### 5.2.3 Structure 3

The relative error of the angles of the resultant force, the result forces and the moments of the structure 1 are shown in the tables 5.17, 5.18, 5.19, 5.20 and 5.21. In the first column it can be seen the angle of rotation of the structure and in the other columns the relative error are presented for every value of the parameter $R1$, $R2$, $L1$, $L2$ and $L3$.

Table 5.17: Relative errors between the analytical and FEM models of structure 3 with respect to the variation of the parameter R1

| Ang | R1 = 6mm | | | R1 = 7mm | | | R1 = 8mm | | | R1 = 9mm | | | R1 = 10mm | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (Rad) | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ |
| -0,4 | 0,2 | -3,6 | 0,8 | 0,1 | -3,7 | -2,3 | 0,4 | -3,2 | 29,4 | 0,0 | -3,3 | 3,3 | 0,1 | -3,0 | -4,2 |
| -0,3 | 0,2 | -3,1 | 1,3 | 0,0 | -3,2 | -2,5 | 0,2 | -2,7 | 14,3 | -0,1 | -2,8 | 17,5 | 0,0 | -2,6 | -2,7 |
| -0,2 | 0,3 | -2,3 | 2,7 | 0,0 | -2,9 | -2,6 | 0,3 | -2,2 | 14,7 | -0,1 | -2,5 | 575,2 | 0,0 | -2,1 | -1,8 |
| -0,1 | 0,4 | -1,6 | 5,5 | 0,0 | -2,4 | -2,3 | 0,5 | -1,6 | 22,9 | -0,2 | -2,5 | -55,8 | 0,0 | -1,8 | -3,2 |
| 0,1 | 0,6 | -3,6 | -6,7 | -0,2 | -2,5 | -1,7 | 1,5 | -3,1 | -15,7 | -0,7 | -2,0 | 10,5 | 0,2 | -2,1 | 5,9 |
| 0,2 | 0,1 | -3,4 | -4,2 | -0,2 | -2,7 | -2,0 | 0,5 | -3,0 | -7,0 | -0,4 | -2,3 | 2,2 | 0,0 | -2,1 | 1,3 |
| 0,3 | -0,1 | -3,6 | -3,5 | -0,2 | -3,0 | -2,2 | 0,3 | -3,2 | -4,3 | -0,3 | -2,6 | 1,0 | -0,1 | -2,5 | 32,9 |
| 0,4 | -0,2 | -4,1 | -2,8 | -0,3 | -3,5 | -2,5 | -0,1 | -3,7 | -3,4 | -0,5 | -3,1 | -0,1 | -0,3 | -3,0 | 7,0 |

Table 5.18: Relative errors between the analytical and FEM models of structure 3 with respect to the variation of the parameter R2

| Ang | R2 = 3mm | | | R2 = 4mm | | | R2 = 5mm | | | R2 = 6mm | | | R2 = 7mm | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (Rad) | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ |
| -0,4 | 0,2 | -3,6 | 0,8 | 0,2 | -3,5 | 5,3 | 0,1 | -3,5 | 57,8 | 0,0 | -3,4 | -2,6 | 0,1 | -3,3 | -6,5 |
| -0,3 | 0,2 | -3,1 | 1,3 | 0,2 | -2,9 | 5,5 | 0,2 | -2,9 | 21,6 | 0,0 | -3,0 | 0,0 | 0,1 | -2,8 | -6,5 |
| -0,2 | 0,3 | -2,3 | 2,7 | 0,3 | -2,4 | 7,0 | 0,2 | -2,5 | 17,5 | 0,0 | -2,7 | 2,9 | 0,2 | -2,5 | -8,5 |
| -0,1 | 0,4 | -1,6 | 5,5 | 0,5 | -1,5 | 12,6 | 0,4 | -1,9 | 24,5 | 0,0 | -2,2 | 6,2 | 0,4 | -2,3 | -16,7 |
| 0,1 | 0,6 | -3,6 | -6,7 | 1,1 | -3,7 | -11,1 | 1,2 | -3,1 | -16,5 | 0,1 | -2,2 | -23,3 | 2,8 | -2,0 | 19,7 |
| 0,2 | 0,1 | -3,4 | -4,2 | 0,4 | -3,4 | -5,6 | 0,4 | -3,0 | -7,2 | 0,0 | -2,5 | -4,7 | 1,2 | -2,3 | 9,8 |
| 0,3 | -0,1 | -3,6 | -3,5 | 0,1 | -3,6 | -4,0 | 0,1 | -3,2 | -4,0 | -0,1 | -2,9 | -1,2 | 0,6 | -2,7 | 4,7 |
| 0,4 | -0,2 | -4,1 | -2,8 | -0,1 | -4,0 | -3,5 | -0,2 | -3,7 | -3,2 | -0,2 | -3,4 | -0,7 | 0,2 | -3,2 | 0,4 |

Table 5.19: Relative errors between the analytical and FEM models of structure 3 with respect to the variation of the parameter L1

| Ang | L1 = 25mm | | | L1 = 26mm | | | L1 = 27mm | | | L1 = 28mm | | | L1 = 29mm | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (Rad) | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ |
| -0,4 | 0,2 | -3,6 | 0,8 | 0,1 | -3,8 | -2,2 | 0,1 | -3,9 | -3,0 | 0,1 | -3,8 | -2,0 | 0,2 | -3,7 | 0,2 |
| -0,3 | 0,2 | -3,1 | 1,3 | 0,1 | -3,3 | -1,9 | 0,4 | -0,1 | 4,8 | 0,1 | -3,3 | -1,6 | 0,3 | -3,1 | 1,0 |
| -0,2 | 0,3 | -2,3 | 2,7 | 0,1 | -2,9 | -1,6 | -0,0 | -3,0 | -2,7 | 0,1 | -2,7 | -1,0 | 0,3 | -2,3 | 2,4 |
| -0,1 | 0,4 | -1,6 | 5,5 | 0,1 | -2,3 | -0,8 | 0,0 | -2,5 | -2,6 | 0,1 | -2,4 | -0,6 | 0,5 | -1,4 | 5,4 |
| 0,1 | 0,6 | -3,6 | -6,7 | -0,1 | -2,4 | -1,9 | -0,3 | -2,3 | -0,9 | -0,1 | -2,8 | -2,6 | 0,5 | -3,7 | -6,6 |
| 0,2 | 0,1 | -3,4 | -4,2 | -0,2 | -2,7 | -2,1 | -0,3 | -2,6 | -1,6 | -0,2 | -2,8 | -2,2 | 0,1 | -3,4 | -4,0 |
| 0,3 | -0,1 | -3,6 | -3,5 | -0,2 | -3,1 | -2,3 | -0,3 | -3,0 | -2,2 | -0,2 | -2,4 | -1,9 | 0,1 | -2,7 | -2,8 |
| 0,4 | -0,2 | -4,1 | -2,8 | -0,4 | -3,9 | -2,9 | -0,4 | -3,8 | -2,7 | -0,3 | -3,9 | -2,9 | -0,3 | -4,3 | -3,6 |

Table 5.20: Relative errors between the analytical and FEM models of structure 3 with respect to the variation of the parameter L2

| Ang | L2 = 12mm | | | L2 = 13mm | | | L2 = 14mm | | | L2 = 15mm | | | L2 = 16mm | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (Rad) | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ |
| -0,4 | 0,2 | -3,6 | 0,8 | 0,0 | -4,0 | -5,2 | 0,1 | -3,7 | 0,3 | 0,1 | -3,8 | -2,7 | 0,0 | -3,9 | -3,4 |
| -0,3 | 0,2 | -3,1 | 1,3 | 0,0 | -3,6 | -5,2 | 0,2 | -3,1 | 0,7 | 0,1 | -3,3 | -2,5 | 0,0 | -3,2 | -3,6 |
| -0,2 | 0,3 | -2,3 | 2,7 | -0,1 | -3,1 | -5,5 | 0,2 | -2,5 | 1,7 | 0,1 | -2,9 | -2,3 | 0,0 | -3,0 | -3,5 |
| -0,1 | 0,4 | -1,6 | 5,5 | 0,0 | -0,9 | -2,5 | 0,3 | -1,8 | 4,3 | 0,1 | -2,7 | -2,2 | 0,0 | -3,0 | -4,6 |
| 0,1 | 0,6 | -3,6 | -6,7 | -0,8 | -1,5 | 2,3 | 0,5 | -3,2 | -6,1 | -0,2 | -2,7 | -2,1 | -0,3 | -2,4 | -1,2 |
| 0,2 | 0,1 | -3,4 | -4,2 | -0,5 | -2,2 | -0,3 | 0,1 | -3,1 | -3,8 | -0,2 | -2,9 | -2,2 | -0,6 | -8,7 | -8,1 |
| 0,3 | -0,1 | -3,6 | -3,5 | -0,4 | -2,8 | -1,3 | 0,0 | -3,4 | -3,1 | -0,2 | -3,0 | -2,1 | -0,4 | -8,9 | -8,3 |
| 0,4 | -0,2 | -4,1 | -2,8 | -0,4 | -3,4 | -1,3 | -0,3 | -4,0 | -3,3 | -0,3 | -3,7 | -2,6 | -0,5 | -9,5 | -9,2 |

Table 5.21: Relative errors between the analytical and FEM models of structure 3 with respect to the variation of the parameter L3

| Ang | L3 = 35mm | | | L3 = 36mm | | | L3 = 37mm | | | L3 = 38mm | | | L3 = 39mm | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (Rad) | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ |
| -0,4 | 0,2 | -3,6 | 0,8 | 0,2 | -3,6 | 0,6 | 0,2 | -3,5 | 0,5 | 0,2 | -3,7 | 0,2 | 0,2 | -3,5 | 0,2 |
| -0,3 | 0,2 | -3,1 | 1,3 | 0,2 | -3,0 | 1,1 | 0,2 | -3,0 | 1,1 | 0,2 | -3,0 | 1,0 | 0,2 | -3,0 | 0,8 |
| -0,2 | 0,3 | -2,3 | 2,7 | 0,3 | -2,4 | 2,5 | 0,3 | -2,2 | 2,6 | 0,3 | -2,2 | 2,4 | 0,3 | -2,3 | 2,3 |
| -0,1 | 0,4 | -1,6 | 5,5 | 0,5 | -1,6 | 5,6 | 0,5 | -1,6 | 5,3 | 0,4 | -1,4 | 5,2 | 0,4 | -1,2 | 5,3 |
| 0,1 | 0,6 | -3,6 | -6,7 | 0,6 | -3,8 | -6,8 | 0,6 | -3,5 | -6,5 | 0,6 | -3,5 | -6,3 | 0,6 | -3,7 | -6,4 |
| 0,2 | 0,1 | -3,4 | -4,2 | 0,1 | -3,4 | -4,1 | 0,1 | -3,3 | -4,1 | 0,1 | -3,2 | -3,8 | 0,1 | -3,4 | -4,0 |
| 0,3 | -0,1 | -3,6 | -3,5 | -0,1 | -3,6 | -3,5 | -0,1 | -3,5 | -3,3 | -0,1 | -3,6 | -3,5 | 0,0 | -2,8 | -2,8 |
| 0,4 | -0,2 | -4,1 | -2,8 | -0,2 | -4,1 | -2,6 | -0,3 | -4,3 | -3,5 | -0,2 | -4,3 | -3,7 | -0,2 | -4,3 | -3,7 |

### 5.2.4 Structure 4

The relative error of the angles of the resultant force, the result forces and the moments of the structure 1 are shown in the tables 5.22, 5.23 and 5.24. In the first column it can be seen the angle of rotation of the structure and in the other columns the relative error are presented for every value of the parameter $R1$, $\alpha$ and $R2$.

Table 5.22: Relative errors between the analytical and FEM models of structure 4 with respect to the variation of the parameter R1

| Ang | R1 = 30mm | | | R1 = 31mm | | | R1 = 32mm | | | R1 = 33mm | | | R1 = 34mm | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (Rad) | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ |
| -0,4 | -0,9 | -3,0 | -1,8 | -0,9 | -2,7 | -1,6 | 1,1 | -9,1 | -5,1 | 1,1 | -9,1 | -4,9 | -0,9 | -2,1 | -1,5 |
| -0,3 | -0,1 | -5,6 | -3,8 | -0,1 | -5,5 | -3,6 | 0,0 | -6,6 | -3,4 | 0,6 | -5,0 | -3,5 | -0,1 | -4,7 | -3,4 |
| -0,2 | -0,6 | -4,1 | -2,5 | -0,7 | -3,9 | -2,4 | -0,7 | -3,7 | -2,3 | -0,4 | -4,3 | -2,2 | -0,2 | -3,3 | -2,2 |
| -0,1 | -0,4 | -3,5 | -1,7 | -0,6 | -3,5 | -1,6 | -0,3 | -3,3 | -1,6 | -0,3 | -3,0 | -1,5 | -0,6 | -3,0 | -1,5 |
| 0,1 | -0,4 | -3,6 | -1,7 | -0,5 | -3,6 | -1,6 | -0,7 | -3,3 | -1,5 | -0,4 | -3,0 | -1,5 | -0,5 | -3,0 | -1,5 |
| 0,2 | -0,6 | -4,1 | -2,5 | -0,7 | -3,9 | -2,4 | -0,8 | -3,6 | -2,3 | -0,5 | -4,3 | -2,1 | -0,7 | -3,4 | -2,2 |
| 0,3 | 0,0 | -5,6 | -3,8 | -0,1 | -5,6 | -3,6 | -0,2 | -5,1 | -3,5 | -0,1 | -5,1 | -3,4 | 0,0 | -4,8 | -3,4 |
| 0,4 | 1,0 | -8,3 | -5,5 | -0,9 | -2,8 | -1,7 | -1,0 | -2,5 | -1,6 | -1,0 | -2,3 | -1,5 | -0,9 | -2,2 | -1,5 |

Table 5.23: Relative errors between the analytical and FEM models of structure 4 with respect to the variation of the parameter $\alpha$

| Ang | $\alpha$ = 0.8rad | | | $\alpha$ = 1.0rad | | | $\alpha$ = 1.2rad | | | $\alpha$ = 1.4rad | | | $\alpha$ = 1.6rad | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (Rad) | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ |
| -0,4 | 0,4 | -4,5 | -1,5 | 2,2 | -9,8 | -5,4 | -0,4 | -3,7 | -1,7 | -0,7 | -3,2 | -1,7 | -0,9 | -3,0 | -1,8 |
| -0,3 | 1,6 | -7,5 | -3,6 | 0,9 | -6,6 | -3,7 | 0,3 | -6,2 | -3,7 | 0,1 | -5,7 | -3,8 | -0,1 | -5,6 | -3,8 |
| -0,2 | 0,9 | -5,0 | -2,4 | 0,4 | -4,5 | -2,5 | -0,3 | -4,3 | -2,5 | -0,6 | -4,2 | -2,5 | -0,6 | -4,1 | -2,5 |
| -0,1 | 0,0 | -4,0 | -1,6 | -0,4 | -3,9 | -1,7 | -0,5 | -3,7 | -1,7 | -0,6 | -3,7 | -1,7 | -0,4 | -3,5 | -1,7 |
| 0,1 | -0,3 | -3,8 | -1,6 | -0,4 | -3,9 | -1,7 | -0,6 | -3,7 | -1,7 | -0,7 | -3,7 | -1,7 | -0,4 | -3,6 | -1,7 |
| 0,2 | 0,7 | -4,8 | -2,4 | 0,3 | -4,5 | -2,5 | -0,4 | -4,3 | -2,5 | -0,6 | -4,1 | -2,5 | -0,6 | -4,1 | -2,5 |
| 0,3 | 1,5 | -7,3 | -3,6 | 0,9 | -6,5 | -3,7 | 0,3 | -6,2 | -3,7 | 0,4 | -5,9 | -3,8 | 0,0 | -5,6 | -3,8 |
| 0,4 | 0,3 | -4,4 | -1,5 | 2,3 | -9,8 | -5,4 | -0,5 | -3,6 | -1,7 | -0,8 | -3,2 | -1,7 | 1,0 | -8,3 | -5,5 |

Table 5.24: Relative errors between the analytical and FEM models of structure 4 with respect to the variation of the parameter R2

| Ang | L1 = 25mm | | | L1 = 26mm | | | L1 = 27mm | | | L1 = 28mm | | | L1 = 29mm | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (Rad) | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ | $\alpha_F$ | $F_R$ | $M_e$ |
| -0,4 | -0,9 | -3,0 | -1,8 | -0,7 | -3,4 | -1,9 | -0,5 | -3,8 | -2,0 | 0,9 | -8,1 | -5,7 | 0,9 | -8,3 | -5,7 |
| -0,3 | -0,1 | -5,6 | -3,8 | 0,1 | -5,8 | -3,9 | 0,1 | -6,0 | -3,9 | 0,2 | -5,9 | -4,0 | 0,3 | -6,6 | -3,8 |
| -0,2 | -0,6 | -4,1 | -2,5 | -0,4 | -4,4 | -2,6 | -0,1 | -4,5 | -2,6 | -0,2 | -4,6 | -2,6 | 0,6 | -4,3 | -2,8 |
| -0,1 | -0,4 | -3,5 | -1,7 | -0,5 | -3,7 | -1,5 | -0,3 | -3,7 | -1,5 | -0,2 | -4,0 | -1,8 | -0,2 | -4,1 | -1,8 |
| 0,1 | -0,4 | -3,6 | -1,7 | -0,3 | -3,8 | -1,8 | -0,3 | -3,7 | -1,5 | -0,2 | -4,0 | -1,8 | -0,2 | -4,1 | -1,8 |
| 0,2 | -0,6 | -4,1 | -2,5 | -0,4 | -4,4 | -2,6 | -0,1 | -4,5 | -2,6 | -0,2 | -4,6 | -2,6 | 0,6 | -4,2 | -2,8 |
| 0,3 | 0,0 | -5,6 | -3,8 | 0,1 | -5,8 | -3,9 | 0,1 | -6,0 | -3,9 | 0,2 | -5,9 | -3,9 | 0,3 | -6,6 | -3,8 |
| 0,4 | 1,0 | -8,3 | -5,5 | 1,4 | -9,8 | -5,4 | -0,5 | -3,8 | -2,0 | 1,0 | -9,0 | -5,5 | 1,0 | -9,0 | -5,5 |

## 5.3 Behavior of the structures

In this section the behavior of the structures based on the results from the section 3.3.2 are analyzed. So, it is important to calculate the normalized moment at the first nodes of the structures. This can be done with the combination of the equations 3.5 and 3.13. After that, the results obtained are going to be presented in graphics $\theta$ vs $\tilde{M}$ with the corresponding variation of the parameters. Here, the row when the $\theta$ is zero is added in order to complete the graphics. In the graphics $\theta$ vs $\tilde{M}$ can be seen the generated adjusted polynomial equation which can be found too in the A.3.

### 5.3.1 Structure 1

The normalized moment of the structure 1 can be seen in the table 5.25 and the behavior of the normalized moment when the parameter R1 varies can be seen in the Fig. 5.1.

Table 5.25: Normalized moments in the node 1 of the structure 1 with respect to the Parameter R1

| $\theta$ (Rad) | $\tilde{M}$ R1=5mm | R1=6mm | R1=7mm | R1=8mm | R1=9mm |
|---|---|---|---|---|---|
| -0,4 | -0,57 | -0,53 | -0,50 | -0,48 | -0,46 |
| -0,3 | -0,39 | -0,37 | -0,35 | -0,33 | -0,32 |
| -0,2 | -0,24 | -0,22 | -0,21 | -0,21 | -0,20 |
| -0,1 | -0,11 | -0,10 | -0,10 | -0,10 | -0,09 |
| 0 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| 0,1 | 0,09 | 0,09 | 0,08 | 0,08 | 0,08 |
| 0,2 | 0,16 | 0,16 | 0,16 | 0,15 | 0,15 |
| 0,3 | 0,23 | 0,22 | 0,22 | 0,21 | 0,21 |
| 0,4 | 0,29 | 0,28 | 0,27 | 0,27 | 0,27 |

### 5.3.2 Structure 2

The normalized moment of the structure 2 can be seen in the tables 5.26, 5.27, 5.28, 5.29 and 5.30 and the behavior of the normalized moments when the parameters R1, R2, L1, L2 and L3 vary can be seen in the Fig. 5.2, 5.3, 5.4, 5.5 and 5.6 respectively.

