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Trend-Cycle Decomposition for Latin American and G7 Countries: Application and Empirical Comparison of Old and New Univariate Methodologies

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Resumen

Utilizando datos trimestrales del producto de los países del G7 y América Latina, comparamos empíricamente los estimados de ciclos económicos obtenidos usando diez métodos de descomposición tendencia-ciclo del producto. Los resultados indican los siguiente: (i) la descomposición de Beveridge y Nelson (1981) y los modelos UCUR de Grant y Chan (2017a) estiman ciclos volátiles que no permiten identificar los periodos de recesión; (ii) los filtros estadísticos (HP, BK, CF, KMW) identifican las recesiones y expansiones de mejor manera; (iii) los mejores procedimientos son los de Perron y Wada (2009, 2016), Perron, Shintani y Yabu (2017) y Hamilton (2018), los cuales presentan ciclos con mayor persistencia y profundidad que permiten una adecuada identificación de los períodos recesivos; (iv) los mejores modelos atribuyen un rol más importante a los choques que afectan al componente cíclico; (v) existe una similitud en la persistencia y profundidad de los ciclos económicos de Brasil, Chile, México y los países del G7 comparados con los hallados para Argentina y Perú; y (vi) el modelo de tendencia determinística con quiebres, a pesar de su simplicidad, se aproxima en varios periodos a las estimaciones de los mejores métodos.

Palabras Claves: Comparación de modelos Bayesianos, Descomposición Tendencia-Ciclo, Modelos UC, Filtros, América Latina, G7.

Clasificacion JEL: C11, C52, E32, N16.

Abstract

Using quarterly output data from the G7 and Latin American countries, we empirically compare estimates of business cycles obtained using ten trend-cycle output decomposition methods. The results indicate the following: (i) the Beveridge and Nelson (1981) decomposition and the Grant and Chan (2017a) UCUR models estimate volatile cycles that do not allow recession periods to be identified; (ii) statistical filters (HP, BK, CF, KMW) better identify recessions and expansions; (iii) the best procedures are those of Perron and Wada (2009, 2016), Perron, Shintani and Yabu (2017) and Hamilton (2018), which present cycles with greater persistence and depth that allow adequate identification of recessive periods; (iv) the best models attribute a more important role to the shocks that affect the cyclical component; (v) there is a similarity in the persistence and depth of the economic cycles of Brazil, Chile, Mexico and the G7 countries compared to those found for Argentina and Peru; and (vi) the deterministic trend model with breaks, despite its simplicity, is similar in several periods to the estimates of the best methods.

Key Words: Trend-Cycle output Decomposition, Unobserved Component Models, Filters, Latin America, G7.

JEL Classification: C11, C52, E32, N16.

Contents

1 Introduction

The measurement or estimation of the business cycle (also called the output gap) is of significant importance for the academic context and for policy makers. Its estimation is useful, for example, to predict inflation, the evaluation of the business cycle and also to determine monetary and fiscal policy. The size and persistence of the output gap provide an indicator of inflationary pressures and is frequently used in the estimation of monetary reaction functions. On the other hand, the trend component provides information on the growth patterns of the economy. Despite the existence of a wide variety of methods to decompose the output into its trend (permanent) and cyclical (transitory) components, the debate on which is the most appropriate decomposition is far from over.

Since the work of Burns and Mitchell (1946), different methods have been proposed to decompose the output, starting with the decomposition of Beveridge and Nelson (1981) based on an unrestricted ARIMA model and the unobserved component (UC) models, highlighting the contributions from Harvey (1985), Watson (1986) and Clark (1987). Likewise, there are statistical filters such as the Hodrick and Prescott (1997) filter, the Baxter and King (1999) and Christiano and Fitzgerald (2003) band-pass type filters. Following Stock and Watson (1988), Morley et al. (2003) identify the correlation between component innovations as the source of the differences between the estimates obtained by the decomposition of Beveridge and Nelson (1981) and the basic UC model. On the other hand, there are UC models with a mixture of Normals proposed by Wada and Perron (2006) and Perron and Wada (2009, 2016) that allow introducing asymmetries in the cyclical component and nonlinearities in the trend component. One of the few studies that has compared different UC models (with/without correlation between innovations) is Grant and Chan (2017a) using Bayesian techniques. None of the mentioned methods is without problems or disadvantages, one of the most criticized being the frequently used Hodrick and Prescott (1997) filter; see Harvey and Jaeger (1993), Cogley and Nason (1995a,b), among others. In this line, Hamilton (2018) has recently proposed a filter that aims to solve the disadvantages of the HP filter. Another recent method that allows obtaining flexible nonlinear deterministic trends using Fourier expansions is proposed by Perron et al. (2017, 2020). Although this method has been used to model trends in global temperature series and unemployment rates, it is plausible to decompose the output obtaining the cycle as a residual.

The objective of this document is the updating and empirical comparison of ten univariate output decomposition techniques applied to two groups of different countries: the G7 (Canada, France, Germany, Italy, Japan, UK and US) and five Latin American countries-LA (Argentina, Brazil, Chile, Mexico and Peru). Is the behavior of the trend and cyclical components similar in both groups of countries? If there are differences, are they linked to domestic and/or international events and which are these? How important are these differences? What are the techniques that allow for a better identification of the periods of recession in both groups of countries according to the periods published by official business cycle research institutions? Are the innovations of the trend more or less important than the innovations of the business cycle? Are there some characteristics of industrialized countries versus developing countries? These are the main questions we answer in this document. On the other hand, there is a wide literature on empirical applications of trend-cycle output decomposition methods. However, few works make a comparison between several of these methods as well as for a large sample of countries. Likewise, the literature has proposed new and recent methodological alternatives to decompose the output. To our knowledge, these procedures have not yet been applied empirically or compared with approaches already established in the literature and for a large sample of countries. This document contributes to this discussion using ten methods and a group of heterogeneous countries (G7 and LA).