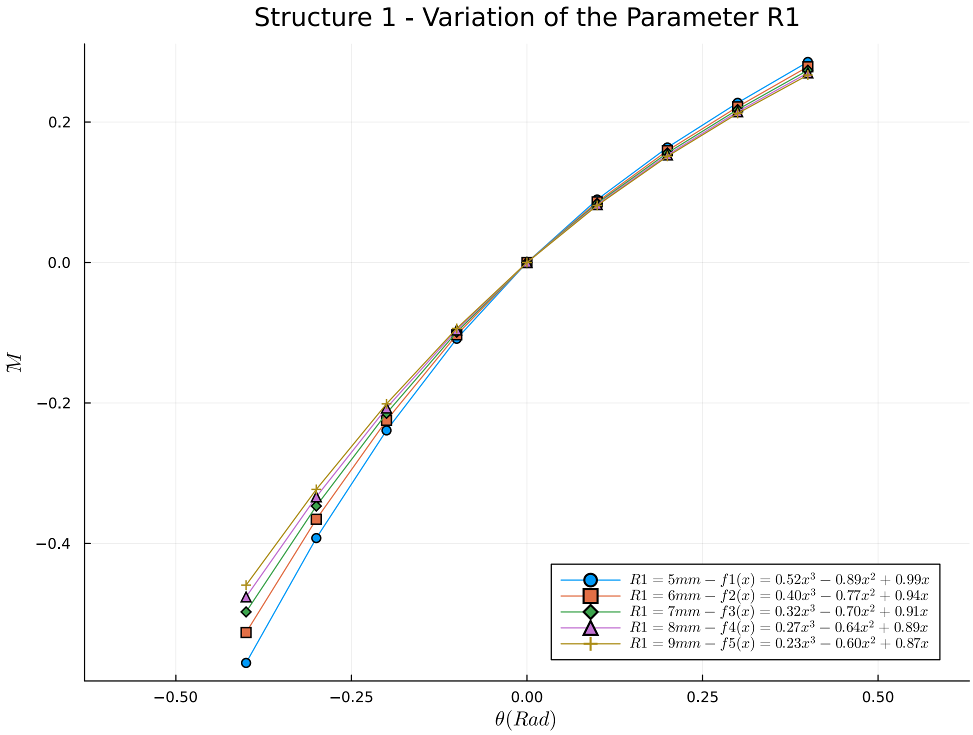



Figure 5.1: Behavior of the parameter R1 of the structure 1 and the adjusted polynomial functions

Table 5.26: Normalized moments in the node 1 of the structure 2 with respect to the Parameter R1

| $\theta$ (Rad) | $\tilde{M}$ R1=8mm | R1=9mm | R1=10mm | R1=11mm | R1=12mm |
|---|---|---|---|---|---|
| -0.4 | -0.58 | -0.53 | -0.48 | -0.43 | -0.39 |
| -0.3 | -0.43 | -0.40 | -0.36 | -0.33 | -0.29 |
| -0.2 | -0.28 | -0.26 | -0.24 | -0.22 | -0.19 |
| -0.1 | -0.14 | -0.13 | -0.12 | -0.11 | -0.10 |
| 0.0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 0.1 | 0.14 | 0.13 | 0.12 | 0.11 | 0.10 |
| 0.2 | 0.28 | 0.25 | 0.24 | 0.21 | 0.20 |
| 0.3 | 0.41 | 0.38 | 0.35 | 0.32 | 0.30 |
| 0.4 | 0.55 | 0.51 | 0.47 | 0.43 | 0.40 |

Table 5.27: Normalized moments in the node 1 of the structure 2 with respect to the Parameter R2

| $\theta$ (Rad) | $\tilde{M}$ R2=4mm | R2=5mm | R2=6mm | R2=7mm | R2=8mm |
|---|---|---|---|---|---|
| -0.4 | -0.58 | -0.54 | -0.50 | -0.47 | -0.44 |
| -0.3 | -0.43 | -0.40 | -0.37 | -0.35 | -0.33 |
| -0.2 | -0.28 | -0.27 | -0.24 | -0.23 | -0.22 |
| -0.1 | -0.14 | -0.13 | -0.12 | -0.12 | -0.11 |
| 0.0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 0.1 | 0.14 | 0.13 | 0.12 | 0.11 | 0.10 |
| 0.2 | 0.28 | 0.25 | 0.23 | 0.21 | 0.20 |
| 0.3 | 0.41 | 0.37 | 0.34 | 0.32 | 0.29 |
| 0.4 | 0.55 | 0.49 | 0.45 | 0.42 | 0.39 |

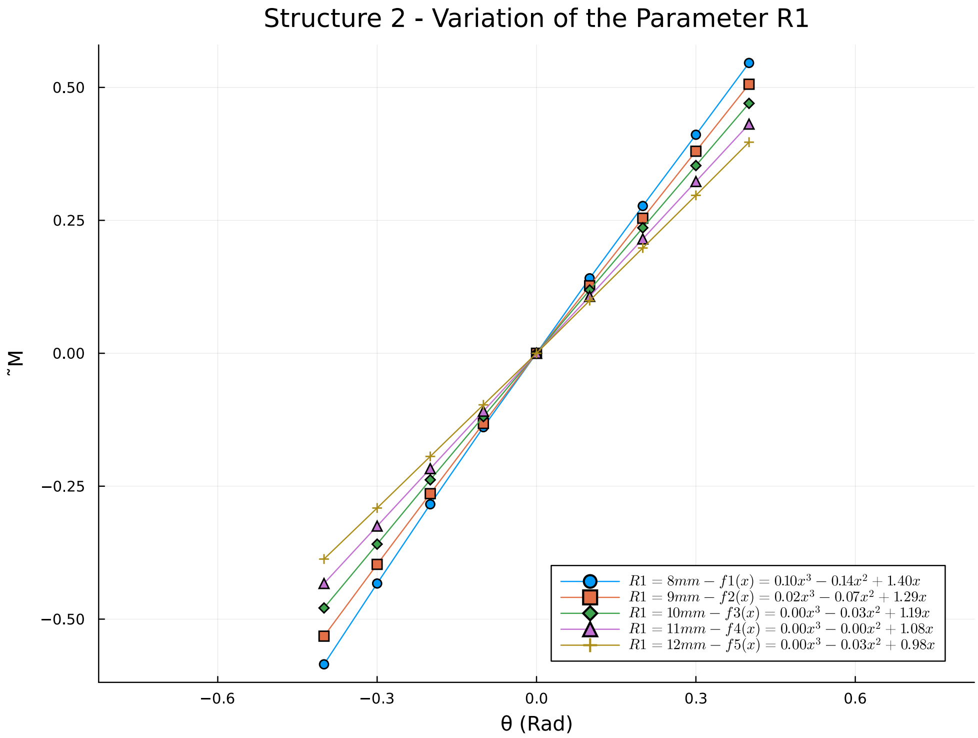



Figure 5.2: Behavior of the parameter R1 of the structure 2 and the adjusted polynomial functions

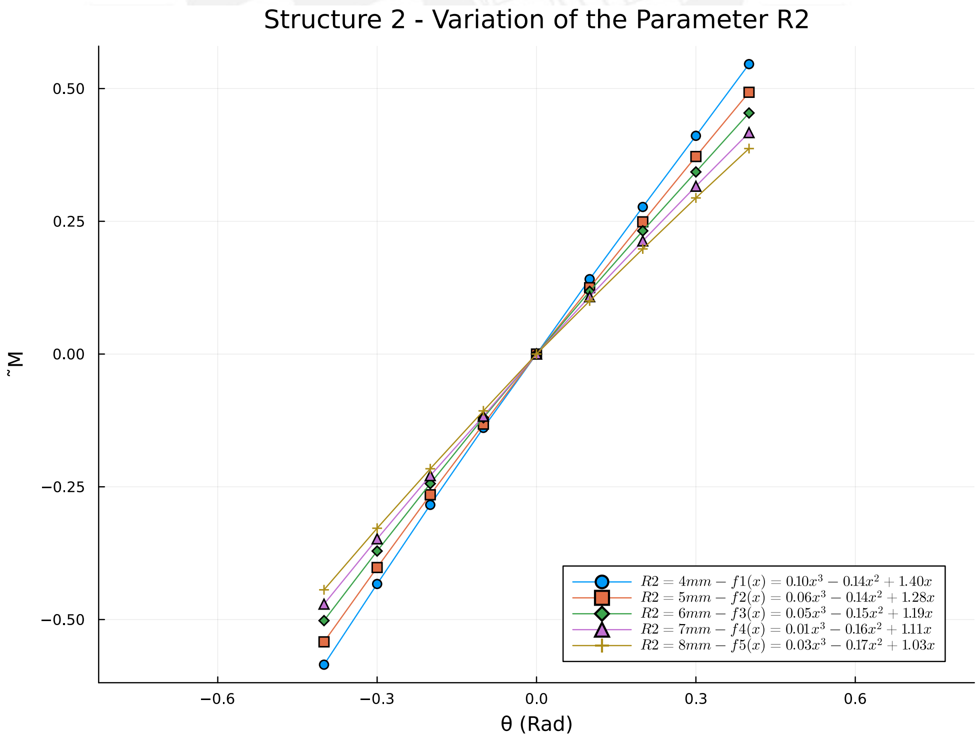



Figure 5.3: Behavior of the parameter R2 of the structure 2 and the adjusted polynomial functions

Table 5.28: Normalized moments in the node 1 of the structure 2 with respect to the Parameter L1

| $\theta$ (Rad) | $\tilde{M}$ L1=12mm | L1=13mm | L1=14mm | L1=15mm | L1=16mm |
|---|---|---|---|---|---|
| -0.4 | -0.58 | -0.65 | -0.70 | -0.76 | -0.82 |
| -0.3 | -0.43 | -0.48 | -0.52 | -0.57 | -0.61 |
| -0.2 | -0.28 | -0.32 | -0.35 | -0.37 | -0.40 |
| -0.1 | -0.14 | -0.16 | -0.17 | -0.19 | -0.20 |
| 0.0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 0.1 | 0.14 | 0.15 | 0.17 | 0.19 | 0.20 |
| 0.2 | 0.28 | 0.30 | 0.34 | 0.37 | 0.40 |
| 0.3 | 0.41 | 0.46 | 0.50 | 0.55 | 0.59 |
| 0.4 | 0.55 | 0.61 | 0.67 | 0.73 | 0.79 |

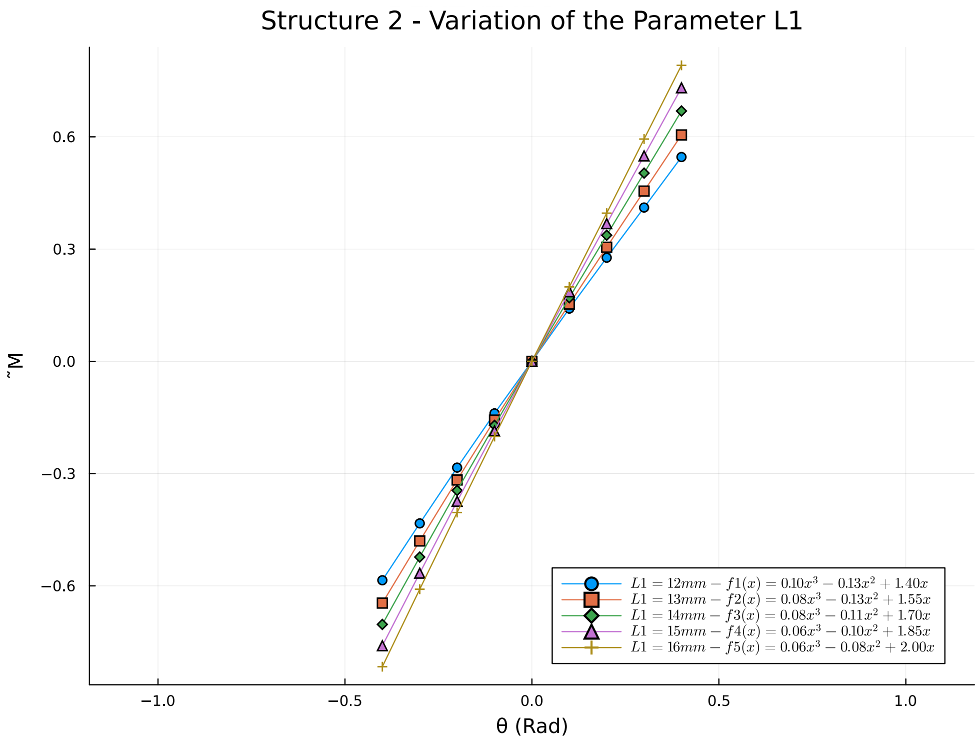



Figure 5.4: Behavior of the parameter L1 of the structure 2 and the adjusted polynomial functions

Table 5.29: Normalized moments in the node 1 of the structure 2 with respect to the Parameter L2

| $\theta$ (Rad) | $\tilde{M}$ L2=5mm | L2=6mm | L2=7mm | L2=8mm | L2=9mm |
|---|---|---|---|---|---|
| -0.4 | -0.58 | -0.55 | -0.52 | -0.48 | -0.45 |
| -0.3 | -0.43 | -0.41 | -0.38 | -0.36 | -0.34 |
| -0.2 | -0.28 | -0.27 | -0.25 | -0.24 | -0.22 |
| -0.1 | -0.14 | -0.13 | -0.13 | -0.12 | -0.11 |
| 0.0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 0.1 | 0.14 | 0.13 | 0.12 | 0.11 | 0.11 |
| 0.2 | 0.28 | 0.26 | 0.24 | 0.23 | 0.21 |
| 0.3 | 0.41 | 0.38 | 0.36 | 0.34 | 0.31 |
| 0.4 | 0.55 | 0.51 | 0.48 | 0.45 | 0.42 |

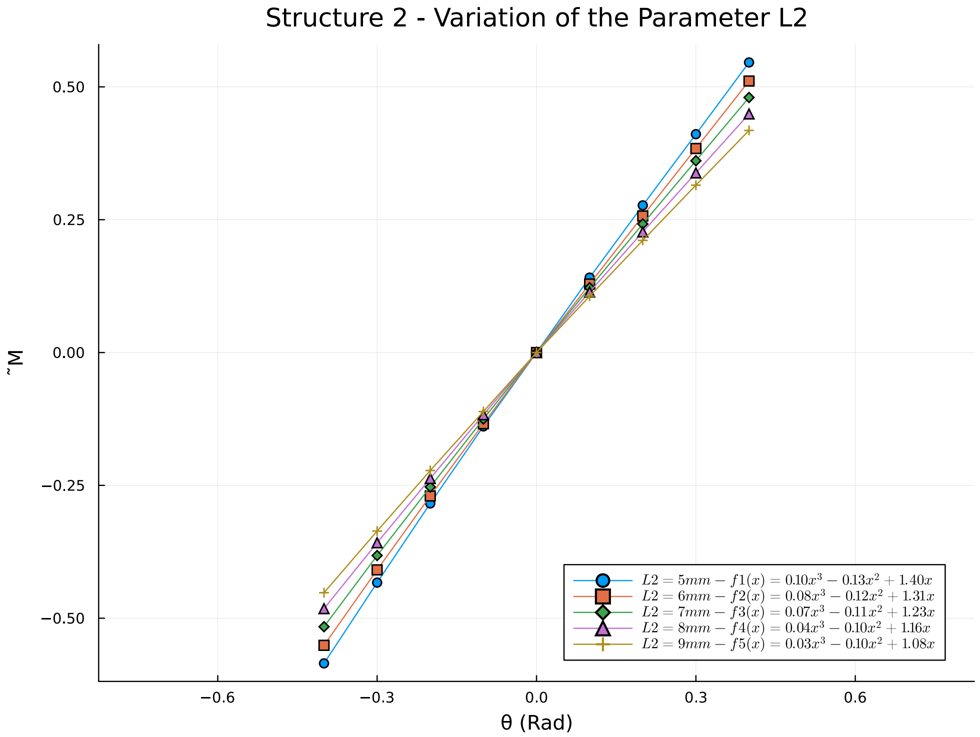



Figure 5.5: Behavior of the parameter L2 of the structure 2 and the adjusted polynomial functions

Table 5.30: Normalized moments in the node 1 of the structure 2 with respect to the Parameter L3

| $\theta$ | | | $\tilde{M}$ | | |
|---|---|---|---|---|---|
| (Rad) | L3=20mm | L3=21mm | L3=22mm | L3=23mm | L3=24mm |
| -0.4 | -0.58 | -0.57 | -0.56 | -0.55 | -0.54 |
| -0.3 | -0.43 | -0.42 | -0.42 | -0.41 | -0.40 |
| -0.2 | -0.28 | -0.28 | -0.27 | -0.27 | -0.26 |
| -0.1 | -0.14 | -0.14 | -0.13 | -0.13 | -0.13 |
| 0.0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 0.1 | 0.14 | 0.14 | 0.13 | 0.13 | 0.13 |
| 0.2 | 0.28 | 0.27 | 0.26 | 0.25 | 0.24 |
| 0.3 | 0.41 | 0.40 | 0.39 | 0.37 | 0.36 |
| 0.4 | 0.55 | 0.53 | 0.51 | 0.49 | 0.47 |

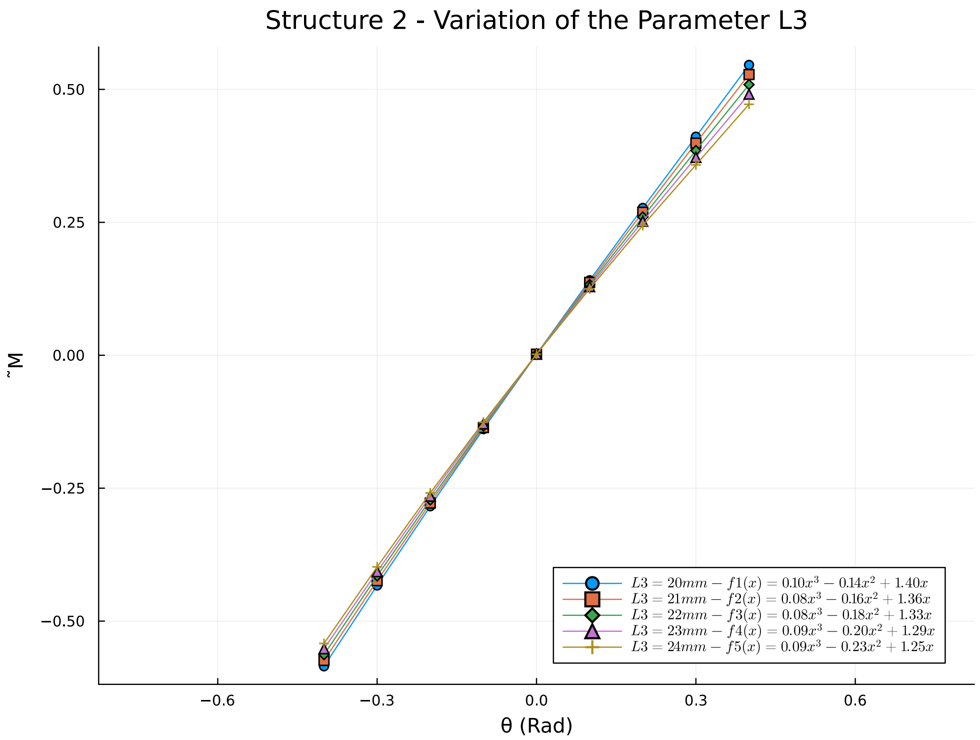



Figure 5.6: Behavior of the parameter L3 of the structure 2 and the adjusted polynomial functions

### 5.3.3 Structure 3

The normalized moment of the structure 3 can be seen in the tables 5.31, 5.32, 5.33, 5.34 and 5.35. And the behaviors of the normalized moments when the parameters R1, R2, L1, L2 and L3 vary can be seen in the Figures 5.7, 5.8, 5.9, 5.10 and 5.11.

Table 5.31: Normalized moments in the node 1 of the structure 3 with respect to the Parameter R1

| $\theta$ | $\tilde{M}$ | | | | |
|---|---|---|---|---|---|
| (Rad) | R1=6mm | R1=7mm | R1=8mm | R1=9mm | R1=10mm |
| -0,4 | -0,48 | -0,41 | -0,34 | -0,28 | -0,23 |
| -0,3 | -0,37 | -0,31 | -0,26 | -0,22 | -0,18 |
| -0,2 | -0,25 | -0,22 | -0,18 | -0,15 | -0,13 |
| -0,1 | -0,13 | -0,11 | -0,10 | -0,08 | -0,07 |
| 0 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| 0,1 | 0,13 | 0,12 | 0,10 | 0,09 | 0,07 |
| 0,2 | 0,26 | 0,24 | 0,20 | 0,18 | 0,15 |
| 0,3 | 0,40 | 0,36 | 0,31 | 0,27 | 0,23 |
| 0,4 | 0,53 | 0,49 | 0,43 | 0,37 | 0,32 |

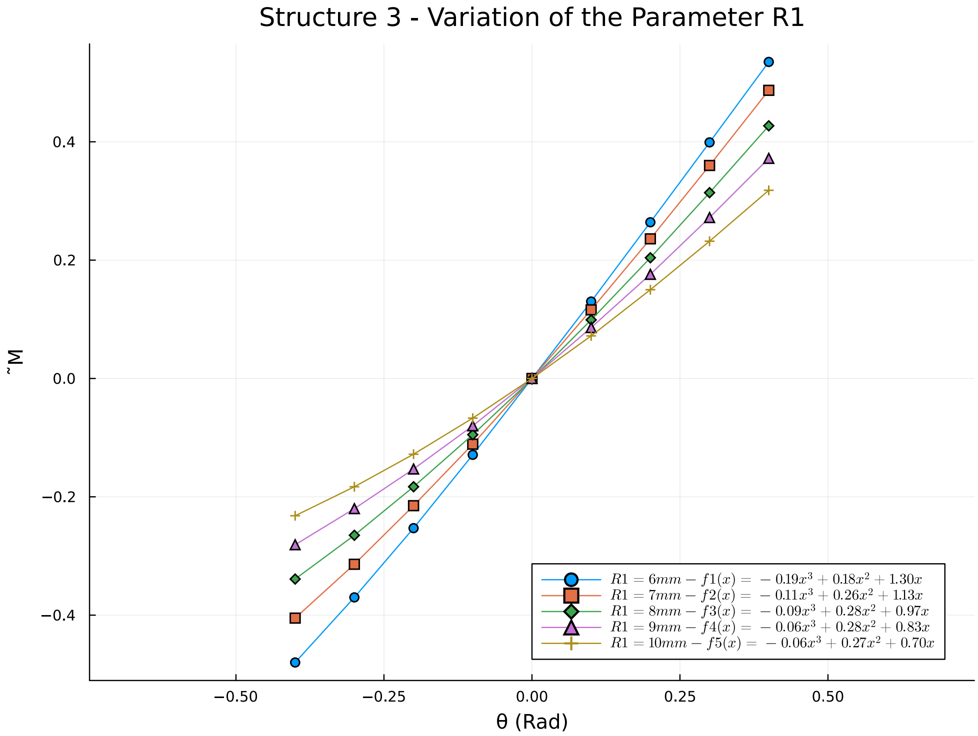



Figure 5.7: Behavior of the parameter R1 of the structure 3 and the adjusted polynomial functions

Table 5.32: Normalized moments in the node 1 of the structure 3 with respect to the Parameter R2

| $\theta$ (Rad) | $\tilde{M}$ R2=3mm | R2=4mm | R2=5mm | R2=6mm | R2=7mm |
|---|---|---|---|---|---|
| -0,4 | -0,48 | -0,41 | -0,34 | -0,28 | -0,23 |
| -0,3 | -0,37 | -0,31 | -0,26 | -0,22 | -0,18 |
| -0,2 | -0,25 | -0,22 | -0,18 | -0,15 | -0,13 |
| -0,1 | -0,13 | -0,11 | -0,10 | -0,08 | -0,07 |
| 0 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| 0,1 | 0,13 | 0,12 | 0,10 | 0,09 | 0,07 |
| 0,2 | 0,26 | 0,24 | 0,20 | 0,18 | 0,15 |
| 0,3 | 0,40 | 0,36 | 0,31 | 0,27 | 0,23 |
| 0,4 | 0,53 | 0,49 | 0,43 | 0,37 | 0,32 |

Table 5.33: Normalized moments in the node 1 of the structure 3 with respect to the Parameter L1

| $\theta$ (Rad) | $\tilde{M}$ L1=25mm | L1=26mm | L1=27mm | L1=28mm | L1=29mm |
|---|---|---|---|---|---|
| -0,4 | -0,48 | -0,52 | -0,56 | -0,60 | -0,63 |
| -0,3 | -0,37 | -0,40 | -0,43 | -0,46 | -0,48 |
| -0,2 | -0,25 | -0,27 | -0,29 | -0,31 | -0,33 |
| -0,1 | -0,13 | -0,14 | -0,15 | -0,16 | -0,17 |
| 0 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| 0,1 | 0,13 | 0,14 | 0,15 | 0,16 | 0,17 |
| 0,2 | 0,26 | 0,28 | 0,30 | 0,32 | 0,34 |
| 0,3 | 0,40 | 0,43 | 0,46 | 0,48 | 0,51 |
| 0,4 | 0,53 | 0,57 | 0,61 | 0,65 | 0,68 |

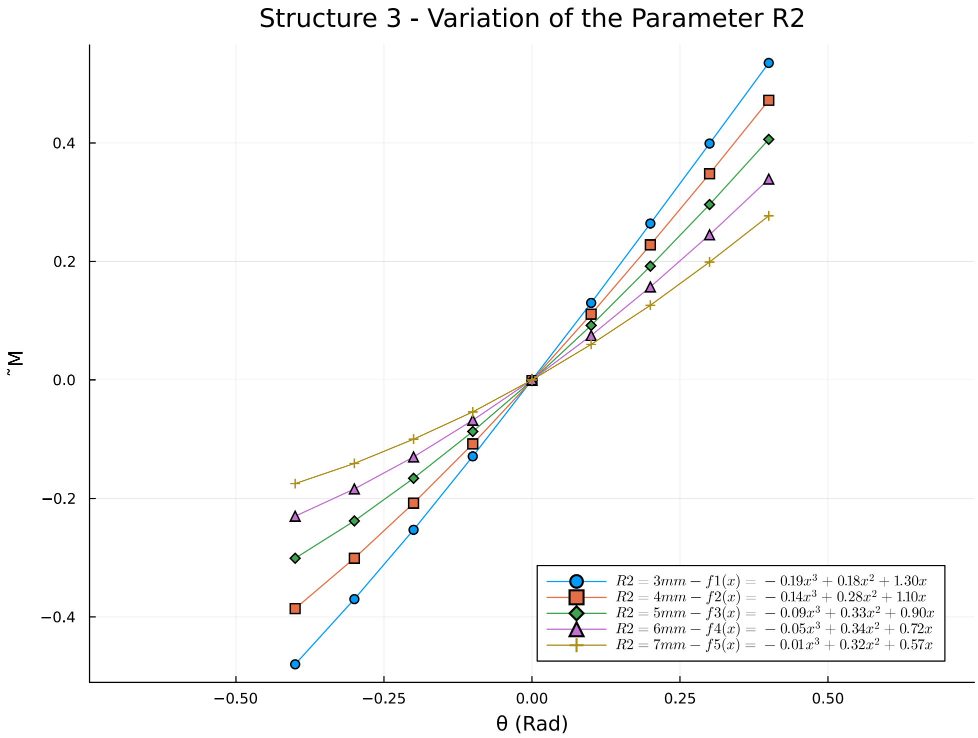



Figure 5.8: Behavior of the parameter R2 of the structure 3 and the adjusted polynomial functions

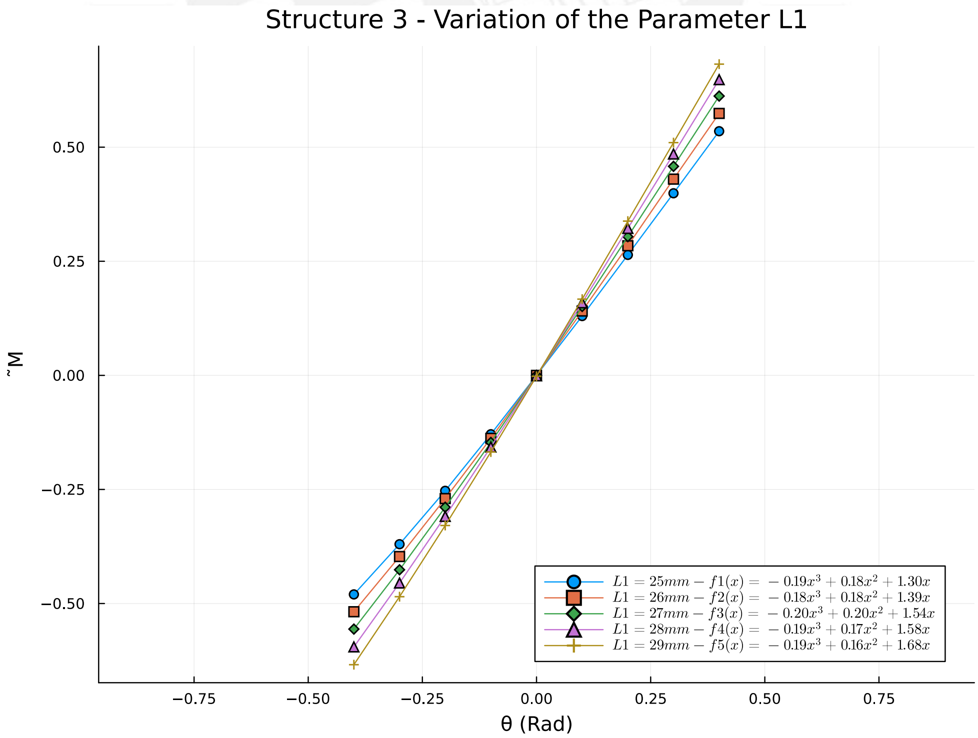



Figure 5.9: Behavior of the parameter L1 of the structure 3 and the adjusted polynomial functions

Table 5.34: Normalized moments in the node 1 of the structure 3 with respect to the Parameter L2

| $\theta$ (Rad) | $\tilde{M}$ L2=12mm | L2=13mm | L2=14mm | L2=15mm | L2=16mm |
|---|---|---|---|---|---|
| -0,4 | -0,48 | -0,44 | -0,41 | -0,37 | -0,34 |
| -0,3 | -0,37 | -0,34 | -0,31 | -0,29 | -0,26 |
| -0,2 | -0,25 | -0,23 | -0,21 | -0,20 | -0,18 |
| -0,1 | -0,13 | -0,12 | -0,11 | -0,10 | -0,09 |
| 0 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| 0,1 | 0,13 | 0,12 | 0,11 | 0,11 | 0,10 |
| 0,2 | 0,26 | 0,25 | 0,23 | 0,21 | 0,20 |
| 0,3 | 0,40 | 0,38 | 0,35 | 0,33 | 0,30 |
| 0,4 | 0,53 | 0,51 | 0,47 | 0,44 | 0,41 |

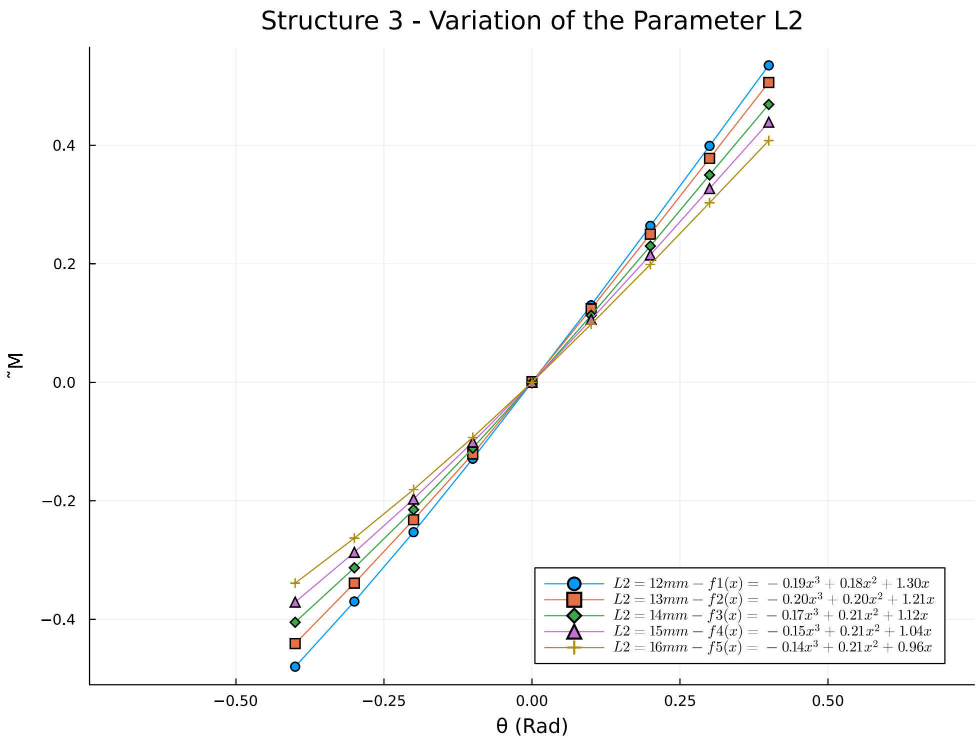



Figure 5.10: Behavior of the parameter L2 of the structure 3 and the adjusted polynomial functions

Table 5.35: Normalized moments in the node 1 of the structure 3 with respect to the Parameter L3

| $\theta$ (Rad) | $\tilde{M}$ L3=35mm | L3=36mm | L3=37mm | L3=38mm | L3=39mm |
|---|---|---|---|---|---|
| -0,4 | -0,48 | -0,48 | -0,48 | -0,48 | -0,47 |
| -0,3 | -0,37 | -0,37 | -0,37 | -0,37 | -0,36 |
| -0,2 | -0,25 | -0,25 | -0,25 | -0,25 | -0,25 |
| -0,1 | -0,13 | -0,13 | -0,13 | -0,13 | -0,13 |
| 0 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| 0,1 | 0,13 | 0,13 | 0,13 | 0,13 | 0,12 |
| 0,2 | 0,26 | 0,26 | 0,26 | 0,26 | 0,25 |
| 0,3 | 0,40 | 0,39 | 0,39 | 0,38 | 0,38 |
| 0,4 | 0,53 | 0,53 | 0,52 | 0,51 | 0,50 |

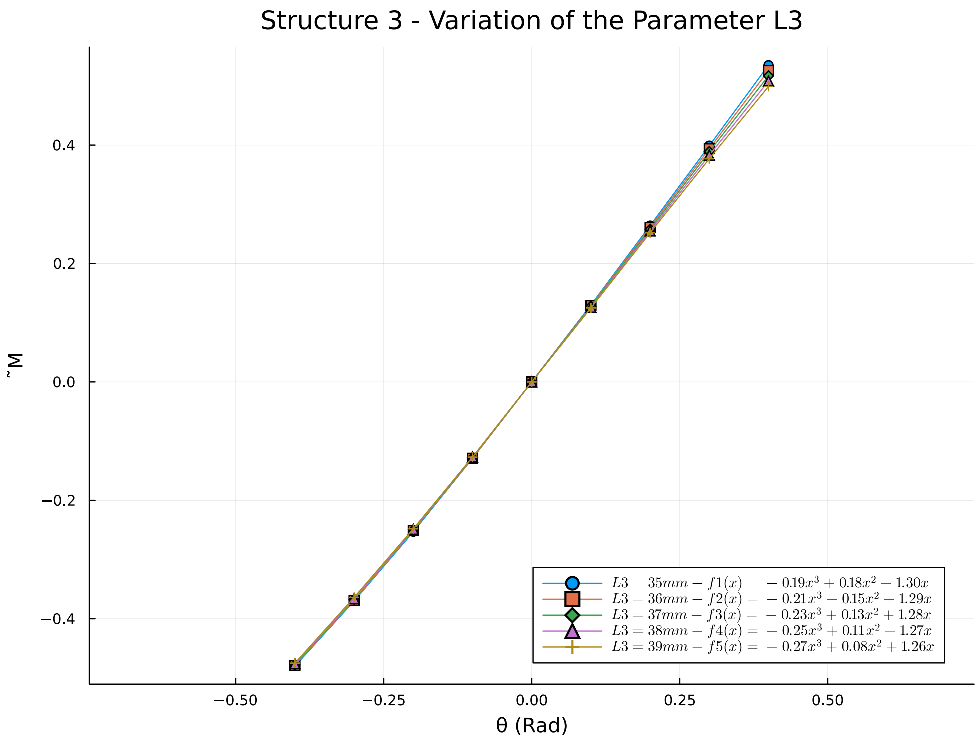



Figure 5.11: Behavior of the parameter L3 of the structure 3 and the adjusted polynomial functions

### 5.3.4 Structure 4

The normalized moment of the structure 4 can be seen in the tables 5.36, 5.37 and 5.38. And the behaviors of the normalized moments when the parameters R1, $\alpha$ and R2 vary can be seen in the Figures 5.12, 5.13 and 5.14.

Table 5.36: Normalized moments in the node 1 of the structure 4 with respect to the Parameter R1

| $\theta$ (Rad) | $\tilde{M}$ R1=30mm | R1=31mm | R1=32mm | R1=33mm | R1=34mm |
|---|---|---|---|---|---|
| -0,4 | -0,59 | -0,59 | -0,59 | -0,60 | -0,60 |
| -0,3 | -0,44 | -0,44 | -0,44 | -0,44 | -0,44 |
| -0,2 | -0,29 | -0,29 | -0,29 | -0,29 | -0,29 |
| -0,1 | -0,15 | -0,15 | -0,14 | -0,14 | -0,14 |
| 0 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| 0,1 | 0,15 | 0,15 | 0,14 | 0,14 | 0,14 |
| 0,2 | 0,29 | 0,29 | 0,29 | 0,29 | 0,29 |
| 0,3 | 0,44 | 0,44 | 0,44 | 0,44 | 0,44 |
| 0,4 | 0,59 | 0,59 | 0,60 | 0,60 | 0,60 |

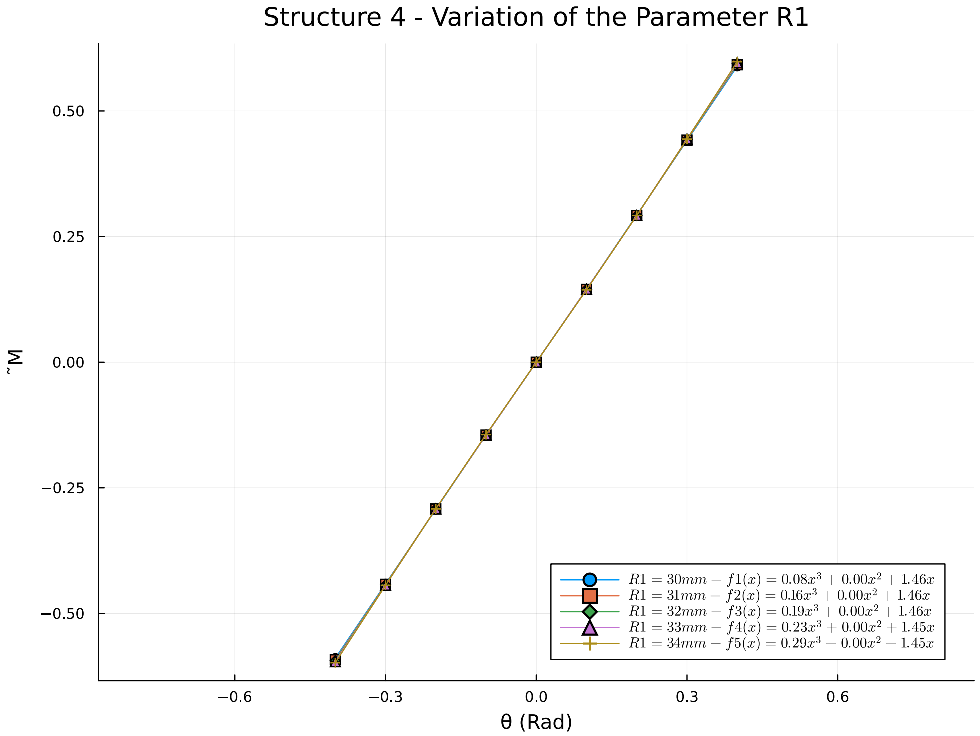



Figure 5.12: Behavior of the parameter R1 of the structure 4 and the adjusted polynomial functions

Table 5.37: Normalized moments in the node 1 of the structure 4 with respect to the Parameter $\alpha$

| $\theta$ (Rad) | $\tilde{M}$ $\alpha$=0.8rad | $\alpha$=1.0rad | $\alpha$=1.2rad | $\alpha$=1.4rad | $\alpha$=1.6rad |
|---|---|---|---|---|---|
| -0,4 | -0,66 | -0,63 | -0,61 | -0,60 | -0,59 |
| -0,3 | -0,49 | -0,47 | -0,46 | -0,45 | -0,44 |
| -0,2 | -0,32 | -0,31 | -0,30 | -0,30 | -0,29 |
| -0,1 | -0,16 | -0,15 | -0,15 | -0,15 | -0,15 |
| 0 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| 0,1 | 0,16 | 0,15 | 0,15 | 0,15 | 0,15 |
| 0,2 | 0,32 | 0,31 | 0,30 | 0,30 | 0,29 |
| 0,3 | 0,49 | 0,47 | 0,46 | 0,45 | 0,44 |
| 0,4 | 0,66 | 0,63 | 0,61 | 0,60 | 0,59 |