The main results can be summarized as follows: (i) the Beveridge and Nelson (1981) decomposition and the Grant and Chan (2017a) UCUR models estimate volatile cycles that do not allow recession periods to be identified; (ii) statistical filters (HP, BK, CF, KMW) better identify recessions and expansions; (iii) the best procedures are those of Perron and Wada (2009, 2016), Perron et al. (2017) and Hamilton (2018), which present cycles with greater persistence and depth that allow for an adequate identification of recessive periods; (iv) the best models attribute a more important role to the shocks that affect the cyclical component; (v) there is a similarity in the persistence and depth of the economic cycles of Brazil, Chile, Mexico and the G7 countries compared to those found for Argentina and Peru; and (vi) the deterministic trend model with breaks, despite its simplicity, is similar in several periods to the estimates of the best methods.

The rest of the paper is structured as follows. Section 2 reviews the literature. Section 3 presents the ten trend-cycle decomposition methods. Section 4 presents and discusses the results of the estimations. Finally, Section 5 presents the conclusions.

2 Literature Review

The first approach to the analysis of business cycles dates back to Mitchell (1927) and Burns and Mitchell (1946), who defined business cycles as fluctuations in economic activity that last between 1.5 and 8 years. Burns and Mitchell (1946) compile a group of time series for the US and construct cyclical indicators to analyze changes in economic activity. This methodology was adopted by the National Bureau of Economic Research (NBER) to classify periods of expansion and recession. However, this approach lacks a metric or model to measure the business cycle and has delays in identifying recession periods. Regarding the first aspect, Fellner (1956) estimates the economic cycle as the residual between the output and its trend modeled in a deterministic way; see also, Zarnowitz and Boschan (1975), who find long expansive cycles and short recessive cycles, being among the first to suggest the presence of asymmetries in the fluctuations.

Unlike Fellner (1956), Beveridge and Nelson (1981), hereinafter BN decomposition, consider the existence of a stochastic trend, which is supported by the findings of Nelson and Plosser (1982). Using an ARIMA(p,1,q) model and imposing a negative correlation between trend and cyclical component innovations, Beveridge and Nelson (1981) find that fluctuations in output are dominated by trend component innovations which is equivalent to say that the real shocks are the determinants of the variations in the output; see Proietti (2006), who supports the idea that supply shocks are predominant. Likewise, the authors find that the expansions and contractions of the cycle have the same duration implying symmetrical fluctuations, which differs with the NBER chronology and the Zarnowitz and Boschan (1975) identification.

On the other hand, there are the models of unobserved components, hereinafter UC models, by Harvey (1985), Watson (1986) and Clark (1987), who adopt the null correlation assumption between the disturbances of the cycle and the trend. Watson (1986) finds that innovations in the cyclical component are more important in explaining output fluctuations, which indicates that transitory (demand) shocks are relevant. For his part, Clark (1987) finds that fluctuations in the US output depend in equal proportion on both innovations; see also Morley and Piger (2012).

The opposition of the results found by the BN decomposition and the UC model is observed in Stock and Watson (1988), who state that there is no empirical evidence of the similarity of results between both models and that this suggests that the correlation between the innovations determines the relative importance of the shocks on output fluctuations. The formalization and proof of this argument, as well as the reconciliation of these apparently different results, appears in Morley et al. (2003), who find that the ARIMA and UC models are theoretically equivalent since they have the same autocovariance structure. To reconcile both approaches, the authors propose a UC model that relaxes the existence of correlation between innovations, which is estimated together with the rest of the parameters. Thus, this model allows to generate results identical to those obtained using the BN decomposition, that is, short cycles, predominance of trend innovations and lack of agreement with the NBER chronology.

Another group of methods to decompose the output are the so-called statistical filters. Among these, the most used is the filter proposed by Hodrick and Prescott (1997), hereinafter the HP filter, which allows obtaining symmetric cycles and a smoothed trend. However, research has found that its use to identify the cyclical component of most time series cannot be justified on the basis of optimal filtering arguments due to four problems detailed by Guay and St-Amant (2005) based on King and Rebelo (1993): (i) does not admit correlation between the cyclical and trend components; (ii) the output is an I(2) process when the output is generally assumed to be $I(0)$ or $I(1)$; (iii) the cyclical component is a white noise, which is unlikely in macroeconomic time series; and (iv) the smoothing parameter is appropriate[1]. Another criticized aspect is the problem of the valuescalculated for the end of the sample; see Orphanides and van Norden (2002), Cayen and van Norden (2005) and Mise et al. (2007). This aspect implies a problem to make estimates in real time, which is problematic if we are at the peak of a cycle. Some solutions have been to use predictions to generate additional data at the end of the sample or the proposal of St-Amant and van Norden (1997), who use a variant of the HP filter to smooth the trend in the last periods of the sample. Based on simulations, Guay and St-Amant (2005) find

[1]Grant and Chan (2017b) also argue that the HP filter is equivalent to having an uncorrelated cyclical component, the correlation of component innovations is zero, and the smoothing parameter corresponds to the ratio between variances of both components.

evidence in favor of these questions and point out that the performance of the HP filter depends on the persistence of the economic cycle (given by the sum of the autoregressive coefficients) as well as on the variance ratio of the innovations of the two components. An important aspect is that most macroeconomic series are integrated or strongly persistent series so that they are characterized as non-stationary processes. For authors such as Harvey and Jaeger (1993) and Cogley and Nason (1995a,b), the HP filter tends to amplify the cycles in the cyclical frequencies; that is, it can generate cyclic frequency-type periodicities even if none exist in the data[2]. The authors conclude that the HP filter amplifies cyclic frequencies and produces spurious dynamics and its performance will depend on whether we want to obtain the cyclic frequencies of y_t or Δy_t . Likewise, it is indicated that the spectrum of the cyclical component of the HP filter has a peak in the frequency (cyclical) equivalent to 30 quarters, which is absent from the original series. For additional criticism, see Hamilton (2018).