Table 5.38: Normalized moments in the node 1 of the structure 4 with respect to the Parameter R2

| $\theta$ (Rad) | $\tilde{M}$ R2=2.5mm | R2=3.0mm | R2=3.5mm | R2=4.0mm | R2=4.5mm |
|---|---|---|---|---|---|
| -0,4 | -0,59 | -0,56 | -0,53 | -0,51 | -0,49 |
| -0,3 | -0,44 | -0,42 | -0,40 | -0,38 | -0,37 |
| -0,2 | -0,29 | -0,28 | -0,27 | -0,26 | -0,25 |
| -0,1 | -0,15 | -0,14 | -0,13 | -0,13 | -0,12 |
| 0 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| 0,1 | 0,15 | 0,14 | 0,13 | 0,13 | 0,12 |
| 0,2 | 0,29 | 0,28 | 0,27 | 0,26 | 0,25 |
| 0,3 | 0,44 | 0,42 | 0,40 | 0,38 | 0,37 |
| 0,4 | 0,59 | 0,56 | 0,53 | 0,51 | 0,49 |

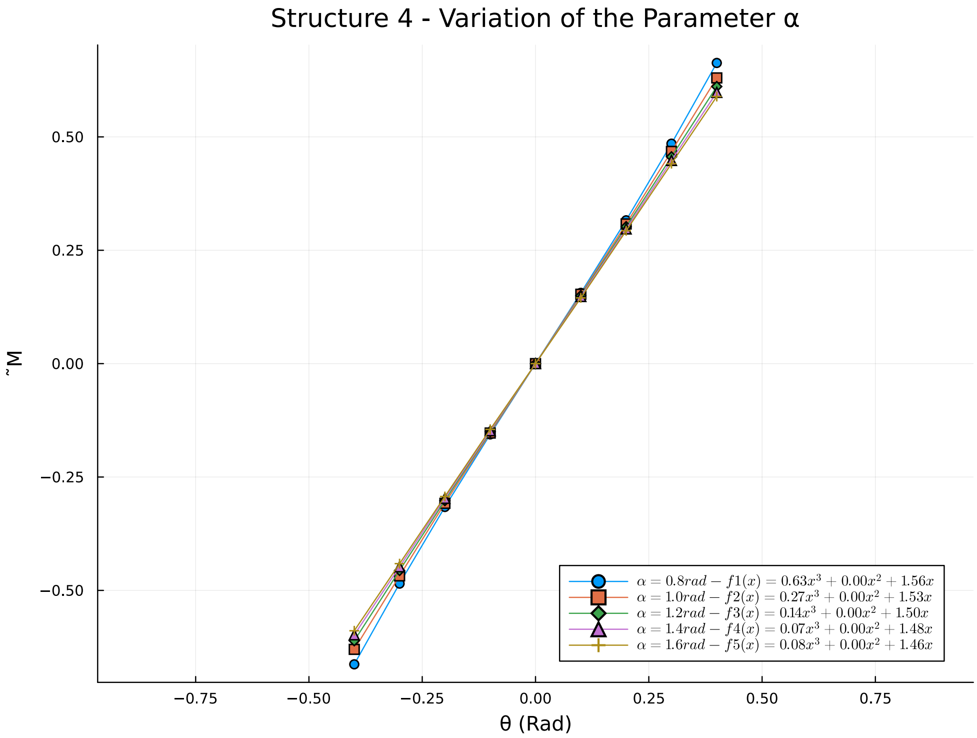



Figure 5.13: Behavior of the parameter $\alpha$ of the structure 4 and the adjusted polynomial functions

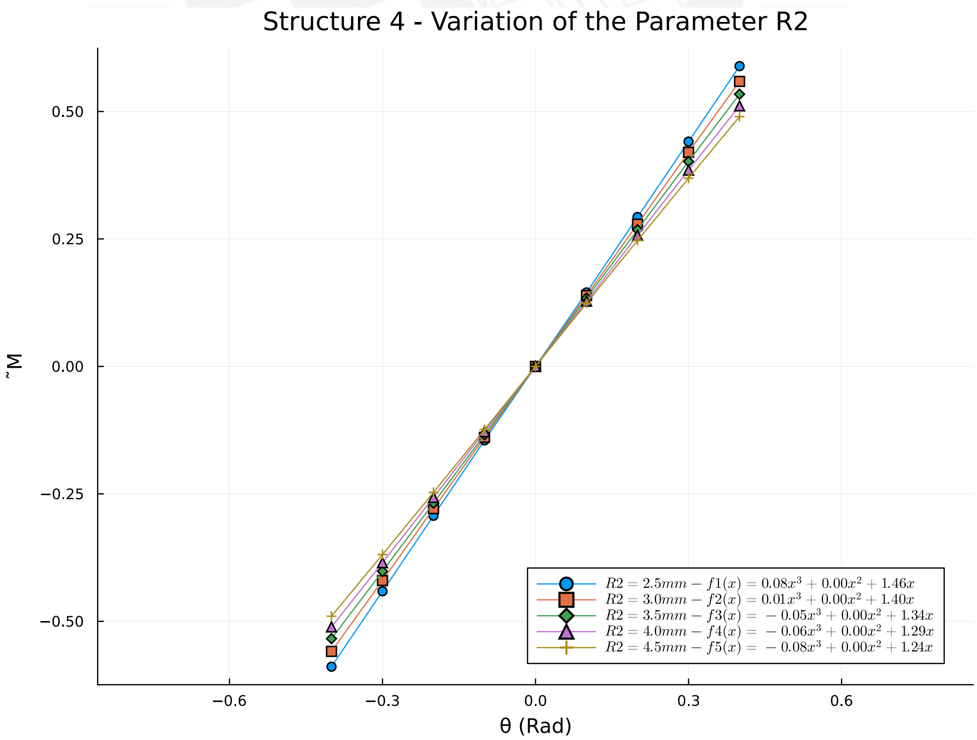



Figure 5.14: Behavior of the parameter R2 of the structure 4 and the adjusted polynomial functions

## 5.4 Discussion of the results

In the section 5.1 the relative and absolute errors when the structures are under external forces were presented. It can be seen that in most of the cases, the relatives error is less than 10%, but in few cases these values are higher. This happens because the absolute values of displacement that are compared are small. For this reason, the absolute errors are presented, too. In the columns of absolute errors it can be seen that in all the cases, the error is less than $1mm$, which is acceptable when no precision applications are required. Avoiding this peaks, an average absolute values of relative error of displacement in x and y axis are 5.69% and 4.28% respectively. Finally, in the table 5.10 it can be seen that the relatives errors of moments in all the cases are less than 10% and the average of the absolute values of the relatives errors is 1.74%.

In the section 5.2 the relative error of the external forces required for the rotation of the structures are presented. Those external forces are represented by the angle of the result forces, the resultant force and the moment. The relatives error can be seen for every step of the rotation of the structures between -0.4 $rad$ and 0.4 $rad$. Similarly, as explained in the previous paragraph, there are peaks of relative errors that are mostly due to small moment values, which make them sensitive to get relative errors. An example of this happens in the table 5.17 where a relative error of 575.19% appears when R1 = $9mm$ and the rotation of the structure is -0.2$rad$. But the values of the moments obtained by Julia and Ansys are -78.59 N.mm and -11.64 N.mm, respectively, which very small values. In order to have a valid representation of the relative errors, the table 5.39 was made, in which, the peak values were omitted to have a better representation. In this table, it can be seen that the maximal average of relative error is 5.05%. It is important to mention too, that the errors can be generated because in one hand the analytical model works on the plane, and on the other hand, the FEM model works in three dimensions. This implies also that the restrictions made in the analytical model are applied to nodes, but in the case of the FEM model, the restrictions are applied to areas. The same happens when the external forces and external displacements are applied.

Finally, the values of the normalized moment due to the rotation of the structure and the variation of the parameters are presented in the section 5.3. In addition to that a representation of those values are represented and for every curve the adjusted polynomial function was generated. For the structures 1, 2 and 3, the curves are not symmetrical because the structure itself is not. When the structure 1 is analyzed, it can be seen that the moment required to rotate it to -0.4 $rad$ is almost twice as much as if it is rotated to 0.4 $rad$. In the case of the curves generated by the variation of the parameters in the structure 2, it can be seen that almost the behavior of the function are nearly lineal to the defined range. Even the adjusted polynomial equations are of third degree, the coefficients that correspond to the cubic and square variables are 0.1 and 0.25, respectively, which are small enough to have an influence on the behavior of the curves. Even in the Fig. 5.2 when R1 = $11mm$, the adjusted equation is linear. Now in

Table 5.39: Average of the values of the relative errors of the 4 structures with their respective parameters.

| Structure | Parameter | Angle (%) | Resultant Force (%) | Moment (%) |
|---|---|---|---|---|
| 1 | R1 | 0,31 | 4,28 | 3,18 |
| 2 | R1 | 0,40 | 2,49 | 2,49 |
| | R2 | 0,47 | 2,57 | 2,05 |
| | L1 | 0,39 | 2,64 | 1,45 |
| | L2 | 0,39 | 2,46 | 1,81 |
| | L3 | 0,50 | 2,42 | 2,52 |
| 3 | R1 | 0,22 | 2,49 | 4,84 |
| | R2 | 0,31 | 2,61 | 5,05 |
| | L1 | 0,20 | 2,66 | 2,33 |
| | L2 | 0,20 | 3,09 | 3,00 |
| | L3 | 0,24 | 2,78 | 2,95 |
| 4 | R1 | 0,49 | 3,75 | 2,24 |
| | $\alpha$ | 0,56 | 4,40 | 2,37 |
| | R2 | 0,37 | 4,63 | 2,74 |

reference to structure 3 it can be seen that the parameters R1 and R2 have good influence in the behavior of the curves. In contrast, the parameters $L1$, $L2$ and $L3$ have little influence in the behavior of the curves. Important to mention are curves generated due to the variation of the parameter $L3$. It can be seen that the curves coincide in the range from -0.4 $rad$ to 0.1 $rad$ and they start to separate from 0.2 $rad$. Lastly, with respect to the adjusted equation with respect to the structure 4, it can be seen that the coefficient of the quadratic variable is zero. This implies that they are odd functions, so the curves are symmetrical. In contrast, it can be seen in the figures of the behavior of the curves of the structure 4, that the variation of the parameters have little influence in their behavior. An special case are the curve respect to the variation of the parameter $R1$, where it can be seen that the points almost coincide.

# 6 Conclusions and Outlook

## 6.1 Conclusions

The analytical model was developed in a programming language called Julia. In this model different kinds of structures were simulated and the results were presented. The same was made in Ansys, a software that use the FEM to make the simulations. The advantages that offer the analytical model is that it requires less computational resources, so the time of process to obtain results is shorter than the use of the FEM model. However the FEM model was useful to make a comparison with the analytical result. In total, nine structures were simulated and, in order to define the dimensions of the structures, the “beamfeatures” and “nodefeatures” matrices were needed.

With respect to the results of this investigation, both the analytical and FEM model were used. The first results were the displacement of the nodes due to the external forces. To do this, nine structures where build in the analytical model and simulated. The common restriction that have the structures were that the first node were fixed. Then, external forces were applied at different nodes of the structures and the displacement were obtained. The average of the absolute values of the relative errors are 5.69% and 4.28% respectively with the displacement in x and y. After that the moments at the first node where obtained where the average of the absolute values of the relative errors are 1.74%.

After that the force determination necessary to rotate the structure in the range of -0.4 $rad$ and 0.4 $rad$ was carried out. In addition to that, it was important to vary the parameters that describe the geometry of the structures. In the case of the structure 1, the parameter was only $R1$ which is the radius of curvature of the beams. In the case of the structure 2, the parameters were $R1$, $R2$, $L1$, $L2$, $L3$, which are related radius of curvature and length of the beams. Similarly with the structure 3, the parameters $R1$, $R2$, $L1$, $L2$, $L3$ were defined. And with respect to the structure 4, there were 3 parameters, $R1$, $\alpha$ and $R2$. In total, 14 parameters were analyzed, which were varied in 8 different positions because we wanted to study of the behavior of the normalized moment at node 1 of each structure. The average of the absolute values of the relatives error of the angle of the resultant force, the resultant force and moment are presented in the table 5.39, where the maximal relative error is 5.05%.

Finally, the behavior of the normalized moment in the first node were studied. To do this, it was necessary to vary the 14 parameters of the structures in 8 different positions. Also the adjusted polynomial functions were generated to see the behavior of the curves. It was observed that the where parameters which have more or less influence in the behavior of the curves. For example, the parameters $R1$ and $\alpha$ from the structure 4 have almost no influence in the behavior of the curve. In contrast, the parameters $R1$ and $R2$ have a relative good influence in the behavior of the curves.

## 6.2 Outlook

Future works that can be derived from this investigation are related to the improvement of the analytical model presented. The following are some limitations found in the present work that may represent as an opportunity for improvement or future work:

- It the present investigation the beams are considered to have a constant cross-sectional area, so it is pending to implement this new attribute to the elements of the compliant mechanism.
- Only elements with constant radius of curvature were used. So the use of element with elliptical or hyperbolic form could bring other kind of behaviors.
- The EBBT was used to developed the analytical model. So it would be an improvement to implement the TBT to the model, in which the shear forces are considered.
- Finally, the search of compliant mechanisms that follow and specific behavior for a specific purpose can be realized by the definition of other parameters than the length of the beams or radius of curvature, and study their influence.

# A Appendix

## A.1 Complete Results of the forces and moments due to movement in Julia

Table A.1: Parameter variation of the parameter R1 of the structure 1

| Parameter R1 (mm) | Rotation $\theta$ (rad) | Pos. end node x (mm) | y (mm) | Angle (rad) | External Forces $F_x$ (N) | $F_y$ (N) | $M_e$ (N.mm) | Moment (node 1) $M_c$ (N.mm) |
|---|---|---|---|---|---|---|---|---|
| | -0.4 | 57.21 | -18.76 | 1.17 | 5391.57 | -5655.97 | 40802.14 | -96790.47 |
| | -0.3 | 58.80 | -12.95 | 1.27 | 3949.27 | -3353.17 | 29112.53 | -66613.27 |
| | -0.2 | 59.80 | -7.02 | 1.37 | 2480.66 | -1710.14 | 18612.32 | -40587.82 |
| | -0.1 | 60.20 | -1.01 | 1.47 | 1119.61 | -631.20 | 8970.22 | -18429.31 |
| 5 | 0 | 60.00 | 5.00 | 1.57 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 59.20 | 10.97 | 1.67 | -796.67 | 340.21 | -8550.23 | 15226.34 |
| | 0.2 | 57.81 | 16.82 | 1.77 | -1221.66 | 559.14 | -16680.39 | 27804.60 |
| | 0.3 | 55.84 | 22.51 | 1.87 | -1276.88 | 836.34 | -24302.76 | 38596.04 |
| | 0.4 | 53.32 | 27.97 | 1.97 | -1001.58 | 1344.02 | -31039.31 | 48477.65 |
| | -0.4 | 65.89 | -21.34 | 1.17 | 3456.45 | -3515.51 | 30609.73 | -74533.77 |
| | -0.3 | 67.69 | -14.66 | 1.27 | 2551.06 | -2109.73 | 22042.92 | -51720.49 |
| | -0.2 | 68.82 | -7.83 | 1.37 | 1623.32 | -1090.68 | 14198.64 | -31791.00 |
| | -0.1 | 69.25 | -0.92 | 1.47 | 742.05 | -407.90 | 6893.58 | -14552.55 |
| 6 | 0 | 69.00 | 6.00 | 1.57 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 68.06 | 12.86 | 1.67 | -560.24 | 219.18 | -6617.30 | 12217.28 |
| | 0.2 | 66.43 | 19.59 | 1.77 | -915.95 | 342.26 | -13008.79 | 22537.14 |
| | 0.3 | 64.15 | 26.12 | 1.87 | -1041.39 | 472.17 | -19047.61 | 31360.53 |
| | 0.4 | 61.22 | 32.40 | 1.97 | -972.34 | 703.40 | -24582.90 | 39431.82 |
| | -0.4 | 74.57 | -23.93 | 1.17 | 2397.19 | -2382.90 | 24255.78 | -60328.96 |
| | -0.3 | 76.58 | -16.36 | 1.27 | 1776.18 | -1439.75 | 17550.78 | -42050.58 |
| | -0.2 | 77.84 | -8.64 | 1.37 | 1142.88 | -752.55 | 11378.80 | -26036.99 |
| | -0.1 | 78.31 | -0.82 | 1.47 | 531.89 | -284.29 | 5549.55 | -12012.79 |
| 7 | 0 | 78.00 | 7.00 | 1.57 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 76.91 | 14.75 | 1.67 | -420.21 | 151.71 | -5371.37 | 10219.37 |
| | 0.2 | 75.05 | 22.36 | 1.77 | -705.77 | 228.75 | -10601.26 | 18916.22 |
| | 0.3 | 72.45 | 29.74 | 1.87 | -844.23 | 294.16 | -15603.12 | 26403.64 |
| | 0.4 | 69.12 | 36.82 | 1.97 | -854.12 | 406.61 | -20259.90 | 33194.77 |
| | -0.4 | 83.25 | -26.52 | 1.17 | 1756.35 | -1715.12 | 19956.71 | -50521.99 |
| | -0.3 | 85.48 | -18.07 | 1.27 | 1310.26 | -1044.10 | 14512.53 | -35399.05 |
| | -0.2 | 86.86 | -9.44 | 1.37 | 846.45 | -548.29 | 9429.62 | -21979.84 |
| | -0.1 | 87.36 | -0.73 | 1.47 | 396.74 | -208.78 | 4625.07 | -10192.75 |
| 8 | 0 | 87.00 | 8.00 | 1.57 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 85.77 | 16.65 | 1.67 | -322.26 | 111.43 | -4502.37 | 8749.31 |
| | 0.2 | 83.68 | 25.12 | 1.77 | -555.17 | 163.08 | -8896.47 | 16249.72 |
| | 0.3 | 80.75 | 33.35 | 1.87 | -690.77 | 196.35 | -13167.85 | 22779.63 |
| | 0.4 | 77.02 | 41.25 | 1.97 | -729.21 | 253.98 | -17188.89 | 28643.00 |
| | -0.4 | 91.93 | -29.09 | 1.17 | 1338.27 | -1288.31 | 16854.63 | -43324.49 |
| | -0.3 | 94.37 | -19.77 | 1.27 | 1002.37 | -788.63 | 12307.42 | -30469.55 |
| | -0.2 | 95.87 | -10.25 | 1.37 | 649.91 | -416.30 | 8025.14 | -18979.32 |
| | -0.1 | 96.42 | -0.63 | 1.47 | 309.37 | -159.68 | 3948.93 | -8857.21 |
| 9 | 0 | 96.00 | 9.00 | 1.57 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 94.62 | 18.54 | 1.67 | -257.70 | 84.77 | -3865.14 | 7662.15 |
| | 0.2 | 92.30 | 27.89 | 1.77 | -449.27 | 121.41 | -7658.53 | 14256.85 |
| | 0.3 | 89.05 | 36.97 | 1.87 | -571.18 | 138.78 | -11371.91 | 20021.02 |
| | 0.4 | 84.92 | 45.67 | 1.97 | -621.90 | 166.96 | -14898.65 | 25177.49 |