Baxter and King (1999) propose another filter, hereinafter the BK filter, which is of the band-pass type. The main objective of the BK filter is to properly extract a specific range of frequencies without altering the properties of the extracted component. If the spectra of the cyclic component identified by the BK filter are compared with that of an ideal band-pass filter, the conclusion depends on whether one is interested in recovering the component of the cyclic frequencies for y_t or for Δy_t . In the latter case, as noted by Harvey and Jaeger (1993) and Cogley and Nason (1995a) for the HP filter, the BK filter greatly amplifies the cyclic frequencies and creates spurious cycles. For example, the filter amplifies by a factor of 10 the variance of cycles with a periodicity of about 20 quarters (frequency $\pi/10$) which is absent in the spectrum of an ideal filter. Murray (2003) argues that the band-pass filter leaves some of the trend shock in the estimated cycle. Guay and St-Amant (2005) show that, when the peak of the spectral density function of the series is found at cyclic frequencies, the HP and BK filters provide a good approximation to the cyclic component. However, if the peak is located near zero frequency such that

[2]Harvey and Jaeger (1993) and Cogley and Nason (1995a) calculate the squared gain of the cyclic component of the HP filter for $(1-L)y_t$. In this case, the squared gain is $(1-L)^3H(L)$ given that $(1-L)^4y_t = (1-L)^3H(L)(1-L)y_t$. Thus, the squared gain is $|HP(\omega)|^2|1-\exp(-i\omega)|^{-2}$, where ω denotes the respective frequency.

the variance weight is concentrated at low frequencies, the two filters cannot identify the cyclic component adequately. This case is known as a time series with the typical Granger shape; see Guay and St-Amant (2005). An alternative way of looking at this aspect is when the sum of the autoregressive coefficients is very close to unity. In this case, the filter performance is poor. If the sum of the two parameters is far from unity (the energy of the series is concentrated in the cyclic frequencies) the filters perform well. Another filter is the one proposed by Christiano and Fitzgerald (2003), hereinafter the CF filter. Like the BK filter, this filter is of the band-pass type with the same objectives as the BK filter[3].

Recently, Kamber et al. (2018), hereinafter the KMW filter, propose a simple modification to the BN decomposition in order to extract more consistent cycles with the stylized facts and the official NBER chronology for the US. Kamber et al. (2018) find that using the BN decomposition based on an AR(1) model, it is evident that the output gap has a small amplitude or lack of persistence, and the movements do not coincide at all with the reference of cycles of expansions and recessions in the US determined by the NBER. On the other hand, if the output gap calculated by the Congressional Budget Office (CBO) is observed, it can be found that this gap is much more persistent (greater amplitude) and its movements are strongly procyclical in terms of the reference cycle of the NBER; see CBO (1995). An important reason for these differences is that the estimation of the autoregressive coefficient for the AR(1) model used in the BN decomposition implies a high signal-to-noise ratio, while the opposite happens in the case of the CBO (1995) estimation. When applying the KMW filter to US output using a low signal-to-noise ratio, the resulting gap is persistent (large amplitude) and its movements match well with the NBER reference cycle.

Two missing and important features in the above methods are asymmetries and nonlinearities. Regarding the asymmetries, Neftçi (1984) shows that the behavior of the unemployment rate is characterized by abrupt jumps and sudden and slow decreases. Additional evidence that recessions are steeper than expansions can be found in Delong and Summers (1986), Falk (1986), Sichel (1991, 1993) and Diebold et al. (1993)[4]. Wada

[3]Other butterworth type filters are proposed by Harvey and Trimbur (2003).

[4]The aspect of asymmetries dates back to Friedman (1964, 1993), who proposed the so-called plucking model which is formalized in Kim and Nelson (1999); see also Goodwin and Sweeney (1993). Empirical

and Perron (2006) and Perron and Wada (2009, 2016) propose an approach that overcomes the identification problems of the UC and ARIMA models. These authors consider a non-linear UC model, which allows mixtures of Normal distributions in innovations (hereinafter UCMN model), asymmetries in the cyclical component and structural changes in the trend. The latter may reflect abrupt or sudden changes in the trend of the output. For example, it is possible to admit changes in the level that could be caused by shocks of large scale but with low probability of occurrence, while most of the dynamics of the trend is driven by shocks of smaller magnitude. This assumes the existence of high and low variance regimes where each of which is associated with a Normal distribution with a probability of occurrence. It is important to mention that, unlike Hamilton (1989), the transitions between regimes are not determined by a Markov process. Perron and Wada (2009) show the importance of taking into account the existence of structural changes in the trend function because, when a structural break is used in 1973Q1, the BN decomposition and the UC model produce similar trends and cycles, and the trend is a broken or segmented deterministic function. In other words, not specifying a break produces artificial results. When a break in the output growth rate is allowed, asymmetric cycles are found, persistent and in accordance with the NBER chronology. Likewise, the innovations of the cyclical component are more important to explain the fluctuations of the output.

A comparison from the Bayesian point of view of correlated UC models (hereinafter UCUR model) using Markov Chain Monte Carlo (MCMC) methods is made by Grant and Chan (2017a); see also Luo and Starz (2014) and Morley and Piger (2012). Like Perron and Wada (2009, 2016), Grant and Chan (2017a) allow for one or two breaks in the trend. Thus, for the US, Grant and Chan (2017a) select a UC model with a break in 2007 where trend innovations are more important to explain output fluctuations[5] which is opposite to Perron and Wada (2009, 2016).

Recently, Perron et al. (2017), hereinafter PSY (2017), propose a method that estimates the nonlinear trend component in a flexible way approximated by Fourier expan-

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evidence on the plucking model can be found in Sichel (1993) for the US, Diebold and Rudebusch (1990) for the US, UK and France, Kim and Nelson (1999) and Mills and Wang (2002) for the G7 countries; see also Rodríguez (2010) for Peru.