Table A.2: Parameter variation of the parameter R1 of the structure 2

| Parameter R1 (mm) | Rotation $\theta$ (rad) | Pos. end node x (mm) | y (mm) | Angle (rad) | External Forces $F_x$ (N) | $F_y$ (N) | $M_e$ (N.mm) | Moment (node 1) $M_c$ (N.mm) |
|---|---|---|---|---|---|---|---|---|
| 8 | -0.4 | -5.22 | -40.85 | -1.34 | -2798.81 | -3550.65 | 30821.98 | -64974.89 |
| | -0.3 | -1.19 | -39.67 | -1.24 | -1901.98 | -2736.64 | 24092.91 | -48102.06 |
| | -0.2 | 2.70 | -38.09 | -1.14 | -1134.13 | -1864.46 | 16632.44 | -31600.64 |
| | -0.1 | 6.41 | -36.13 | -1.04 | -495.07 | -945.76 | 8453.45 | -15495.85 |
| | 0 | 9.91 | -33.81 | -0.94 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 13.16 | -31.16 | -0.84 | 404.86 | 993.63 | -10067.96 | 15623.74 |
| | 0.2 | 16.13 | -28.19 | -0.74 | 669.11 | 2001.61 | -20383.91 | 30764.32 |
| | 0.3 | 18.79 | -24.94 | -0.64 | 811.88 | 3024.27 | -31353.09 | 45721.09 |
| | 0.4 | 21.11 | -21.44 | -0.54 | 835.02 | 4057.10 | -42921.56 | 60626.80 |
| 9 | -0.4 | -7.99 | -41.41 | -1.40 | -2601.80 | -2834.99 | 25924.70 | -59164.23 |
| | -0.3 | -3.89 | -40.51 | -1.30 | -1806.58 | -2210.86 | 20445.51 | -44138.61 |
| | -0.2 | 0.09 | -39.19 | -1.20 | -1110.36 | -1523.27 | 14373.45 | -29278.54 |
| | -0.1 | 3.93 | -37.49 | -1.10 | -511.68 | -786.49 | 7651.76 | -14622.09 |
| | 0 | 7.58 | -35.41 | -1.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 11.00 | -32.98 | -0.90 | 386.41 | 814.98 | -7594.39 | 14114.21 |
| | 0.2 | 14.16 | -30.22 | -0.80 | 685.98 | 1665.41 | -16086.09 | 28226.39 |
| | 0.3 | 17.04 | -27.16 | -0.70 | 887.99 | 2542.11 | -25168.63 | 42266.55 |
| | 0.4 | 19.59 | -23.83 | -0.60 | 990.06 | 3439.11 | -34718.81 | 56246.65 |
| 10 | -0.4 | -10.74 | -41.78 | -1.47 | -2339.58 | -2257.33 | 20238.48 | -53265.45 |
| | -0.3 | -6.59 | -41.15 | -1.37 | -1639.75 | -1774.73 | 15929.03 | -39851.18 |
| | -0.2 | -2.52 | -40.10 | -1.27 | -1013.62 | -1231.50 | 11088.47 | -26454.39 |
| | -0.1 | 1.42 | -38.66 | -1.17 | -466.59 | -639.42 | 5762.49 | -13183.77 |
| | 0 | 5.20 | -36.82 | -1.07 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 8.77 | -34.62 | -0.97 | 393.14 | 681.77 | -6412.43 | 13177.37 |
| | 0.2 | 12.11 | -32.08 | -0.87 | 703.12 | 1395.98 | -13230.07 | 26231.17 |
| | 0.3 | 15.18 | -29.21 | -0.77 | 933.25 | 2143.42 | -20525.40 | 39271.91 |
| | 0.4 | 17.94 | -26.05 | -0.67 | 1081.29 | 2916.70 | -28218.87 | 52274.38 |
| 11 | -0.4 | -13.43 | -41.97 | -1.53 | -2102.70 | -1796.23 | 16025.65 | -48101.24 |
| | -0.3 | -9.25 | -41.60 | -1.43 | -1491.17 | -1423.73 | 12726.80 | -36136.24 |
| | -0.2 | -5.13 | -40.82 | -1.33 | -938.11 | -998.02 | 9012.22 | -24161.46 |
| | -0.1 | -1.10 | -39.63 | -1.23 | -443.24 | -521.92 | 4851.75 | -12139.74 |
| | 0 | 2.79 | -38.05 | -1.13 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 6.50 | -36.08 | -1.03 | 358.29 | 558.92 | -4682.32 | 11877.94 |
| | 0.2 | 9.99 | -33.75 | -0.93 | 662.29 | 1155.79 | -10028.17 | 23870.54 |
| | 0.3 | 13.24 | -31.09 | -0.83 | 901.41 | 1784.06 | -15773.89 | 35871.84 |
| | 0.4 | 16.20 | -28.12 | -0.73 | 1073.16 | 2440.27 | -21846.52 | 47862.96 |
| 12 | -0.4 | -16.06 | -41.99 | -1.60 | -1841.50 | -1434.03 | 11266.52 | -43027.47 |
| | -0.3 | -11.87 | -41.88 | -1.50 | -1310.28 | -1145.47 | 8912.24 | -32365.53 |
| | -0.2 | -7.70 | -41.36 | -1.40 | -821.72 | -807.75 | 6174.35 | -21592.44 |
| | -0.1 | -3.61 | -40.43 | -1.30 | -381.50 | -425.19 | 3113.56 | -10775.69 |
| | 0 | 0.37 | -39.09 | -1.20 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 4.19 | -37.36 | -1.10 | 350.99 | 463.50 | -4013.63 | 11041.46 |
| | 0.2 | 7.83 | -35.25 | -1.00 | 640.95 | 963.05 | -8090.72 | 22043.34 |
| | 0.3 | 11.23 | -32.80 | -0.90 | 875.20 | 1491.37 | -12437.39 | 33017.15 |
| | 0.4 | 14.38 | -30.02 | -0.80 | 1056.08 | 2049.13 | -17111.75 | 44058.27 |

Table A.3: Parameter variation of the parameter R2 of the structure 2

| Parameter R2 (mm) | Rotation $\theta$ (rad) | Pos. end node x (mm) | y (mm) | Angle (rad) | External Forces $F_x$ (N) | $F_y$ (N) | $M_e$ (N.mm) | Moment (node 1) $M_c$ (N.mm) |
|---|---|---|---|---|---|---|---|---|
| 4 | -0.4 | -5.22 | -40.85 | -1.34 | -2798.81 | -3550.65 | 30821.98 | -64974.89 |
| | -0.3 | -1.19 | -39.67 | -1.24 | -1901.98 | -2736.64 | 24092.91 | -48102.06 |
| | -0.2 | 2.70 | -38.09 | -1.14 | -1134.13 | -1864.46 | 16632.44 | -31600.64 |
| | -0.1 | 6.41 | -36.13 | -1.04 | -495.07 | -945.76 | 8453.45 | -15495.85 |
| | 0 | 9.91 | -33.81 | -0.94 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 13.16 | -31.16 | -0.84 | 404.86 | 993.63 | -10067.96 | 15623.74 |
| | 0.2 | 16.13 | -28.19 | -0.74 | 669.11 | 2001.61 | -20383.91 | 30764.32 |
| | 0.3 | 18.79 | -24.94 | -0.64 | 811.88 | 3024.27 | -31353.09 | 45721.09 |
| | 0.4 | 21.11 | -21.44 | -0.54 | 835.02 | 4057.10 | -42921.56 | 60626.80 |
| 5 | -0.4 | -8.82 | -41.54 | -1.42 | -2825.58 | -3102.48 | 29747.68 | -60263.20 |
| | -0.3 | -4.70 | -40.72 | -1.32 | -1951.05 | -2408.42 | 23449.54 | -44677.85 |
| | -0.2 | -0.69 | -39.49 | -1.22 | -1192.28 | -1654.84 | 16463.07 | -29478.27 |
| | -0.1 | 3.18 | -37.86 | -1.12 | -546.19 | -850.00 | 8750.60 | -14631.26 |
| | 0 | 6.87 | -35.86 | -1.02 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 10.34 | -33.49 | -0.92 | 407.35 | 869.40 | -8703.80 | 13927.96 |
| | 0.2 | 13.56 | -30.80 | -0.82 | 718.01 | 1766.82 | -18371.17 | 27701.64 |
| | 0.3 | 16.49 | -27.79 | -0.72 | 921.68 | 2686.85 | -28637.38 | 41282.22 |
| | 0.4 | 19.11 | -24.51 | -0.62 | 1020.43 | 3619.70 | -39437.82 | 54745.57 |
| 6 | -0.4 | -12.39 | -41.92 | -1.51 | -2790.64 | -2730.39 | 27354.99 | -55799.29 |
| | -0.3 | -8.22 | -41.45 | -1.41 | -1937.08 | -2128.20 | 21524.81 | -41273.21 |
| | -0.2 | -4.11 | -40.56 | -1.31 | -1183.82 | -1462.67 | 14945.28 | -27058.87 |
| | -0.1 | -0.12 | -39.28 | -1.21 | -538.91 | -752.86 | 7747.08 | -13330.97 |
| | 0 | 3.73 | -37.59 | -1.11 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 7.39 | -35.54 | -1.01 | 447.99 | 788.09 | -8669.63 | 13075.86 |
| | 0.2 | 10.82 | -33.12 | -0.91 | 790.28 | 1601.58 | -17751.39 | 25751.86 |
| | 0.3 | 14.00 | -30.38 | -0.81 | 1035.66 | 2433.02 | -27371.52 | 38154.08 |
| | 0.4 | 16.89 | -27.33 | -0.71 | 1187.41 | 3283.12 | -37490.86 | 50412.99 |
| 7 | -0.4 | -15.90 | -41.99 | -1.59 | -2786.18 | -2422.48 | 26135.16 | -52339.13 |
| | -0.3 | -11.70 | -41.87 | -1.49 | -1949.00 | -1899.45 | 20669.13 | -38711.70 |
| | -0.2 | -7.54 | -41.33 | -1.39 | -1207.48 | -1315.18 | 14531.06 | -25457.45 |
| | -0.1 | -3.45 | -40.38 | -1.29 | -560.53 | -560.53 | 7705.05 | -12995.48 |
| | 0 | 0.52 | -39.03 | -1.19 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 4.34 | -37.28 | -1.09 | 447.68 | 705.63 | -7791.66 | 11960.16 |
| | 0.2 | 7.97 | -35.16 | -0.99 | 813.72 | 1441.96 | -16403.77 | 23698.94 |
| | 0.3 | 11.36 | -32.70 | -0.89 | 1087.96 | 2194.72 | -25431.52 | 35076.78 |
| | 0.4 | 14.50 | -29.90 | -0.79 | 1279.10 | 2969.61 | -34981.08 | 46323.43 |
| 8 | -0.4 | -19.31 | -41.78 | -1.67 | -2770.17 | -2160.05 | 24747.59 | -49279.65 |
| | -0.3 | -15.12 | -42.00 | -1.57 | -1950.49 | -1704.75 | 19677.43 | -36467.58 |
| | -0.2 | -10.93 | -41.80 | -1.47 | -1217.17 | -1186.79 | 13915.96 | -23990.11 |
| | -0.1 | -6.77 | -41.19 | -1.37 | -570.87 | -617.52 | 7455.98 | -11877.36 |
| | 0 | -2.70 | -40.16 | -1.27 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 1.24 | -38.73 | -1.17 | 451.72 | 638.32 | -7192.07 | 11094.58 |
| | 0.2 | 5.03 | -36.92 | -1.07 | 833.11 | 1307.71 | -15330.74 | 22005.40 |
| | 0.3 | 8.62 | -34.73 | -0.97 | 1132.21 | 2000.01 | -23944.51 | 32617.33 |
| | 0.4 | 11.97 | -32.20 | -0.87 | 1350.03 | 2707.07 | -32921.02 | 42953.68 |

Table A.4: Parameter variation of the parameter L1 of the structure 2

| Parameter L1 (mm) | Rotation $\theta$ (rad) | Pos. end node x (mm) | y (mm) | Angle (rad) | External Forces $F_x$ (N) | $F_y$ (N) | $M_e$ (N.mm) | Moment (node 1) $M_c$ (N.mm) |
|---|---|---|---|---|---|---|---|---|
| | -0.4 | -5.22 | -40.85 | -1.34 | -2798.81 | -3550.65 | 30821.98 | -64974.89 |
| | -0.3 | -1.19 | -39.67 | -1.24 | -1901.98 | -2736.64 | 24092.91 | -48102.06 |
| | -0.2 | 2.7 | -38.09 | -1.14 | -1134.13 | -1864.46 | 16632.44 | -31600.64 |
| | -0.1 | 6.41 | -36.13 | -1.04 | -495.07 | -945.76 | 8453.45 | -15495.85 |
| 12 | 0 | 9.91 | -33.81 | -0.94 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 13.16 | -31.16 | -0.84 | 404.86 | 993.63 | -10067.96 | 15623.74 |
| | 0.2 | 16.13 | -28.19 | -0.74 | 669.11 | 2001.61 | -20383.91 | 30764.32 |
| | 0.3 | 18.79 | -24.94 | -0.64 | 811.88 | 3024.27 | -31353.09 | 45721.09 |
| | 0.4 | 21.11 | -21.44 | -0.54 | 835.02 | 4057.10 | -42921.56 | 60626.80 |
| | -0.4 | -3.54 | -41.44 | -1.3 | -2647.29 | -3542.47 | 30927.86 | -66235.34 |
| | -0.3 | 0.54 | -40.09 | -1.2 | -1796.71 | -2731.84 | 24300.17 | -49204.98 |
| | -0.2 | 4.47 | -38.34 | -1.1 | -1071.55 | -1865.22 | 16935.20 | -32485.66 |
| | -0.1 | 8.2 | -36.21 | -1 | -474.73 | -953.93 | 8883.08 | -16129.01 |
| 13 | 0 | 11.7 | -33.71 | -0.9 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 14.93 | -30.88 | -0.8 | 340.69 | 975.49 | -9375.11 | 15709.35 |
| | 0.2 | 17.86 | -27.73 | -0.7 | 559.61 | 1977.95 | -19563.27 | 31281.05 |
| | 0.3 | 20.47 | -24.31 | -0.6 | 654.13 | 2991.23 | -30431.83 | 46700.43 |
| | 0.4 | 22.72 | -20.65 | -0.5 | 623.09 | 4009.31 | -41893.50 | 62064.79 |
| | -0.4 | -1.91 | -42.01 | -1.27 | -2467.41 | -3518.95 | 29965.23 | -66969.51 |
| | -0.3 | 2.22 | -40.49 | -1.17 | -1660.94 | -2709.94 | 23484.26 | -49783.37 |
| | -0.2 | 6.18 | -38.57 | -1.07 | -979.20 | -1848.29 | 16302.51 | -32887.56 |
| | -0.1 | 9.92 | -36.26 | -0.97 | -424.45 | -940.37 | 8443.62 | -16275.32 |
| 14 | 0 | 13.42 | -33.59 | -0.87 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 16.63 | -30.59 | -0.77 | 307.77 | 973.06 | -9423.18 | 16173.68 |
| | 0.2 | 19.53 | -27.28 | -0.67 | 483.16 | 1963.24 | -19419.33 | 32103.26 |
| | 0.3 | 22.08 | -23.69 | -0.57 | 529.33 | 2966.16 | -30098.74 | 47933.78 |
| | 0.4 | 24.26 | -19.87 | -0.47 | 445.79 | 3968.35 | -41406.72 | 63723.39 |
| | -0.4 | 1.22 | -43.04 | -1.21 | -2145.54 | -3433.54 | 28528.64 | -68004.37 |
| | -0.3 | 5.44 | -41.21 | -1.11 | -1425.79 | -2643.05 | 22386.62 | -50748.24 |
| | -0.2 | 9.45 | -38.96 | -1.01 | -827.80 | -1799.99 | 15608.02 | -33652.77 |
| | -0.1 | 13.22 | -36.33 | -0.91 | -351.96 | -918.09 | 8160.97 | -16762.97 |
| 15 | 0 | 15.09 | -33.46 | -0.81 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 18.28 | -30.29 | -0.71 | 221.72 | 939.77 | -8763.58 | 16561.17 |
| | 0.2 | 21.13 | -26.82 | -0.61 | 315.21 | 1894.85 | -18269.64 | 33036.42 |
| | 0.3 | 23.63 | -22.47 | -0.51 | 276.46 | 2855.46 | -28508.05 | 49490.33 |
| | 0.4 | 25.74 | -18.35 | -0.41 | 100.64 | 3805.89 | -39379.30 | 65911.61 |
| | -0.4 | 1.22 | -43.04 | -1.21 | -2145.54 | -3433.54 | 28528.64 | -68004.37 |
| | -0.3 | 5.44 | -41.21 | -1.11 | -1425.79 | -2643.05 | 22386.62 | -50748.24 |
| | -0.2 | 9.45 | -38.96 | -1.01 | -827.80 | -1799.99 | 15608.02 | -33652.77 |
| | -0.1 | 13.22 | -36.33 | -0.91 | -351.96 | -918.09 | 8160.97 | -16762.97 |
| 16 | 0 | 16.7 | -33.33 | -0.81 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 19.87 | -30 | -0.71 | 221.72 | 939.77 | -8763.58 | 16561.17 |
| | 0.2 | 22.69 | -26.37 | -0.61 | 315.21 | 1894.85 | -18269.64 | 33036.42 |
| | 0.3 | 25.14 | -22.47 | -0.51 | 276.46 | 2855.46 | -28508.05 | 49490.33 |
| | 0.4 | 27.18 | -18.35 | -0.41 | 100.64 | 3805.89 | -39379.30 | 65911.61 |

Table A.5: Parameter variation of the parameter L2 of the structure 2

| Parameter L2 (mm) | Rotation $\theta$ (rad) | Pos. end node x (mm) | y (mm) | Angle (rad) | External Forces $F_x$ (N) | $F_y$ (N) | $M_e$ (N.mm) | Moment (node 1) $M_c$ (N.mm) |
|---|---|---|---|---|---|---|---|---|
| 5 | -0.4 | -5.22 | -40.85 | -1.34 | -2798.81 | -3550.65 | 30821.98 | -64974.89 |
| | -0.3 | -1.19 | -39.67 | -1.24 | -1901.98 | -2736.64 | 24092.91 | -48102.06 |
| | -0.2 | 2.70 | -38.09 | -1.14 | -1134.13 | -1864.46 | 16632.44 | -31600.64 |
| | -0.1 | 6.41 | -36.13 | -1.04 | -495.07 | -945.76 | 8453.45 | -15495.85 |
| | 0 | 9.91 | -33.81 | -0.94 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 13.16 | -31.16 | -0.84 | 404.86 | 993.63 | -10067.96 | 15623.74 |
| | 0.2 | 16.13 | -28.19 | -0.74 | 669.11 | 2001.61 | -20383.91 | 30764.32 |
| | 0.3 | 18.79 | -24.94 | -0.64 | 811.88 | 3024.27 | -31353.09 | 45721.09 |
| | 0.4 | 21.11 | -21.44 | -0.54 | 835.02 | 4057.10 | -42921.56 | 60626.80 |
| 6 | -0.4 | -6.12 | -40.03 | -1.35 | -2775.41 | -3241.93 | 30013.66 | -61245.36 |
| | -0.3 | -2.17 | -38.94 | -1.25 | -1913.57 | -2509.45 | 23612.90 | -45455.98 |
| | -0.2 | 1.65 | -37.47 | -1.15 | -1167.34 | -1722.60 | 16536.63 | -30045.98 |
| | -0.1 | 5.31 | -35.62 | -1.05 | -532.70 | -884.50 | 8741.29 | -14930.36 |
| | 0 | 8.76 | -33.41 | -0.95 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 11.98 | -30.87 | -0.85 | 398.43 | 905.88 | -8814.59 | 14337.49 |
| | 0.2 | 14.93 | -28.02 | -0.75 | 699.94 | 1844.11 | -18550.96 | 28593.82 |
| | 0.3 | 17.58 | -24.90 | -0.65 | 894.20 | 2800.30 | -28833.95 | 42660.80 |
| | 0.4 | 19.90 | -21.52 | -0.55 | 985.30 | 3778.17 | -39661.99 | 56727.18 |
| 7 | -0.4 | -7.06 | -39.20 | -1.37 | -2710.65 | -2947.82 | 28155.42 | -57290.63 |
| | -0.3 | -3.18 | -38.21 | -1.27 | -1880.47 | -2288.21 | 22079.28 | -42496.85 |
| | -0.2 | 0.57 | -36.84 | -1.17 | -1155.44 | -1572.80 | 15404.47 | -28058.42 |
| | -0.1 | 4.17 | -35.11 | -1.07 | -530.76 | -810.33 | 8078.84 | -13935.07 |
| | 0 | 7.58 | -33.02 | -0.97 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 10.77 | -30.60 | -0.87 | 423.49 | 839.29 | -8455.30 | 13542.58 |
| | 0.2 | 13.69 | -27.87 | -0.77 | 753.99 | 1711.66 | -17534.66 | 26911.53 |
| | 0.3 | 16.33 | -24.87 | -0.67 | 992.62 | 2607.53 | -27130.34 | 40137.22 |
| | 0.4 | 18.66 | -21.62 | -0.57 | 1143.27 | 3527.58 | -37191.66 | 53350.40 |
| 8 | -0.4 | -8.03 | -38.37 | -1.39 | -2648.20 | -2677.65 | 26505.02 | -53604.76 |
| | -0.3 | -4.23 | -37.48 | -1.29 | -1851.31 | -2084.77 | 20787.40 | -39780.98 |
| | -0.2 | -0.55 | -36.22 | -1.19 | -1146.11 | -1312.92 | 13860.91 | -24695.40 |
| | -0.1 | 1.78 | -34.60 | -1.09 | -541.13 | -678.18 | 7365.61 | -12288.67 |
| | 0 | 6.36 | -32.63 | -0.99 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 9.51 | -30.08 | -0.89 | 432.71 | 705.81 | -7086.21 | 11731.40 |
| | 0.2 | 12.42 | -27.73 | -0.79 | 791.34 | 1580.24 | -16363.73 | 25206.64 |
| | 0.3 | 15.05 | -24.87 | -0.69 | 1066.65 | 2412.31 | -25250.75 | 37571.40 |
| | 0.4 | 17.38 | -21.73 | -0.59 | 1267.91 | 3274.41 | -34532.94 | 49928.11 |
| 9 | -0.4 | -9.05 | -37.53 | -1.41 | -2593.97 | -2428.11 | 25154.52 | -50222.88 |
| | -0.3 | -5.33 | -36.75 | -1.31 | -1829.73 | -1898.37 | 19789.16 | -37335.07 |
| | -0.2 | -1.71 | -35.60 | -1.21 | -1146.11 | -1312.92 | 13860.91 | -24695.40 |
| | -0.1 | 1.78 | -34.09 | -1.11 | -541.13 | -678.18 | 7365.61 | -12288.67 |
| | 0 | 5.10 | -32.25 | -1.01 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 8.22 | -30.08 | -0.91 | 432.71 | 705.81 | -7086.21 | 11731.40 |
| | 0.2 | 11.11 | -27.61 | -0.81 | 808.98 | 1448.52 | -14981.12 | 23447.85 |
| | 0.3 | 13.73 | -24.87 | -0.71 | 1113.94 | 2217.64 | -23174.57 | 34977.17 |
| | 0.4 | 16.07 | -21.88 | -0.61 | 1357.09 | 3015.47 | -31681.67 | 46469.97 |

Table A.6: Parameter variation of the parameter L3 of the structure 2

| Parameter L3 (mm) | Rotation $\theta$ (rad) | Pos. end node x (mm) | y (mm) | Angle (rad) | External Forces $F_x$ (N) | $F_y$ (N) | $M_e$ (N.mm) | Moment (node 1) $M_c$ (N.mm) |
|---|---|---|---|---|---|---|---|---|
| | -0.4 | -5.22 | -40.85 | -1.34 | -2798.81 | -3550.65 | 30821.98 | -64974.89 |
| | -0.3 | -1.19 | -39.67 | -1.24 | -1901.98 | -2736.64 | 24092.91 | -48102.06 |
| | -0.2 | 2.70 | -38.09 | -1.14 | -1134.13 | -1864.46 | 16632.44 | -31600.64 |
| | -0.1 | 6.41 | -36.13 | -1.04 | -495.07 | -945.76 | 8453.45 | -15495.85 |
| 20 | 0 | 9.91 | -33.81 | -0.94 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 13.16 | -31.16 | -0.84 | 404.86 | 993.63 | -10067.96 | 15623.74 |
| | 0.2 | 16.13 | -28.19 | -0.74 | 669.11 | 2001.61 | -20383.91 | 30764.32 |
| | 0.3 | 18.79 | -24.94 | -0.64 | 811.88 | 3024.27 | -31353.09 | 45721.09 |
| | 0.4 | 21.11 | -21.44 | -0.54 | 835.02 | 4057.10 | -42921.56 | 60626.80 |
| | -0.4 | -4.99 | -41.82 | -1.34 | -2694.35 | -3527.48 | 31290.65 | -63785.13 |
| | -0.3 | -0.86 | -40.61 | -1.24 | -1819.15 | -2710.60 | 24441.72 | -47102.93 |
| | -0.2 | 3.12 | -39.00 | -1.14 | -1078.26 | -1844.26 | 16893.05 | -30913.07 |
| | -0.1 | 6.92 | -36.99 | -1.04 | -466.19 | -931.84 | 8580.13 | -15112.51 |
| 21 | 0 | 10.51 | -34.62 | -0.94 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 13.83 | -31.90 | -0.84 | 373.75 | 974.87 | -10209.85 | 15195.25 |
| | 0.2 | 16.88 | -28.86 | -0.74 | 608.42 | 1956.37 | -20707.85 | 29874.71 |
| | 0.3 | 19.60 | -25.53 | -0.64 | 720.70 | 2945.60 | -31828.04 | 44305.01 |
| | 0.4 | 21.97 | -21.95 | -0.54 | 715.42 | 3935.24 | -43530.83 | 58629.88 |
| | -0.4 | -4.76 | -42.79 | -1.34 | -2591.39 | -3502.21 | 31625.70 | -62589.29 |
| | -0.3 | -0.54 | -41.55 | -1.24 | -1740.32 | -2683.86 | 24725.95 | -46135.01 |
| | -0.2 | 3.54 | -39.90 | -1.14 | -1022.51 | -1819.00 | 17060.11 | -30177.20 |
| | -0.1 | 7.43 | -37.85 | -1.04 | -437.97 | -917.28 | 8666.34 | -14726.38 |
| 22 | 0 | 11.10 | -35.42 | -0.94 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 14.51 | -32.64 | -0.84 | 345.05 | 955.82 | -10343.58 | 14787.74 |
| | 0.2 | 17.62 | -29.53 | -0.74 | 548.40 | 1908.90 | -20889.97 | 28939.09 |
| | 0.3 | 20.40 | -26.13 | -0.64 | 631.87 | 2860.54 | -32070.70 | 42794.94 |
| | 0.4 | 22.83 | -22.46 | -0.54 | 602.01 | 3810.17 | -43917.65 | 56589.59 |
| | -0.4 | -4.52 | -43.76 | -1.34 | -2487.68 | -3473.98 | 31802.41 | -61356.23 |
| | -0.3 | -0.21 | -42.50 | -1.24 | -1661.63 | -2658.44 | 24887.29 | -45173.59 |
| | -0.2 | 3.96 | -40.81 | -1.14 | -969.03 | -1796.48 | 17177.03 | -29482.98 |
| | -0.1 | 7.94 | -38.71 | -1.04 | -410.58 | -902.22 | 8716.91 | -14340.16 |
| 23 | 0 | 11.69 | -36.23 | -0.94 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 15.18 | -33.38 | -0.84 | 315.77 | 935.50 | -10394.05 | 14347.39 |
| | 0.2 | 18.36 | -30.20 | -0.74 | 490.88 | 1860.29 | -20982.54 | 27996.96 |
| | 0.3 | 21.21 | -26.72 | -0.64 | 548.78 | 2777.64 | -32241.12 | 39816.60 |
| | 0.4 | 23.69 | -22.97 | -0.54 | 495.26 | 3683.13 | -44069.19 | 52426.51 |
| | -0.4 | -4.29 | -44.74 | -1.34 | -2390.33 | -3449.23 | 31931.85 | -60214.18 |
| | -0.3 | 0.12 | -43.44 | -1.24 | -1584.08 | -2627.95 | 24953.13 | -44174.61 |
| | -0.2 | 4.38 | -41.72 | -1.14 | -917.09 | -1773.10 | 17235.57 | -28791.50 |
| | -0.1 | 8.45 | -39.57 | -1.04 | -384.11 | -886.78 | 8736.37 | -13956.16 |
| 24 | 0 | 12.29 | -37.03 | -0.94 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 15.85 | -34.12 | -0.84 | 287.69 | 914.74 | -10407.31 | 13907.33 |
| | 0.2 | 19.10 | -30.87 | -0.74 | 436.09 | 1810.99 | -20995.98 | 27055.92 |
| | 0.3 | 22.01 | -27.32 | -0.64 | 469.11 | 2690.38 | -32214.75 | 39816.60 |
| | 0.4 | 24.55 | -23.49 | -0.54 | 395.61 | 3552.07 | -44069.58 | 52426.51 |

Table A.7: Parameter variation of the parameter R1 of the structure 3

| Parameter R1 (mm) | Rotation $\theta$ (rad) | Pos. end node x (mm) | y (mm) | Angle (rad) | External Forces $F_x$ (N) | $F_y$ (N) | $M_e$ (N.mm) | Moment (node 1) $M_c$ (N.mm) |
|---|---|---|---|---|---|---|---|---|
| 6 | -0.4 | -22.37 | -58.90 | -1.93 | -993.91 | -689.80 | 7138.24 | -25625.05 |
| | -0.3 | -16.38 | -60.83 | -1.83 | -717.92 | -570.50 | 6053.36 | -19715.21 |
| | -0.2 | -10.22 | -62.17 | -1.73 | -458.20 | -415.07 | 4551.15 | -13467.06 |
| | -0.1 | -3.96 | -62.88 | -1.63 | -218.84 | -224.23 | 2604.97 | -6904.03 |
| | 0 | 2.33 | -62.93 | -1.53 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 8.61 | -62.41 | -1.43 | 181.90 | 250.80 | -2812.13 | 6937.38 |
| | 0.2 | 14.79 | -61.24 | -1.33 | 335.84 | 526.71 | -6376.39 | 14079.61 |
| | 0.3 | 20.83 | -59.46 | -1.23 | 455.44 | 822.31 | -10582.92 | 21291.63 |
| | 0.4 | 26.67 | -57.08 | -1.13 | 538.76 | 1132.74 | -15460.44 | 28511.31 |
| 7 | -0.4 | -30.71 | -55.01 | -2.08 | -863.73 | -483.80 | 3774.52 | -21624.74 |
| | -0.3 | -25.06 | -57.80 | -1.98 | -630.98 | -408.92 | 3342.36 | -16747.23 |
| | -0.2 | -19.16 | -60.01 | -1.88 | -406.74 | -302.44 | 2585.90 | -11490.87 |
| | -0.1 | -13.08 | -61.63 | -1.78 | -195.77 | -166.17 | 1486.20 | -5913.03 |
| | 0 | -6.86 | -62.63 | -1.68 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | -0.57 | -63.00 | -1.58 | 176.31 | 193.13 | -1912.46 | 6188.17 |
| | 0.2 | 5.72 | -62.74 | -1.48 | 329.84 | 410.97 | -4271.57 | 12608.81 |
| | 0.3 | 11.95 | -61.86 | -1.38 | 457.56 | 649.81 | -7116.43 | 19206.42 |
| | 0.4 | 18.07 | -60.35 | -1.28 | 558.01 | 907.49 | -10493.38 | 25968.59 |
| 8 | -0.4 | -38.33 | -50.00 | -2.22 | -735.15 | -323.77 | 1401.72 | -18088.88 |
| | -0.3 | -33.14 | -53.58 | -2.12 | -543.87 | -281.13 | 1490.35 | -14117.00 |
| | -0.2 | -27.63 | -56.62 | -2.02 | -356.37 | -212.69 | 1330.81 | -9779.77 |
| | -0.1 | -21.84 | -59.09 | -1.92 | -174.90 | -118.34 | 895.49 | -5079.54 |
| | 0 | -15.83 | -60.98 | -1.82 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | -9.66 | -62.25 | -1.72 | 156.20 | 144.13 | -883.92 | 5285.49 |
| | 0.2 | -3.40 | -62.91 | -1.62 | 299.39 | 309.19 | -2263.45 | 10882.07 |
| | 0.3 | 2.90 | -62.93 | -1.52 | 424.80 | 495.08 | -4003.67 | 16738.48 |
| | 0.4 | 9.17 | -62.33 | -1.42 | 529.09 | 698.21 | -6115.09 | 22792.56 |
| 9 | -0.4 | -45.08 | -44.01 | -2.37 | -605.95 | -210.58 | -953.30 | -14969.71 |
| | -0.3 | -40.47 | -48.29 | -2.27 | -452.45 | -189.45 | -414.98 | -11755.11 |
| | -0.2 | -35.44 | -52.08 | -2.17 | -297.68 | -147.14 | -78.59 | -8160.04 |
| | -0.1 | -30.07 | -55.36 | -2.07 | -146.16 | -84.31 | 48.70 | -4242.85 |
| | 0 | -24.39 | -58.09 | -1.97 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | -18.47 | -60.23 | -1.87 | 139.70 | 103.58 | -382.21 | 4564.86 |
| | 0.2 | -12.36 | -61.78 | -1.77 | 268.27 | 226.81 | -965.53 | 9402.73 |
| | 0.3 | -6.13 | -62.70 | -1.67 | 384.12 | 368.13 | -1803.53 | 14502.13 |
| | 0.4 | 0.16 | -63.00 | -1.57 | 484.55 | 525.20 | -2908.11 | 19824.52 |
| 10 | -0.4 | -50.86 | -37.18 | -2.51 | -494.35 | -127.08 | -2360.74 | -12371.57 |
| | -0.3 | -46.89 | -42.07 | -2.41 | -372.18 | -120.05 | -1529.47 | -9757.25 |
| | -0.2 | -42.46 | -46.55 | -2.31 | -247.98 | -96.98 | -854.31 | -6825.39 |
| | -0.1 | -37.60 | -50.55 | -2.21 | -123.03 | -56.85 | -336.10 | -3565.03 |
| | 0 | -32.36 | -54.05 | -2.11 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | -26.81 | -57.01 | -2.01 | 118.47 | 73.72 | 175.81 | 3847.91 |
| | 0.2 | -20.98 | -59.40 | -1.91 | 230.83 | 163.65 | 153.31 | 7976.88 |
| | 0.3 | -14.94 | -61.20 | -1.81 | 334.44 | 268.80 | -64.02 | 12356.13 |
| | 0.4 | -8.76 | -62.39 | -1.71 | 427.04 | 387.72 | -476.92 | 16953.71 |

Table A.8: Parameter variation of the parameter R2 of the structure 3

| Parameter R2 (mm) | Rotation $\theta$ (rad) | Pos. end node x (mm) | y (mm) | Angle (rad) | External Forces $F_x$ (N) | $F_y$ (N) | $M_e$ (N.mm) | Moment (node 1) $M_c$ (N.mm) |
|---|---|---|---|---|---|---|---|---|
| | -0.4 | -22.37 | -58.90 | -1.93 | -993.91 | -689.80 | 7138.24 | -25625.05 |
| | -0.3 | -16.38 | -60.83 | -1.83 | -717.92 | -570.50 | 6053.36 | -19715.21 |
| | -0.2 | -10.22 | -62.17 | -1.73 | -458.20 | -415.07 | 4551.15 | -13467.06 |
| | -0.1 | -3.96 | -62.88 | -1.63 | -218.84 | -224.23 | 2604.97 | -6904.03 |
| 3 | 0 | 2.33 | -62.93 | -1.53 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 8.61 | -62.41 | -1.43 | 181.90 | 250.80 | -2812.13 | 6937.38 |
| | 0.2 | 14.79 | -61.24 | -1.33 | 335.84 | 526.71 | -6376.39 | 14079.61 |
| | 0.3 | 20.83 | -59.46 | -1.23 | 455.44 | 822.31 | -10582.92 | 21291.63 |
| | 0.4 | 26.67 | -57.08 | -1.13 | 538.76 | 1132.74 | -15460.44 | 28511.31 |
| | -0.4 | -34.2 | -52.91 | -2.14 | -889.22 | -464.58 | 3603.41 | -20587.87 |
| | -0.3 | -28.75 | -56.06 | -2.04 | -653.87 | -396.40 | 3272.92 | -16040.48 |
| | -0.2 | -23.01 | -58.65 | -1.94 | -425.18 | -295.73 | 2612.82 | -11083.62 |
| | -0.1 | -17.04 | -60.65 | -1.84 | -207.21 | -163.03 | 1597.05 | -5746.90 |
| 4 | 0 | -10.9 | -62.05 | -1.74 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | -4.65 | -62.83 | -1.64 | 180.35 | 192.17 | -1649.71 | 5905.58 |
| | 0.2 | 1.64 | -62.98 | -1.54 | 342.76 | 410.64 | -3962.49 | 12138.22 |
| | 0.3 | 7.92 | -62.50 | -1.44 | 480.31 | 652.78 | -6811.40 | 18586.61 |
| | 0.4 | 14.12 | -61.40 | -1.34 | 589.80 | 914.13 | -10236.71 | 25172.68 |
| | -0.4 | -44.43 | -44.66 | -2.35 | -750.61 | -285.52 | 488.36 | -16065.40 |
| | -0.3 | -39.75 | -48.88 | -2.25 | -560.53 | -254.43 | 794.70 | -12673.91 |
| | -0.2 | -34.67 | -52.60 | -2.15 | -369.78 | -196.23 | 852.00 | -8851.78 |
| | -0.1 | -29.25 | -55.80 | -2.05 | -182.88 | -111.31 | 640.04 | -4639.42 |
| 5 | 0 | -23.53 | -58.44 | -1.95 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | -17.58 | -60.50 | -1.85 | 167.76 | 139.19 | -694.82 | 4920.03 |
| | 0.2 | -11.45 | -61.95 | -1.75 | 324.52 | 303.07 | -1873.30 | 10214.69 |
| | 0.3 | -5.21 | -62.78 | -1.65 | 464.30 | 490.35 | -3436.31 | 15802.80 |
| | 0.4 | 1.08 | -62.99 | -1.55 | 583.97 | 698.31 | -5424.60 | 21639.54 |
| | -0.4 | -52.64 | -34.61 | -2.56 | -600.12 | -155.22 | -2022.28 | -12292.87 |
| | -0.3 | -48.92 | -39.70 | -2.46 | -454.83 | -148.38 | -1233.60 | -9805.94 |
| | -0.2 | -44.71 | -44.38 | -2.36 | -304.15 | -120.62 | -621.61 | -6918.03 |
| | -0.1 | -40.06 | -48.62 | -2.26 | -151.74 | -71.43 | -205.23 | -3650.16 |
| 6 | 0 | -35 | -52.38 | -2.16 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | -29.6 | -55.61 | -2.06 | 148.00 | 93.90 | -23.23 | 4019.24 |
| | 0.2 | -23.9 | -58.29 | -1.96 | 288.87 | 209.82 | -289.00 | 8386.90 |
| | 0.3 | -17.96 | -60.39 | -1.86 | 419.71 | 347.06 | -820.06 | 13086.91 |
| | 0.4 | -11.84 | -61.88 | -1.76 | 537.80 | 504.50 | -1637.94 | 18100.35 |
| | -0.4 | -58.54 | -23.27 | -2.76 | -459.22 | -64.67 | -3411.21 | -9341.13 |
| | -0.3 | -55.93 | -29.00 | -2.66 | -354.35 | -71.75 | -2338.67 | -7525.44 |
| | -0.2 | -52.75 | -34.44 | -2.56 | -240.99 | -63.88 | -1387.40 | -5359.45 |
| | -0.1 | -49.05 | -39.53 | -2.46 | -122.30 | -39.76 | -565.68 | -2853.62 |
| 7 | 0 | -44.86 | -44.23 | -2.36 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | -40.22 | -48.49 | -2.26 | 122.24 | 60.88 | 626.86 | 3192.28 |
| | 0.2 | -35.18 | -52.26 | -2.16 | 243.05 | 138.61 | 979.09 | 6725.31 |
| | 0.3 | -29.78 | -55.52 | -2.06 | 359.38 | 234.59 | 1143.43 | 10591.24 |
| | 0.4 | -24.09 | -58.21 | -1.96 | 468.55 | 348.46 | 1118.40 | 14771.41 |