[5]Grant and Chan (2017b) also reconcile the small and large gaps found in the UC models of Morley et al. (2003) and the HP filter, respectively. To do this, they model the trend as a second-order Markov process.

sions. The authors apply this method to a set of three global temperature series, while Perron et al. (2020) apply this approximation to the G7 unemployment series, sustaining that the trend obtained is robust to the possibility of mean reversion (ie, process $I(0)$) as well as compatible with the hysteresis hypothesis in the unemployment rate. (ie, process $I(1)$). Although this procedure has not been used previously to decompose output, we adopt this approach to estimate the trend component and obtain the cyclical component per residual.

Based on the various criticisms of the HP filter, Hamilton (2018) proposes an alternative approach that overcomes the disadvantages of the HP filter by estimating a one-variable regression at date t on the four most recent values to date $t - h$, with $h = 8$ for quarterly series, so as to include shocks whose effects last substantially more than two years, but which are still transitory. According to the author, this procedure preserves the underlying dynamic relationships and consistently estimates the population characteristics in such a way that the use of linear projections is advisable instead of the use of the HP filter. Jönsson (2020) argues that the Hamilton (2018) method performs better than the HP filter in terms of real-time estimates. However, Schüler (2021) and Donayre (2021) consider that, when addressing the deficiencies of the HP filter, the Hamilton (2018) method has other disadvantages, such as: absence of asymmetries in the cycles and alteration of the variations in the extracted frequencies which generate amplified business cycles. Using simulations, Donayre (2021) discusses the ability to properly identify asymmetries in business cycle fluctuations. When considering different linear and asymmetric process specifications consistent with previous estimates, the results reveal that the Hamilton (2018) approach cannot preserve true asymmetric behavior or reproduce features of business cycles. This flaw is aggravated in processes that exhibit a high degree of persistence, a slight asymmetric effect, or in small samples. These findings are robust to the complexity of the autoregressive dynamics and the type of nonlinearity. In addition, the analysis reveals that the Hamilton (2018) approach generates spurious expansionary periods when they do not exist in the data generating process that distort the results of the linearity tests and produce characteristics of the economic cycles that are in disagreement with those of the output of the US. Likewise, Schüller (2021) maintains that Hamilton (2018) avoids spurious cycles and problems at the end of the sample, but the cost is the introduction of fluctuations, altering the variances and inducing a change in the phases of the economic cycle.

Regarding the empirical applications, these are extensive for some of the methods mentioned, so in the following lines we only consider some of them. For the case of the United Kingdom, Hindrayanto et al. (2018) use a set of UC models to reject the null hypothesis of no correlation between trend and cycle innovations. A similar result is found by Melolinna and Tóth (2019) who, using Bayesian techniques, find the existence of a negative correlation between the innovations approaching the results of the BN decomposition. For Canada, Kichian (1999) uses a UC model with a break in trend in 1976Q2, while Wakerly et al. (2004), based on Stock and Watson (1988), find that in three of the five Canadian regions there is a predominance of the shocks of the trend component in the short and long term fluctuations of the output. For Germany, Flaig (2002) uses UC models with three breaks in the trend (1973, 1982, and 1992). For their part, Doz et al. (1995) find that the UC model obtains a reasonable estimate of the cyclical fluctuations for France, compared to the BN decomposition, which is the opposite of what Garnier and Jean-Baptiste (2016) found. Kamber et al. (2018) apply their filter to the US output and manage to obtain persistent and broad cycles, consistent with the NBER chronology. See also Alqaralleh (2019), who uses the UCUR for G7 countries but using industrial production.

For Germany, Brandner and Neusser (1992) use the HP filter and find that the evolution of the business cycle follows a pattern similar to that of the US. Marczak and Beissinger (2012) use the HP and BK filters to model the comovement of the business cycle with wages. For the case of France, Benati (2001) uses these filters to show that cross-correlations between the cyclical components of output, inflation, and unemployment can lead to distorted stylized facts. For the case of Italy, Busseti and Caivano (2013) obtain identical cycles using the HP filter and the UC model. For their part, Bulligan et al. (2019) use the UC model and determine that economic and financial cycles present persistence; see also Zizza (2006). For the case of Japan, Iiboshi (2011) uses the approach of Morley et al. (2003) and Perron and Wada (2009) to decompose the output and the unemployment rate, finding that the results depend on the variance ratio of the trend and cycle innovations, as well as the correlation between both innovations; see Guay and St-Amant (2005). For the UK, see Blackburn and Ravn (1992). In several cases, the results suggest that the output is mainly driven by permanent shocks; however, during recessions and recoveries, transitory shocks are dominant. This supports the use of models that emphasize demand shocks as opposed to models that prioritize supply shocks.

For Argentina, Rabanal and Baronio (2010) find a high correlation between the cycles estimated by the HP and BK filters, which does not happen with the cycle estimated by the BN decomposition. Similar results are found by Trajtenberg (2004) and Krysa and Lanteri (2018). In the case of Brazil, Araujo et al. (2008) use the HP filter and find that the business cycle shows more persistence after the second world war. For their part, Tiecher et al. (2010) use the HP filter and the UC model to show that there is mean reversion by finding changes in the sign of the output gap in 30% of the sample. Recently, Sabioni et al. (2017) use the HP filter with different smoothing parameters to obtain asymmetric cycles. For the case of Chile, Gallego and Johnson (2001) carry out a comparison of methods concluding that the best estimate is provided by the HP filter since the cycle is within the confidence intervals for a longer period of time. On the opposite side are Chumacero and Gallego (2002), who evaluate alternative methods and find that the HP filter is not very robust and unstable at the end of the sample. For the case of Mexico, Faal (2005) uses UC models and finds that the disturbances in the trend are at least twice as large in the unrestricted model as in the restricted model, which implies that a greater proportion of the variance of the output it is explained by the innovations of the trend component. For their part, Catalán and Romero (2018) use the HP filter that reports six cycles, the largest being 38 quarters (1985Q1-1994Q3), while the BK filter reports seven cycles, the largest of which comprises 29 quarters (2000Q3-2007Q4). Recently, Galindo et al. (2019) find that the HP filter and the BN decomposition estimate expansionary cycles, while the production function estimates recessive cycles. For the case of Peru, Terrones and Calderón (1993) and Jiménez (1997) use the HP filter and obtain a maximum drop in the cycle of -8% in 1983 and -11% in 1993, respectively; see also Dancourt and Jiménez