Table A.9: Parameter variation of the parameter L1 of the structure 3

| Parameter L1 (mm) | Rotation $\theta$ (rad) | Pos. end node x (mm) | y (mm) | Angle (rad) | External Forces $F_x$ (N) | $F_y$ (N) | $M_e$ (N.mm) | Moment (node 1) $M_c$ (N.mm) |
|---|---|---|---|---|---|---|---|---|
| | -0.4 | -22.37 | -58.90 | -1.93 | -993.91 | -689.80 | 7138.24 | -25625.05 |
| | -0.3 | -16.38 | -60.83 | -1.83 | -717.92 | -570.50 | 6053.36 | -19715.21 |
| | -0.2 | -10.22 | -62.17 | -1.73 | -458.20 | -415.07 | 4551.15 | -13467.06 |
| | -0.1 | -3.96 | -62.88 | -1.63 | -218.84 | -224.23 | 2604.97 | -6904.03 |
| 25 | 0 | 2.33 | -62.93 | -1.53 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 8.61 | -62.41 | -1.43 | 181.90 | 250.80 | -2812.13 | 6937.38 |
| | 0.2 | 14.79 | -61.24 | -1.33 | 335.84 | 526.71 | -6376.39 | 14079.61 |
| | 0.3 | 20.83 | -59.46 | -1.23 | 455.44 | 822.31 | -10582.92 | 21291.63 |
| | 0.4 | 26.67 | -57.08 | -1.13 | 538.76 | 1132.74 | -15460.44 | 28511.31 |
| | -0.4 | -20.11 | -60.76 | -1.89 | -974.03 | -717.84 | 7405.26 | -26573.37 |
| | -0.3 | -13.94 | -62.46 | -1.79 | -699.29 | -590.67 | 6202.62 | -20381.39 |
| | -0.2 | -7.63 | -63.54 | -1.69 | -442.51 | -427.07 | 4593.43 | -13859.25 |
| | -0.1 | -1.25 | -63.99 | -1.59 | -208.49 | -229.60 | 2548.92 | -7061.23 |
| 26 | 0 | 5.14 | -63.79 | -1.49 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 11.49 | -62.96 | -1.39 | 177.87 | 257.36 | -3057.30 | 7238.12 |
| | 0.2 | 17.71 | -61.50 | -1.29 | 322.43 | 537.72 | -6697.00 | 14589.55 |
| | 0.3 | 23.77 | -59.42 | -1.19 | 431.81 | 837.58 | -10978.11 | 22025.67 |
| | 0.4 | 29.58 | -56.75 | -1.09 | 503.03 | 1149.76 | -15882.22 | 29428.20 |
| | -0.4 | -17.88 | -62.49 | -1.85 | -953.77 | -741.03 | 7759.47 | -27476.27 |
| | -0.3 | -11.55 | -63.97 | -1.75 | -682.10 | -607.77 | 6451.04 | -21046.50 |
| | -0.2 | -5.10 | -65.80 | -1.65 | -429.16 | -437.57 | 4737.58 | -15667.74 |
| | -0.1 | 1.39 | -64.99 | -1.55 | -200.74 | -234.43 | 2598.94 | -7256.75 |
| 27 | 0 | 7.87 | -64.52 | -1.45 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 14.27 | -63.41 | -1.35 | 170.59 | 261.53 | -3173.41 | 7452.59 |
| | 0.2 | 20.53 | -61.67 | -1.25 | 306.10 | 545.12 | -6888.46 | 15003.49 |
| | 0.3 | 26.59 | -59.31 | -1.15 | 404.92 | 846.58 | -11212.07 | 22615.86 |
| | 0.4 | 32.38 | -56.36 | -1.05 | 464.78 | 1159.74 | -16145.21 | 30206.32 |
| | -0.4 | -15.68 | -64.11 | -1.81 | -933.50 | -760.18 | 8185.96 | -28338.29 |
| | -0.3 | -9.20 | -65.36 | -1.71 | -665.29 | -621.11 | 6776.79 | -21675.56 |
| | -0.2 | -2.63 | -65.95 | -1.61 | -417.60 | -452.10 | 5292.28 | -15132.74 |
| | -0.1 | 3.97 | -65.88 | -1.51 | -194.54 | -237.84 | 2733.20 | -7459.41 |
| 28 | 0 | 10.52 | -65.16 | -1.41 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 16.97 | -63.78 | -1.31 | 160.83 | 263.49 | -3099.46 | 7671.09 |
| | 0.2 | 23.26 | -61.77 | -1.21 | 287.30 | 548.59 | -6915.08 | 15533.49 |
| | 0.3 | 29.31 | -59.14 | -1.11 | 375.86 | 850.21 | -11298.48 | 23457.66 |
| | 0.4 | 35.06 | -55.91 | -1.01 | 425.06 | 1163.45 | -16242.48 | 31358.93 |
| | -0.4 | -13.52 | -65.62 | -1.77 | -913.70 | -775.35 | 8686.19 | -29157.93 |
| | -0.3 | -6.90 | -66.64 | -1.67 | -649.54 | -631.28 | 7180.37 | -22279.81 |
| | -0.2 | -0.22 | -67.00 | -1.57 | -407.73 | -452.87 | 5292.28 | -15132.74 |
| | -0.1 | 6.47 | -66.69 | -1.47 | -191.32 | -241.96 | 2970.75 | -7724.46 |
| 29 | 0 | 13.10 | -65.71 | -1.37 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 19.59 | -64.07 | -1.27 | 149.23 | 263.49 | -3099.46 | 7671.09 |
| | 0.2 | 25.89 | -61.80 | -1.17 | 266.57 | 548.64 | -6915.08 | 15533.49 |
| | 0.3 | 31.93 | -58.90 | -1.07 | 345.73 | 850.21 | -11298.48 | 23457.66 |
| | 0.4 | 37.65 | -55.42 | -0.97 | 384.37 | 1161.14 | -16242.48 | 31358.93 |

Table A.10: Parameter variation of the parameter L2 of the structure 3

| Parameter L2 (mm) | Rotation $\theta$ (rad) | Pos. end node x (mm) | y (mm) | Angle (rad) | External Forces $F_x$ (N) | $F_y$ (N) | $M_e$ (N.mm) | Moment (node 1) $M_c$ (N.mm) |
|---|---|---|---|---|---|---|---|---|
| 12 | -0.4 | -22.37 | -58.90 | -1.93 | -993.91 | -689.80 | 7138.24 | -25625.05 |
| | -0.3 | -16.38 | -60.83 | -1.83 | -717.92 | -570.50 | 6053.36 | -19715.21 |
| | -0.2 | -10.22 | -62.17 | -1.73 | -458.20 | -415.07 | 4551.15 | -13467.06 |
| | -0.1 | -3.96 | -62.88 | -1.63 | -218.84 | -224.23 | 2604.97 | -6904.03 |
| | 0 | 2.33 | -62.93 | -1.53 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 8.61 | -62.41 | -1.43 | 181.90 | 250.80 | -2812.13 | 6937.38 |
| | 0.2 | 14.79 | -61.24 | -1.33 | 335.84 | 526.71 | -6376.39 | 14079.61 |
| | 0.3 | 20.83 | -59.46 | -1.23 | 455.44 | 822.31 | -10582.92 | 21291.63 |
| | 0.4 | 26.67 | -57.08 | -1.13 | 538.76 | 1132.74 | -15460.44 | 28511.31 |
| 13 | -0.4 | -23.37 | -57.43 | -1.96 | -927.85 | -626.23 | 5762.64 | -23495.30 |
| | -0.3 | -17.52 | -59.47 | -1.86 | -671.07 | -519.60 | 4916.83 | -18094.34 |
| | -0.2 | -11.50 | -60.93 | -1.76 | -428.30 | -379.25 | 3685.19 | -12361.16 |
| | -0.1 | -5.46 | -61.77 | -1.66 | -207.70 | -208.61 | 2130.74 | -6430.83 |
| | 0 | 0.84 | -61.99 | -1.56 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 7.02 | -61.60 | -1.46 | 182.74 | 233.95 | -2776.93 | 6612.88 |
| | 0.2 | 13.14 | -60.59 | -1.36 | 334.55 | 492.25 | -6016.82 | 13338.07 |
| | 0.3 | 19.12 | -58.98 | -1.26 | 454.80 | 770.09 | -9863.35 | 20133.39 |
| | 0.4 | 24.91 | -56.77 | -1.16 | 542.24 | 1064.11 | -14368.05 | 26960.33 |
| 14 | -0.4 | -24.42 | -55.90 | -1.98 | -875.51 | -563.25 | 5117.77 | -21619.61 |
| | -0.3 | -18.71 | -58.06 | -1.88 | -637.32 | -469.64 | 4456.70 | -16714.49 |
| | -0.2 | -12.82 | -59.64 | -1.78 | -410.21 | -343.96 | 3430.57 | -11465.56 |
| | -0.1 | -6.81 | -60.62 | -1.68 | -197.77 | -187.11 | 2003.77 | -5904.19 |
| | 0 | -0.72 | -61.00 | -1.58 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 5.37 | -60.76 | -1.48 | 170.49 | 213.53 | -2275.60 | 6027.40 |
| | 0.2 | 11.41 | -59.92 | -1.38 | 319.04 | 451.63 | -5216.61 | 12279.05 |
| | 0.3 | 17.34 | -58.48 | -1.28 | 440.16 | 710.48 | -8758.37 | 18644.61 |
| | 0.4 | 23.09 | -56.46 | -1.18 | 531.03 | 985.01 | -12916.82 | 25033.71 |
| 15 | -0.4 | -25.51 | -54.31 | -2.01 | -816.57 | -507.00 | 4033.94 | -19775.49 |
| | -0.3 | -19.96 | -56.58 | -1.91 | -596.41 | -424.23 | 3594.20 | -15319.50 |
| | -0.2 | -14.21 | -58.29 | -1.81 | -384.59 | -311.65 | 2797.77 | -10516.65 |
| | -0.1 | -8.32 | -59.42 | -1.71 | -184.71 | -169.93 | 1612.98 | -5399.84 |
| | 0 | -2.35 | -59.95 | -1.61 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 3.65 | -59.89 | -1.51 | 166.35 | 196.12 | -2097.61 | 5638.93 |
| | 0.2 | 9.61 | -59.23 | -1.41 | 310.95 | 415.85 | -4717.44 | 11458.57 |
| | 0.3 | 15.48 | -57.97 | -1.31 | 431.63 | 656.88 | -7919.08 | 17417.88 |
| | 0.4 | 21.19 | -56.14 | -1.21 | 525.02 | 913.94 | -11716.52 | 23415.48 |
| 16 | -0.4 | -26.65 | -52.64 | -2.04 | -761.63 | -453.86 | 3117.32 | -18071.67 |
| | -0.3 | -21.26 | -55.04 | -1.94 | -558.62 | -381.65 | 2870.48 | -14037.19 |
| | -0.2 | -15.66 | -56.88 | -1.84 | -361.63 | -281.26 | 2292.10 | -9653.87 |
| | -0.1 | -9.90 | -58.16 | -1.74 | -174.05 | -153.72 | 1337.19 | -4957.93 |
| | 0 | -4.05 | -58.86 | -1.64 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 1.85 | -58.97 | -1.54 | 159.81 | 179.20 | -1840.57 | 5227.08 |
| | 0.2 | 7.73 | -58.49 | -1.44 | 300.45 | 381.44 | -4162.10 | 10637.85 |
| | 0.3 | 13.53 | -57.43 | -1.34 | 419.44 | 603.72 | -7025.80 | 16175.37 |
| | 0.4 | 19.19 | -55.79 | -1.24 | 514.76 | 843.27 | -10468.91 | 21782.89 |

Table A.11: Parameter variation of the parameter L3 of the structure 3

| Parameter L3 (mm) | Rotation $\theta$ (rad) | Pos. end node x (mm) | y (mm) | Angle (rad) | External Forces $F_x$ (N) | $F_y$ (N) | $M_e$ (N.mm) | Moment (node 1) $M_c$ (N.mm) |
|---|---|---|---|---|---|---|---|---|
| 35 | -0.4 | -22.37 | -58.90 | -1.93 | -993.91 | -689.80 | 7138.24 | -25625.05 |
| | -0.3 | -16.38 | -60.83 | -1.83 | -717.92 | -570.50 | 6053.36 | -19715.21 |
| | -0.2 | -10.22 | -62.17 | -1.73 | -458.20 | -415.07 | 4551.15 | -13467.06 |
| | -0.1 | -3.96 | -62.88 | -1.63 | -218.84 | -224.23 | 2604.97 | -6904.03 |
| | 0 | 2.33 | -62.93 | -1.53 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 8.61 | -62.41 | -1.43 | 181.90 | 250.80 | -2812.13 | 6937.38 |
| | 0.2 | 14.79 | -61.24 | -1.33 | 335.84 | 526.71 | -6376.39 | 14079.61 |
| | 0.3 | 20.83 | -59.46 | -1.23 | 455.44 | 822.31 | -10582.92 | 21291.63 |
| | 0.4 | 26.67 | -57.08 | -1.13 | 538.76 | 1132.74 | -15460.44 | 28511.31 |
| 36 | -0.4 | -22.72 | -59.83 | -1.93 | -987.57 | -690.77 | 7486.38 | -25543.86 |
| | -0.3 | -16.64 | -61.80 | -1.83 | -713.08 | -571.43 | 6333.86 | -19654.70 |
| | -0.2 | -10.38 | -63.15 | -1.73 | -454.03 | -414.67 | 4748.95 | -13398.69 |
| | -0.1 | -4.03 | -63.87 | -1.63 | -216.82 | -223.84 | 2720.36 | -6868.58 |
| | 0 | 2.37 | -63.96 | -1.53 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 8.74 | -63.40 | -1.43 | 179.05 | 249.39 | -2923.55 | 6867.10 |
| | 0.2 | 15.03 | -62.21 | -1.33 | 330.53 | 523.66 | -6640.20 | 13937.89 |
| | 0.3 | 21.16 | -60.40 | -1.23 | 446.88 | 815.46 | -10995.55 | 21019.21 |
| | 0.4 | 27.09 | -57.98 | -1.13 | 527.11 | 1120.49 | -16037.41 | 28070.92 |
| 37 | -0.4 | -23.08 | -60.77 | -1.93 | -982.03 | -692.58 | 7819.45 | -25485.15 |
| | -0.3 | -16.90 | -62.77 | -1.83 | -707.99 | -572.27 | 6595.69 | -19589.07 |
| | -0.2 | -10.55 | -64.14 | -1.73 | -450.67 | -415.17 | 4944.88 | -13353.82 |
| | -0.1 | -4.09 | -64.87 | -1.63 | -214.53 | -223.62 | 2818.37 | -6829.13 |
| | 0 | 2.41 | -64.96 | -1.53 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 8.88 | -64.39 | -1.43 | 176.48 | 248.24 | -3037.91 | 6806.47 |
| | 0.2 | 15.26 | -63.18 | -1.33 | 324.67 | 519.88 | -6873.86 | 13773.84 |
| | 0.3 | 21.50 | -61.34 | -1.23 | 438.11 | 808.35 | -11385.91 | 20741.95 |
| | 0.4 | 27.51 | -58.89 | -1.13 | 514.54 | 1106.45 | -16551.24 | 27591.66 |
| 38 | -0.4 | -23.43 | -61.70 | -1.93 | -975.10 | -693.45 | 8120.90 | -25393.41 |
| | -0.3 | -17.16 | -63.73 | -1.83 | -702.29 | -572.46 | 6840.24 | -19431.06 |
| | -0.2 | -10.71 | -65.13 | -1.73 | -446.51 | -415.20 | 5116.41 | -13289.20 |
| | -0.1 | -4.15 | -65.87 | -1.63 | -212.14 | -223.34 | 2909.34 | -6787.12 |
| | 0 | 2.44 | -65.95 | -1.53 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 9.02 | -65.38 | -1.43 | 173.80 | 246.98 | -3143.95 | 6742.11 |
| | 0.2 | 15.50 | -64.15 | -1.33 | 318.86 | 516.18 | -7099.57 | 13613.45 |
| | 0.3 | 22.16 | -63.23 | -1.23 | 418.73 | 790.88 | -12034.78 | 20423.19 |
| | 0.4 | 28.36 | -60.70 | -1.13 | 488.46 | 1076.70 | -17439.68 | 26594.84 |
| 39 | -0.4 | -23.79 | -62.64 | -1.93 | -969.00 | -695.14 | 8409.52 | -25324.51 |
| | -0.3 | -17.42 | -64.70 | -1.83 | -696.74 | -573.14 | 7066.42 | -19431.06 |
| | -0.2 | -10.87 | -66.11 | -1.73 | -441.80 | -414.53 | 5274.60 | -13209.20 |
| | -0.1 | -4.22 | -66.87 | -1.63 | -210.19 | -223.38 | 3005.16 | -6756.63 |
| | 0 | 2.48 | -66.95 | -1.53 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 9.15 | -66.37 | -1.43 | 170.65 | 245.22 | -3230.15 | 6661.63 |
| | 0.2 | 15.73 | -65.13 | -1.33 | 312.19 | 511.07 | -7287.04 | 13419.00 |
| | 0.3 | 22.16 | -63.23 | -1.23 | 418.73 | 790.88 | -12034.78 | 20104.24 |
| | 0.4 | 28.36 | -60.70 | -1.13 | 488.46 | 1076.70 | -17439.68 | 26594.84 |

Table A.12: Parameter variation of the parameter R1 of the structure 4

| Parameter R1 (mm) | Rotation $\theta$ (rad) | Pos. end node x (mm) | y (mm) | Angle (rad) | External Forces $F_x$ (N) | $F_y$ (N) | $M_e$ (N.mm) | Moment (node 1) $M_c$ (N.mm) |
|---|---|---|---|---|---|---|---|---|
| 30 | -0.4 | 64.47 | -27.26 | -0.40 | 147.04 | -1738.44 | 29630.91 | -52361.22 |
| | -0.3 | 66.87 | -20.69 | -0.30 | 163.00 | -1255.81 | 22530.04 | -39236.37 |
| | -0.2 | 68.60 | -13.91 | -0.20 | 101.46 | -794.05 | 15147.22 | -26002.28 |
| | -0.1 | 69.65 | -6.99 | -0.10 | 30.93 | -379.55 | 7605.02 | -12921.40 |
| | 0.0 | 70.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 69.65 | 6.99 | 0.10 | 30.93 | 379.55 | -7605.02 | 12921.40 |
| | 0.2 | 68.60 | 13.91 | 0.20 | 101.46 | 794.05 | -15147.22 | 26002.28 |
| | 0.3 | 66.87 | 20.69 | 0.30 | 163.00 | 1255.81 | -22530.04 | 39236.37 |
| | 0.4 | 64.47 | 27.26 | 0.40 | 147.04 | 1738.44 | -29630.91 | 52361.22 |
| 31 | -0.4 | 67.24 | -28.43 | -0.40 | 193.75 | -1578.85 | 27659.84 | -49311.11 |
| | -0.3 | 69.74 | -21.57 | -0.30 | 185.99 | -1130.34 | 21016.88 | -36846.30 |
| | -0.2 | 71.54 | -14.50 | -0.20 | 109.41 | -708.92 | 14129.64 | -24366.28 |
| | -0.1 | 72.64 | -7.29 | -0.10 | 35.37 | -337.61 | 7100.51 | -12101.29 |
| | 0.0 | 73.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 72.64 | 7.29 | 0.10 | 35.37 | 337.61 | -7100.51 | 12101.29 |
| | 0.2 | 71.54 | 14.50 | 0.20 | 109.41 | 708.92 | -14129.64 | 24366.28 |
| | 0.3 | 69.74 | 21.57 | 0.30 | 185.99 | 1130.34 | -21016.88 | 36846.30 |
| | 0.4 | 67.24 | 28.43 | 0.40 | 193.75 | 1578.85 | -27659.84 | 49311.11 |
| 32 | -0.4 | 70.00 | -29.60 | -0.40 | 227.03 | -1440.62 | 25916.71 | -46597.12 |
| | -0.3 | 72.61 | -22.46 | -0.30 | 204.04 | -1024.42 | 19689.90 | -34743.90 |
| | -0.2 | 74.49 | -15.10 | -0.20 | 120.05 | -638.58 | 13239.22 | -22937.21 |
| | -0.1 | 75.62 | -7.59 | -0.10 | 34.10 | -301.65 | 6654.66 | -11372.25 |
| | 0.0 | 76.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 75.62 | 7.59 | 0.10 | 34.10 | 301.65 | -6654.66 | 11372.25 |
| | 0.2 | 74.49 | 15.10 | 0.20 | 120.05 | 638.58 | -13239.22 | 22937.21 |
| | 0.3 | 72.61 | 22.46 | 0.30 | 204.04 | 1024.42 | -19689.90 | 34743.90 |
| | 0.4 | 70.00 | 29.60 | 0.40 | 227.03 | 1440.62 | -25916.71 | 46597.12 |
| 33 | -0.4 | 72.76 | -30.76 | -0.40 | 251.37 | -1319.62 | 24343.36 | -44145.37 |
| | -0.3 | 75.47 | -23.35 | -0.30 | 214.08 | -932.24 | 18510.98 | -32862.76 |
| | -0.2 | 77.43 | -15.69 | -0.20 | 123.32 | -576.75 | 12431.94 | -21639.56 |
| | -0.1 | 78.61 | -7.89 | -0.10 | 37.16 | -271.50 | 6254.26 | -10722.86 |
| | 0.0 | 79.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 78.61 | 7.89 | 0.10 | 37.16 | 271.50 | -6254.26 | 10722.86 |
| | 0.2 | 77.43 | 15.69 | 0.20 | 123.32 | 576.75 | -12431.94 | 21639.56 |
| | 0.3 | 75.47 | 23.35 | 0.30 | 214.08 | 932.24 | -18510.98 | 32862.76 |
| | 0.4 | 72.76 | 30.76 | 0.40 | 251.37 | 1319.62 | -24343.36 | 44145.37 |
| 34 | -0.4 | 75.53 | -31.93 | -0.40 | 272.80 | -1215.95 | 22935.71 | -41955.39 |
| | -0.3 | 78.34 | -24.23 | -0.30 | 222.97 | -852.35 | 17431.50 | -31153.84 |
| | -0.2 | 80.37 | -16.29 | -0.20 | 126.02 | -524.23 | 11723.18 | -20493.12 |
| | -0.1 | 81.59 | -8.19 | -0.10 | 35.35 | -245.16 | 5897.12 | -10138.51 |
| | 0.0 | 82.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 81.59 | 8.19 | 0.10 | 35.35 | 245.16 | -5897.12 | 10138.51 |
| | 0.2 | 80.37 | 16.29 | 0.20 | 126.02 | 524.23 | -11723.18 | 20493.12 |
| | 0.3 | 78.34 | 24.23 | 0.30 | 222.97 | 852.35 | -17431.50 | 31153.84 |
| | 0.4 | 75.53 | 31.93 | 0.40 | 272.80 | 1215.95 | -22935.71 | 41955.39 |