(2009), Jiménez (2011, 2016). However, Cabredo and Valdivia (1999) conclude that this filter is unreliable because it obtains large differences between the observed and potential output. Seminario et al. (2007) use the HP and BK filters to conclude that the higher the smoothing parameter used in the HP filter, the output gap shows larger fluctuations, with similar results using the BK filter. For their part, Castillo et al. (2006) use the BK filter to determine three complete business cycles: (i) from 1980 to 1986, which is the shortest and has a depth of 13.5%; (ii) from 1986 to 1994, which is the most volatile with a depth of 20%; and (iii) from 1994 to 2003, which shows a depth of 5.5% and is the least volatile. Following the methodology of Perron and Wada (2009), Guillén and Rodríguez (2014) estimate seven UCMN univariate models with different configurations, and find that the best model is the UC model that includes a mixture of Normals in the disturbances of the cycle and the trend. In addition, they find that the cycles show a greater amplitude until before the 1990s, while after the 2000s the cycles show less amplitude and a lower probability of being in a high volatility regime. For their part, Florián and Martínez (2019) use the UC model to identify four complete cycles between 1980 and 2018 (1981Q3-1987Q3, 1987Q4-1997Q1, 1997Q2-2008Q2 and 2008Q3-2013Q3), with the first period being the one with the highest volatility.

In summary, although abundant empirical applications associated with UC models, BN decomposition and HP, BK and CF filters can be noted, it is clear that other procedures have been applied in few cases for individual countries or developed countries. A comparison of a large set of procedures and for a large sample of countries is missing. This document fills that gap[6].

[6]Given the extension of the topic, we decided to focus on a select set of univariate methods. A review, estimation and comparison of multivariate methods is beyond the scope of this research and is the subject of future research. Some references using multivariate methods are Kuttner (1994), Apel and Jansson (1999), Gerlach and Smets (1999), Kichian (1999), Roberts (2001), Basistha (2007) and Basistha and Nelson (2007). Some methods from the point of view of central banks are Laubach and Williams (2003), Pichette et al. (2015), Blagrave et al. (2015), Holston et al. (2017) and Castillo and Florián Hoyle (2019).

3 Methodologies

This Section describes the ten different methods or univariate procedures used in the empirical part. To establish notation, we use the following basic UC model proposed by Watson (1986). In this model, the output series (y_t) is composed of two components: trend (τ_t) and cycle (c_t). Watson (1986) considers a UC-ARIMA model as follows:

$$\begin{aligned} y_t &= \tau_t + c_t\,, && (1)\\ \tau_t &= \mu + \tau_{t-1} + \eta_t\,, && (2)\\ \varphi(L)c_t &= \theta(L)\varepsilon_t\,, && (3) \end{aligned}$$

where $\varphi(L) = 1 - \varphi_1 L - \ldots - \varphi_p L^p$, $\theta(L) = 1 + \theta_1 L + \ldots + \theta_q L^q$, $\eta_t \sim iid(0, \sigma_\eta^2)$, $\varepsilon_t \sim iid(0, \sigma_\varepsilon^2)$. In this way, the trend component is a random walk with drift and the cyclical component is an ARMA(p,q) process with zero mean. In this specification, however, the model parameters are not identified without imposing additional restrictions. Common constraints are: (i) the roots of $\varphi(z) = 0$ are outside the unit circle; (ii) $\theta(L) = 1$; and (iii) $cov(\varepsilon_t, \eta_t) = 0$. The restriction in (iii) establishes that the shocks to τ_t and c_t are not correlated, which is an over-identification restriction in certain models; see Morley et al. (2003)[7].

3.1 BN Decomposition

The BN decomposition decomposes an integrated series of order 1 ($I(1)$) into two components: τ_t and c_t. The former is modeled as a drifting random walk, while the latter is specified as a stationary process with zero mean. In other words, the BN decomposition assumes that y_t is an ARIMA(p,1,q) process or that Δy_t is an ARMA(p,q) process, that is, $\varphi(L)\Delta y_t = \theta(L)\varepsilon_t$. This means that Δy_t admits a Wold representation, that is, $\Delta y_t = \mu + \psi(L)\varepsilon_t$ where $\psi(L) = \sum_{k=0}^{\infty} \psi_k L^k$ with $\psi_0 = 1$. In this case, the trend τ_t is a random walk without drift: $\tau_t = \tau_{t-1} + \psi(L)\varepsilon_t = \tau_0 + \psi(1)\sum_{j=1}^{t} \varepsilon_j$. Also, we have that $\psi(L) = \psi(1) + (1-L)\tilde{\psi}(L)$ where $\psi(1) = \sum_{k=0}^{\infty} \psi_k$, $\tilde{\psi}(L) = \sum_{k=0}^{\infty} \tilde{\psi}_j L^j$ and $\tilde{\psi}_j = -\sum_{k=j+1}^{\infty} \psi_k$

[7] For example, in the *random walk plus noise* model, it can be verified that constraint (iii) is necessary to achieve identification.

where it is required that $\sum_{k=0}^{\infty} k|\psi_k| < \infty$ which implies that $\sum_{k=0}^{\infty} |\psi_k| < \infty$ all of which is equivalent to requiring the 1-sumability condition, a condition that is satisfied by any stationary ARMA(p,q) process. Using these definitions, we have that: $\Delta y_t = \mu + [\psi(1) + (1 - L)\psi(L)]\varepsilon_t$, that is, $y_t = y_0 + \mu t + [\psi(1) + (1-L)\psi(L)]\sum_{j=1}^{t} \varepsilon_j$. By iterative process and applying the previous formulas, $y_t = y_0 + \mu t + \psi(1)\sum_{j=1}^{t} \varepsilon_j + \varepsilon_t - \varepsilon_0$ where $\varepsilon_t = \psi(L)\varepsilon_t$. Thus, y_t is composed of a deterministic trend ($y_0 + \mu t$), a stochastic trend ($\psi(1)\sum_{j=1}^{t} \varepsilon_j$) and a cyclical component ($\varepsilon_t - \varepsilon_0$). Now, the series y_t with h periods ahead is $y_{t+h} = y_0 + \mu(t+h) + \psi(1)\sum_{j=1}^{t+h} \varepsilon_j + \varepsilon_{t+h}$. Consequently, the prediction of y_{t+h} with information at time t is $y_{t+h|t} = y_0 + \mu(t+h) + \psi(1)\sum_{j=1}^{t} \varepsilon_j + \varepsilon_{t+h|t}$. Finally, the limit prediction when $h \to \infty$, adjusted for average growth is $\lim_{h\to\infty} y_{t+h|t} - \mu h = y_0 + \mu t + \psi(1)\sum_{j=1}^{t} \varepsilon_j$ given that $\lim_{h\to\infty} \varepsilon_{t+h|t} = 0$ because ε_{t+h} is a stationary process with zero mean. In terms of notation and using $\mu = E(\Delta y_t)$, we have:

$$\tau_t = \lim_{h\to\infty} E(y_{t+h|t} - hE[\Delta y_t]), \quad (4)$$

$$\tau_t = y_0 + \mu t + \psi(1)\sum_{j=1}^{t} \varepsilon_j, \quad (5)$$

and the cyclical component is modeled as the difference between the series y_t and its trend level: $c_t = y_t - \tau_t$.[8]

The BN decomposition forces the existence of a perfect (negative) correlation between the two components while the UC model does the opposite. In this way, in no case are the decompositions unique, since they depend on the correlation between the disturbances. It is important to specify that any UC-ARIMA process with a specific correlation between perturbations is observationally equivalent to an ARMA(p,q) model for Δy_t with nonlinear constraints on the parameters. The ARMA(p,q) model restricted for Δy_t it is called the reduced form of the UC-ARIMA model. This is exploited by Morley et al. (2003) who

[8] A simple calculation of the BN decomposition can be done in the following steps: (i) estimating the best ARMA(p,q) model for Δy_t; (ii) estimating $\psi(1)$ from the parameters estimated in (i); (iii) estimation of $\sum_{j=1}^{t} \varepsilon_j$ using the residuals of the ARMA(p,q) model estimated in (i); and (iv) calculating the cycle such that $c_t = y_t - \tau_t$. As an example, we can cite the estimation of Morley et al. (2003) who specify an ARMA(2,2) model for the US output growth rate (Δy_t). The fitted model for the period 1947Q1-1998Q2 is such that $\mu = 0.816$, $\varphi(L) = 1 - 1.342L + 0.706L^2$, $\theta(L) = 1 - 1.054L + 0.519L^2$. From these values we obtain $\psi(1) = \varphi(1)^{-1}\theta(1) = 1.276$. In this way, the trend component $\tau_t = y_0 + 0.816t + 1.276\sum_{j=1}^{t} \varepsilon_j$ and the cyclical component is obtained as $c_t = y_t - \tau_t$.

allow for the correlation between disturbances to be estimated; see empirical results for the UCUR models (Section 4.1.3).

3.2 Filters

3.2.1 HP Filter

The objective of the HP filter is to extract the component τ_t (for a given value of λ), as the solution to the following problem:

$$\min_{\{\tau_t\}_{t=-1}^{T}} \sum_{t=1}^{T}(y_t-\tau_t)^2 + \lambda\sum_{t=2}^{T-1}[(\tau_{t+1}-\tau_t)-(\tau_t-\tau_{t-1})]^2 , \tag{6}$$

which has a unique solution. The parameter λ is arbitrary and reflects the penalty of incorporating fluctuations in the trend component, with the value of 1600 defined for quarterly data. If $\lambda = 0$, the solution of the minimization problem in (6) is $y_t = \tau_t$, that is, the trend component is equal to the observed series. When $\lambda \to \infty$, minimizing the sum of squares (6) happens when $(\tau_{t+1} - \tau_t) = (\tau_t - \tau_{t-1})$ implying that the change in τ_t is constant which results in a linear trend. In intuitive terms, high values of λ force the change in trend $(\Delta\tau_{t+1} - \Delta\tau_t)$ to be as small as possible, which is what happens when the trend is linear. So increasing the value of λ implies smoothing τ_t.

The HP filter is characterized as a high-pass filter in the sense that it must remove low frequencies or long-cycle components and allow high frequencies or short-term components to be obtained. If we compare the squared gain of the HP filter we find that it is zero at zero frequency and it is close to unity from about $\pi/10$ frequency onwards. In this sense, the HP filter seems to be a good approximation of a high-pass filter. The value of $\lambda = 1600$ implies a cycle of duration between 8-10 years (32-40 quarters). However, the cyclical frequencies are generally grouped or concentrated in the interval from 6 to 32 quarters, implying cycles of duration from 1.5 years to 8 years, i.e., fractions of π equal to 0.05 (32 quarters, frequency $\pi/16$) up to 0.35 (6 quarters, frequency $\pi/3$); see Burns and Mitchell (1946) and Guay and St-Amant (2005).

An advantage over the BN decomposition is that it is not necessary to find the appropriate ARMA(p,q) model for Δy_t since the HP filter uses the same approximation for any series under analysis. The parameter λ corresponds to the ratio between the variance of the cyclical and trend components. However, economic theory provides little or no information on what this relationship should be. Since Hodrick and Prescott (1997) argue that this value is reasonable for US data, it may be arbitrary for output series from other countries as well as its use in other macroeconomic series.