Table A.13: Parameter variation of the parameter $\alpha$ of the structure 4

| Parameter $\alpha$ (rad) | Rotation $\theta$ (rad) | Pos. end node x (mm) | y (mm) | Angle (rad) | External Forces $F_x$ (N) | $F_y$ (N) | $M_e$ (N.mm) | Moment (node 1) $M_c$ (N.mm) |
|---|---|---|---|---|---|---|---|---|
| | -0.4 | 64.47 | -27.26 | -0.40 | 1108.31 | -2454.69 | 32280.23 | -58940.49 |
| | -0.3 | 66.87 | -20.69 | -0.30 | 739.70 | -1604.11 | 24783.79 | -43117.19 |
| | -0.2 | 68.60 | -13.91 | -0.20 | 370.75 | -934.21 | 16810.10 | -28106.24 |
| | -0.1 | 69.65 | -6.99 | -0.10 | 101.33 | -421.44 | 8492.23 | -13830.93 |
| 0.8 | 0.0 | 70.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 69.65 | 6.99 | 0.10 | 101.33 | 421.44 | -8492.23 | 13830.93 |
| | 0.2 | 68.60 | 13.91 | 0.20 | 370.75 | 934.21 | -16810.10 | 28106.24 |
| | 0.3 | 66.87 | 20.69 | 0.30 | 739.70 | 1604.11 | -24783.79 | 43117.19 |
| | 0.4 | 64.47 | 27.26 | 0.40 | 1108.31 | 2454.69 | -32280.23 | 58940.49 |
| | -0.4 | 64.47 | -27.26 | -0.40 | 533.38 | -2066.65 | 31721.89 | -55975.43 |
| | -0.3 | 66.87 | -20.69 | -0.30 | 393.05 | -1426.95 | 24327.20 | -41556.45 |
| | -0.2 | 68.60 | -13.91 | -0.20 | 209.59 | -872.85 | 16477.01 | -27392.50 |
| | -0.1 | 69.65 | -6.99 | -0.10 | 59.49 | -408.12 | 8314.56 | -13573.53 |
| 1.0 | 0.0 | 70.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 69.65 | 6.99 | 0.10 | 59.49 | 408.12 | -8314.56 | 13573.53 |
| | 0.2 | 68.60 | 13.91 | 0.20 | 209.59 | 872.85 | -16477.01 | 27392.50 |
| | 0.3 | 66.87 | 20.69 | 0.30 | 393.05 | 1426.95 | -24327.20 | 41556.45 |
| | 0.4 | 64.47 | 27.26 | 0.40 | 533.38 | 2066.65 | -31721.89 | 55975.43 |
| | -0.4 | 64.47 | -27.26 | -0.40 | 287.73 | -1884.48 | 31112.47 | -54269.06 |
| | -0.3 | 66.87 | -20.69 | -0.30 | 248.38 | -1340.34 | 23810.53 | -39852.95 |
| | -0.2 | 68.60 | -13.91 | -0.20 | 142.99 | -838.70 | 16096.66 | -26868.46 |
| | -0.1 | 69.65 | -6.99 | -0.10 | 42.22 | -398.09 | 8111.94 | -13348.84 |
| 1.2 | 0.0 | 70.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 69.65 | 6.99 | 0.10 | 42.22 | 398.09 | -8111.94 | 13348.84 |
| | 0.2 | 68.60 | 13.91 | 0.20 | 142.99 | 838.70 | -16096.66 | 26868.46 |
| | 0.3 | 66.87 | 20.69 | 0.30 | 248.38 | 1340.34 | -23810.53 | 39852.95 |
| | 0.4 | 64.47 | 27.26 | 0.40 | 287.73 | 1884.48 | -31112.47 | 54269.06 |
| | -0.4 | 64.47 | -27.26 | -0.40 | 184.95 | -1791.75 | 30425.80 | -53170.46 |
| | -0.3 | 66.87 | -20.69 | -0.30 | 187.89 | -1290.91 | 23218.95 | -39852.95 |
| | -0.2 | 68.60 | -13.91 | -0.20 | 114.53 | -814.86 | 15658.41 | -26424.74 |
| | -0.1 | 69.65 | -6.99 | -0.10 | 34.66 | -388.96 | 7877.98 | -13136.54 |
| 1.4 | 0.0 | 70.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 69.65 | 6.99 | 0.10 | 34.66 | 388.96 | -7877.98 | 13136.54 |
| | 0.2 | 68.60 | 13.91 | 0.20 | 114.53 | 814.86 | -15658.41 | 26424.74 |
| | 0.3 | 66.87 | 20.69 | 0.30 | 187.89 | 1290.91 | -23218.95 | 39852.95 |
| | 0.4 | 64.47 | 27.26 | 0.40 | 184.95 | 1791.75 | -30425.80 | 53170.46 |
| | -0.4 | 64.47 | -27.26 | -0.40 | 147.04 | -1738.44 | 29630.91 | -52361.22 |
| | -0.3 | 66.87 | -20.69 | -0.30 | 163.00 | -1255.81 | 22530.04 | -39236.37 |
| | -0.2 | 68.60 | -13.91 | -0.20 | 101.46 | -794.05 | 15147.22 | -26002.28 |
| | -0.1 | 69.65 | -6.99 | -0.10 | 30.93 | -379.55 | 7605.02 | -12921.40 |
| 1.6 | 0.0 | 70.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 69.65 | 6.99 | 0.10 | 30.93 | 379.55 | -7605.02 | 12921.40 |
| | 0.2 | 68.60 | 13.91 | 0.20 | 101.46 | 794.05 | -15147.22 | 26002.28 |
| | 0.3 | 66.87 | 20.69 | 0.30 | 163.00 | 1255.81 | -22530.04 | 39236.37 |
| | 0.4 | 64.47 | 27.26 | 0.40 | 147.04 | 1738.44 | -29630.91 | 52361.22 |

Table A.14: Parameter variation of the parameter R2 of the structure 4

| Parameter R2 (mm) | Rotation $\theta$ (rad) | Pos. end node x (mm) | y (mm) | Angle (rad) | External Forces $F_x$ (N) | $F_y$ (N) | $M_e$ (N.mm) | Moment (node 1) $M_c$ (N.mm) |
|---|---|---|---|---|---|---|---|---|
| 2.5 | -0.4 | 64.47 | -27.26 | -0.40 | 147.04 | -1738.44 | 29630.91 | -52361.22 |
| | -0.3 | 66.87 | -20.69 | -0.30 | 163.00 | -1255.81 | 22530.04 | -39236.37 |
| | -0.2 | 68.60 | -13.91 | -0.20 | 101.46 | -794.05 | 15147.22 | -26002.28 |
| | -0.1 | 69.65 | -6.99 | -0.10 | 30.93 | -379.55 | 7605.02 | -12921.40 |
| | 0.0 | 70.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 69.65 | 6.99 | 0.10 | 30.93 | 379.55 | -7605.02 | 12921.40 |
| | 0.2 | 68.60 | 13.91 | 0.20 | 101.46 | 794.05 | -15147.22 | 26002.28 |
| | 0.3 | 66.87 | 20.69 | 0.30 | 163.00 | 1255.81 | -22530.04 | 39236.37 |
| | 0.4 | 64.47 | 27.26 | 0.40 | 147.04 | 1738.44 | -29630.91 | 52361.22 |
| 3.0 | -0.4 | 66.32 | -28.04 | -0.40 | 20.45 | -1522.22 | 27831.42 | -49715.46 |
| | -0.3 | 68.78 | -21.28 | -0.30 | 62.69 | -1112.76 | 21157.82 | -37352.50 |
| | -0.2 | 70.56 | -14.30 | -0.20 | 46.43 | -714.49 | 14210.38 | -24822.61 |
| | -0.1 | 71.64 | -7.19 | -0.10 | 15.51 | -346.63 | 7143.45 | -12378.43 |
| | 0.0 | 72.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 71.64 | 7.19 | 0.10 | 15.51 | 346.63 | -7143.45 | 12378.43 |
| | 0.2 | 70.56 | 14.30 | 0.20 | 46.43 | 714.49 | -14210.38 | 24822.61 |
| | 0.3 | 68.78 | 21.28 | 0.30 | 62.69 | 1112.76 | -21157.82 | 37352.50 |
| | 0.4 | 66.32 | 28.04 | 0.40 | 20.45 | 1522.22 | -27831.42 | 49715.46 |
| 3.5 | -0.4 | 68.16 | -28.82 | -0.40 | -57.08 | -1353.85 | 26193.27 | -47422.38 |
| | -0.3 | 70.69 | -21.87 | -0.30 | 2.49 | -999.16 | 19899.23 | -35689.54 |
| | -0.2 | 72.52 | -14.70 | -0.20 | 13.88 | -649.36 | 13369.00 | -23778.25 |
| | -0.1 | 73.63 | -7.39 | -0.10 | 6.18 | -317.94 | 6718.46 | -11876.44 |
| | 0.0 | 74.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 73.63 | 7.39 | 0.10 | 6.18 | 317.94 | -6718.46 | 11876.44 |
| | 0.2 | 72.52 | 14.70 | 0.20 | 13.88 | 649.36 | -13369.00 | 23778.25 |
| | 0.3 | 70.69 | 21.87 | 0.30 | 2.49 | 999.16 | -19899.23 | 35689.54 |
| | 0.4 | 68.16 | 28.82 | 0.40 | -57.08 | 1353.85 | -26193.27 | 47422.38 |
| 4.0 | -0.4 | 70.00 | -29.60 | -0.40 | -103.94 | -1218.13 | 24688.02 | -45385.64 |
| | -0.3 | 72.61 | -22.46 | -0.30 | -32.22 | -906.14 | 18735.58 | -34191.07 |
| | -0.2 | 74.49 | -15.10 | -0.20 | -3.75 | -594.30 | 12590.00 | -22821.55 |
| | -0.1 | 75.62 | -7.59 | -0.10 | 0.38 | -292.66 | 6327.05 | -11411.04 |
| | 0.0 | 76.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 75.62 | 7.59 | 0.10 | 0.38 | 292.66 | -6327.05 | 11411.04 |
| | 0.2 | 74.49 | 15.10 | 0.20 | -3.75 | 594.30 | -12590.00 | 22821.55 |
| | 0.3 | 72.61 | 22.46 | 0.30 | -32.22 | 906.14 | -18735.58 | 34191.07 |
| | 0.4 | 70.00 | 29.60 | 0.40 | -103.94 | 1218.13 | -24688.02 | 45385.64 |
| 4.5 | -0.4 | 71.84 | -30.37 | -0.40 | -132.44 | -1104.82 | 23287.75 | -43532.49 |
| | -0.3 | 74.52 | -23.05 | -0.30 | -55.29 | -827.08 | 17671.47 | -32831.04 |
| | -0.2 | 76.45 | -15.50 | -0.20 | -16.72 | -546.14 | 11876.92 | -21942.37 |
| | -0.1 | 77.61 | -7.79 | -0.10 | -3.28 | -270.24 | 5966.34 | -10978.74 |
| | 0.0 | 78.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | 0.1 | 77.61 | 7.79 | 0.10 | -3.28 | 270.24 | -5966.34 | 10978.74 |
| | 0.2 | 76.45 | 15.50 | 0.20 | -16.72 | 546.14 | -11876.92 | 21942.37 |
| | 0.3 | 74.52 | 23.05 | 0.30 | -55.29 | 827.08 | -17671.47 | 32831.04 |
| | 0.4 | 71.84 | 30.37 | 0.40 | -132.44 | 1104.82 | -23287.75 | 43532.49 |

## A.2 Adjacency matrices of the structures

- Structure A

$$adj = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

- Structure B

$$adj = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

- Structure C

$$adj = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

- Structure D

$$adj = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

- Structure E

$$adj = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

- Structure 1

$$adj = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

- Structure 2

$$adj = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

- Structure 3

$$adj = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

- Structure 4

$$adj = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

## A.3 Adjusted polynomial equation

1. Structure 1
   - $R1 = 5mm - f1(x) = 0.52x^3 - 0.89x^2 + 0.99x$
   - $R1 = 6mm - f2(x) = 0.40x^3 - 0.77x^2 + 0.94x$
   - $R1 = 7mm - f3(x) = 0.32x^3 - 0.70x^2 + 0.91x$
   - $R1 = 8mm - f4(x) = 0.27x^3 - 0.64x^2 + 0.89x$
   - $R1 = 9mm - f5(x) = 0.23x^3 - 0.60x^2 + 0.87x$
2. Structure 2
   - $R1 = 8mm - f1(x) = 0.10x^3 - 0.14x^2 + 1.40x$
   - $R1 = 9mm - f2(x) = 0.02x^3 - 0.07x^2 + 1.29x$
   - $R1 = 10mm - f3(x) = 0.00x^3 - 0.03x^2 + 1.19x$
   - $R1 = 11mm - f4(x) = 0.00x^3 - 0.00x^2 + 1.08x$
   - $R1 = 12mm - f5(x) = 0.00x^3 - 0.03x^2 + 0.98x$
   - $R2 = 4mm - f1(x) = 0.10x^3 - 0.14x^2 + 1.40x$
   - $R2 = 5mm - f2(x) = 0.06x^3 - 0.14x^2 + 1.28x$
   - $R2 = 6mm - f3(x) = 0.05x^3 - 0.15x^2 + 1.19x$
   - $R2 = 7mm - f4(x) = 0.01x^3 - 0.16x^2 + 1.11x$
   - $R2 = 8mm - f5(x) = 0.03x^3 - 0.17x^2 + 1.03x$
   - $L1 = 12mm - f1(x) = 0.10x^3 - 0.13x^2 + 1.40x$
   - $L1 = 13mm - f2(x) = 0.08x^3 - 0.13x^2 + 1.55x$
   - $L1 = 14mm - f3(x) = 0.08x^3 - 0.11x^2 + 1.70x$
   - $L1 = 15mm - f4(x) = 0.06x^3 - 0.10x^2 + 1.85x$
   - $L1 = 16mm - f5(x) = 0.06x^3 - 0.08x^2 + 2.00x$
   - $L2 = 5mm - f1(x) = 0.10x^3 - 0.13x^2 + 1.40x$
   - $L2 = 6mm - f2(x) = 0.08x^3 - 0.12x^2 + 1.31x$
   - $L2 = 7mm - f3(x) = 0.07x^3 - 0.11x^2 + 1.23x$
   - $L2 = 8mm - f4(x) = 0.04x^3 - 0.10x^2 + 1.16x$
   - $L2 = 9mm - f5(x) = 0.03x^3 - 0.10x^2 + 1.08x$
   - $L3 = 20mm - f1(x) = 0.10x^3 - 0.14x^2 + 1.40x$

- $L3 = 21mm - f2(x) = 0.08x^3 - 0.16x^2 + 1.36x$
- $L3 = 22mm - f3(x) = 0.08x^3 - 0.18x^2 + 1.33x$
- $L3 = 23mm - f4(x) = 0.09x^3 - 0.20x^2 + 1.29x$
- $L3 = 24mm - f5(x) = 0.09x^3 - 0.23x^2 + 1.25x$

3. Structure 3
    - $R1 = 6mm - f1(x) = -0.19x^3 + 0.18x^2 + 1.30x$
    - $R1 = 7mm - f2(x) = -0.11x^3 + 0.26x^2 + 1.13x$
    - $R1 = 8mm - f3(x) = -0.09x^3 + 0.28x^2 + 0.97x$
    - $R1 = 9mm - f4(x) = -0.06x^3 + 0.28x^2 + 0.83x$
    - $R1 = 10mm - f5(x) = -0.06x^3 + 0.27x^2 + 0.70x$
    - $R2 = 3mm - f1(x) = -0.19x^3 + 0.18x^2 + 1.30x$
    - $R2 = 4mm - f2(x) = -0.14x^3 + 0.28x^2 + 1.10x$
    - $R2 = 5mm - f3(x) = -0.09x^3 + 0.33x^2 + 0.90x$
    - $R2 = 6mm - f4(x) = -0.05x^3 + 0.34x^2 + 0.72x$
    - $R2 = 7mm - f5(x) = -0.01x^3 + 0.32x^2 + 0.57x$
    - $L1 = 25mm - f1(x) = -0.19x^3 + 0.18x^2 + 1.30x$
    - $L1 = 26mm - f2(x) = -0.18x^3 + 0.18x^2 + 1.39x$
    - $L1 = 27mm - f3(x) = -0.20x^3 + 0.20x^2 + 1.54x$
    - $L1 = 28mm - f4(x) = -0.19x^3 + 0.17x^2 + 1.58x$
    - $L1 = 29mm - f5(x) = -0.19x^3 + 0.16x^2 + 1.68x$
    - $L2 = 12mm - f1(x) = -0.19x^3 + 0.18x^2 + 1.30x$
    - $L2 = 13mm - f2(x) = -0.20x^3 + 0.20x^2 + 1.21x$
    - $L2 = 14mm - f3(x) = -0.17x^3 + 0.21x^2 + 1.12x$
    - $L2 = 15mm - f4(x) = -0.15x^3 + 0.21x^2 + 1.04x$
    - $L2 = 16mm - f5(x) = -0.14x^3 + 0.21x^2 + 0.96x$
    - $L3 = 35mm - f1(x) = -0.19x^3 + 0.18x^2 + 1.30x$
    - $L3 = 36mm - f2(x) = -0.21x^3 + 0.15x^2 + 1.29x$
    - $L3 = 37mm - f3(x) = -0.23x^3 + 0.13x^2 + 1.28x$
    - $L3 = 38mm - f4(x) = -0.25x^3 + 0.11x^2 + 1.27x$
    - $L3 = 39mm - f5(x) = -0.27x^3 + 0.08x^2 + 1.26x$

4. Structure 4

- $R1 = 30mm - f1(x) = 0.08x^3 + 0.00x^2 + 1.46x$
- $R1 = 31mm - f2(x) = 0.16x^3 + 0.00x^2 + 1.46x$
- $R1 = 32mm - f3(x) = 0.19x^3 + 0.00x^2 + 1.46x$
- $R1 = 33mm - f4(x) = 0.23x^3 + 0.00x^2 + 1.45x$
- $R1 = 34mm - f5(x) = 0.29x^3 + 0.00x^2 + 1.45x$
- $\alpha = 0.8rad - f1(x) = 0.63x^3 + 0.00x^2 + 1.56x$
- $\alpha = 1.0rad - f2(x) = 0.27x^3 + 0.00x^2 + 1.53x$
- $\alpha = 1.2rad - f3(x) = 0.14x^3 + 0.00x^2 + 1.50x$
- $\alpha = 1.4rad - f4(x) = 0.07x^3 + 0.00x^2 + 1.48x$
- $\alpha = 1.6rad - f5(x) = 0.08x^3 + 0.00x^2 + 1.46x$
- $R2 = 2.5mm - f1(x) = 0.08x^3 + 0.00x^2 + 1.46x$
- $R2 = 3.0mm - f2(x) = 0.01x^3 + 0.00x^2 + 1.40x$
- $R2 = 3.5mm - f3(x) = -0.05x^3 + 0.00x^2 + 1.34x$
- $R2 = 4.0mm - f4(x) = -0.06x^3 + 0.00x^2 + 1.29x$
- $R2 = 4.5mm - f5(x) = -0.08x^3 + 0.00x^2 + 1.24x$

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