3.2.2 BK Filter

The BK filter is a band-pass filter, that is, it removes low and high frequencies. This filter uses a finite approximation of the moving average type based on the definition of cycles postulated by Burn and Mitchell (1946), that is, it is designed to isolate the components between 6 and 32 quarters (frequencies between $\pi/3$ and $\pi/16$) removing all other frequencies. When this filter is applied to quarterly data it takes the form of a 24-quarter moving average filter:

$$c_t = \sum_{h=-12}^{12} a_h y_{t-h} = a(L) y_t, \tag{7}$$

where the weights a_h can be derived from the inverse Fourier transform; see Priestley (1981). Also, the filter uses the constraint that the sum of the moving average coefficients must be zero.

When comparing the squared gain of an ideal filter with the BK filter, it can be seen that the latter performs well. The filter manages to pass through most components found between 6 and 32 quarters removing low and high frequencies. However, the BK filter does not correspond to an ideal band-pass filter because it is a finite approximation of an infinite moving average filter. In particular, it can be seen that there are low and high frequencies where there is a compression effect[9].

[9] Following Guay and St-Amant (2005), assume that $y_t = (1-L)^{-r}\varepsilon_t$ where r determine the order of integration of y_t and ε_t is a stationary process with zero mean. Baxter and King (1999) show that their filter can be factored as $a(L) = (1-L)^2 a^*(L)$ so that the filter can make a series containing up to two unit roots stationary. The spectrum of the cyclic component obtained by the BK filter is $f_c(\omega) = |BK(\omega)|^2 f_y(\omega)$, where $|BK(\omega)|^2$ is the squared gain of the filter and $f_y(\omega)$ is the spectrum of y_t ; so that $|BK(\omega)|^2 = |a(\omega)|^2$ where $a(\omega)$ denotes the Fourier transform of $a(L)$ at the frequency of ω. So the pseudo-spectrum of y_t is equal to $f_y(\omega) = |1-\exp(-i\omega)|^{-2r} f_\varepsilon(\omega) = 2^{-2r}(\sin^2(\omega/2))^{-r} f_\varepsilon(\omega)$ for $\omega \neq 0$ and where $f_\varepsilon(\omega)$ is the spectrum of ε_t which is well defined since it is stationary. If we assume that $r = 1$ and that ε_t is a white noise with

3.2.3 CF Filter

Christiano and Fitzgerald (2003) propose another band-pass filter of the moving average type, alternative to the BK filter. These filters differ in the objective function used to select the weights of the moving averages. There are two variants of the CF filter. The first is a fixed-length symmetric filter that uses a fixed lead/lag length. The symmetric form is time invariant in the sense that the weights of the moving averages depend on the specified frequency band and not on the data. In the BK filter notation, the difference is given in the polynomial $a(L)$. The second variant is the most general filter, where the weights of the leads and lags can differ. Thus, the asymmetric filter varies over time and the weights depend on the data and change for each observation.

The CF filter is designed to work with an infinite series of data that eliminates very short and very long term movements. The filter is $c_t = B(L)y_t$ with the ideal filter $B(L)$ given by $B(L) = \sum_{j=-\infty}^{\infty} B_j L^j$, where the particular values of B_j have the same shape as the weights of the best approximation of the ideal filter. The recommended approximation is $\hat{c}_t = B_0 y_t + B_1 y_{t+1} + \ldots + B_{T-1-t} y_{T-1} + \tilde{B}_{T-t} y_T + B_1 y_{t-1} + \ldots + B_{t-2} y_2 + \tilde{B}_{t-1} y_1$ for $t = 3, 4, \ldots\ldots\ldots, T-2$. The first and last values of c_t are not reliably estimated and can therefore be excluded from the formula. The coefficients are defined by $B_j = \frac{\sin(jb)-\sin(ja)}{\pi j}$, $j \geq 1$, $B_0 = \frac{b-a}{\pi}$ where $a = \frac{2\pi}{P_U}$ and $b = \frac{2\pi}{P_L}$, P_L and P_U refer to the minimum and maximum number of specified cycle length periods that are normally $\pi/16$ and $\pi/3$, $\tilde{B}_{T-t} = -0.5B_0 - \sum_{j=1}^{T-t-1} B_j$ for $t = 3, 4, \ldots, T-2$ and where $B_0 + B_1 + \ldots + B_{T-1-t} + \tilde{B}_{T-t} + B_1 + \ldots + B_{t-2} + \tilde{B}_{t-1} = 0$. This expression allows us to obtain the value of $\tilde{B}_{t-1}$ so that the sum of the filter weights equals zero.

3.2.4 KMW Filter

Procedure proposed by Kamber et al. (2018). Since the BN decomposition produces estimates of the output gap that are difficult to reconcile with already established beliefs or stylized facts about transitory movements in economic activity, the authors argue that the problem is the imposition of a high signal-to-noise ratio in terms of the variance of

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variance equal to 2π, the spectrum of ε_t is equal to 1 at all frequencies and the cyclic component obtained with the BK filter corresponds exactly to the squared gain of the BK filter: $|BK(\omega)|^2|1-\exp(-i\omega)|^{-2} = |BK(\omega)|^2 2^{-2}(\sin(\omega/2))^{-1}$.

the trend shocks as a fraction of the variance of the forecast error. On the other hand, when a lower signal-to-noise ratio is imposed, the result is that the KMW filter produces a more intuitive estimate of the output gap that is highly persistent, and is typically wide in expansions and narrow in recessions. Notably, the approach is also reliable in the sense of being subject to smaller revisions. Likewise, the approach is robust to the omission of multivariate information and can be accommodated to allow for structural breaks as proposed by Evans and Reichlin (1994) and Perron and Wada (2009).

More generally, to understand the BN decomposition for an AR(p) model, it is useful to define the following signal-to-noise ratio for a time series in terms of the variance of trend shocks as a fraction of the overall variance of the prediction error: $\delta = \sigma^2_{\Delta\tau} / \sigma^2_{\varepsilon}$. Given a Wold representation for $\{\Delta y_t\}$, $\delta = \psi(1)^2$ which is the long-term multiplier corresponding to the sum of the coefficients of the Wold representation that captures the permanent effect of a forecast error in the long-term horizon and is related to the BN trend as follows: $\Delta\tau_t^{BN} = \psi(1)\varepsilon_t$. For an AR(p) model, this long-run multiplier has the simple form $\psi(1) = \varphi(1)^{-1}$ and, based on a maximum likelihood estimate for a US AR(1) model, the signal-to-noise ratio appears to be quite high with $\delta = 2.22$. That is, trend shocks in the BN decomposition are much more volatile than quarter-to-quarter forecast errors in log real output, leading to an estimated output gap with small amplitude and counterintuitive sign. Therefore, many of the counterintuitive results of a BN decomposition based on an AR(1) model are transferred to a higher order, i.e., AR(p) models. The idea that the signal-to-noise ratio (δ) is mechanically linked to $\varphi(1)$ for an AR(p) model is important because it implies that we can impose a low signal-to-noise ratio by fixing the sum of the autoregressive coefficients when estimating an AR(p) model.

The KMW filter is easy to implement compared to related methods. Also, real-time estimates are subject to smaller revisions and appear to be more accurate in the sense of performing better on out-of-sample forecasts of output growth than real-time estimates from other methods. This makes it possible to address an important criticism of Orphanides and van Norden (2002) in the sense that popular methods for estimating the output gap are not reliable in real terms.

3.3 UCMN Models

Perron and Wada (2009) consider that the differences found between the BN decomposition, the HP filter and the UC models (cycles with little similarity with respect to the NBER chronology, movements in the trend dominate movements in the cycle and some correlation between the components) are artificial consequences caused by the absence of the modeling of a break in the slope of the trend. When this structural change is allowed, all methods estimate the same cycle with a trend that is non-stochastic except for a few periods around 1973 (the break date).

Wada and Perron (2006) propose a UC model with the following specification:

$$y_t = \tau_t + c_t + \omega_t , \tag{8}$$

$$\tau_t = \tau_{t-1} + \beta_t + \eta_t , \tag{9}$$

$$\beta_t = \beta_{t-1} + \upsilon_t , \tag{10}$$

$$\varphi (L)c_t = \varepsilon_t , \tag{11}$$

where y_t is the logarithm of the real GDP, τ_t is the trend component, c_t is the cyclical component, β_t is the component that allows changes in the slope of the trend and ω_t , η_t, υ_t , ε_t are the error terms or innovations. The model (8)-(11) it is called the UC model with mixture of Normals (UCMN). In general, an AR(2) model with zero mean is used for the component c_t in (11). The model is non-linear due to the behavior of the innovations. If these innovations are represented by u_t , then they have the following distribution:

$$u_t = \lambda_t \gamma_{1t} + (1 - \lambda_t)\gamma_{2t}, \tag{12}$$

where $\gamma_{it} \sim$ *i.i.d.* $N\,(0, \sigma_i^2)$ and $\lambda_t \sim$ *i.i.d.* $Bernouilli(\alpha)$. So, the disturbance in t behaves like a $N\,(0, \sigma_1^2)$ with probability α and as a $N\,(0, \sigma_2^2)$ with probability $(1 - \alpha)$. This specification allows capturing nonlinearities in the output in the event that α take a value close to 1 and that $\sigma_2^2 > \sigma_1^2$. In this case there would be, most of the time, "normal periods" or low variance, while periods of large disturbances that alter the level of the series will be characterized as "atypical periods". In general, α_i denotes the probability that the innovation η_t , υ_t , ε_t belong to the low variance regime (σ_1^2) while $(1 - \alpha_i)$ is the probability that this innovation is in the high variance regime (σ_2^2).

The characterization of “normal periods” and “atypical periods” has different implications for the various error terms present in the model. First, the measurement errors ω_t take small values or close to zero in "normal periods" and take values of greater magnitude in atypical cases. Second, the errors η_t generate a stochastic trend in “normal periods” or, conversely, if $\sigma_1^2 = 0$, a deterministic trend with occasional changes in level. Third, the innovation υ_t allows little or no change in slope in “normal periods” with large changes in outliers. Lastly, the error ε_t can have different variations depending on whether the economy is in an expansive or recessive period. Each of these scenarios is not independent of the other and can be combined indistinctly, thus affecting the evolution of economic cycles.

The vector u_t in (12) does not have a Normal distribution. However, it is possible to assign a Normal distribution with possible states in the state-space representation. The variance-covariance matrix of u_t takes M possible states that are generated as a result of the combination of the values taken by the random variables *Bernoulli*. For example, in the case that a model with a single mixture of Normals is estimated, there are only two possible states linked to the periods of low and high variance associated with combinations of low and high variance for each innovation, while in the case that estimate a model with two mixtures of Normals, there will be four possible states and in the case that a model with three mixtures of Normals is estimated, there will be eight possible states. Therefore, we have 2^m possible states, where m is the number of disturbances with mixture of Normals.

In the Section 4.1.2, a variety of models with all possible combinations of mixtures of Normals are estimated. As an example, the model denoted as UC-C is a model with a mixture in the cyclical component (ε_t). The matrix Q of variances and covariances for this model is specified as follows[10]:

$$Q = \left\{ \begin{pmatrix} \sigma_\eta^2 & 0 & 0 & 0 \\ 0 & \sigma_{\varepsilon_1}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_\upsilon^2 \end{pmatrix} \begin{pmatrix} \sigma_\eta^2 & 0 & 0 & 0 \\ 0 & \sigma_{\varepsilon_2}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_\upsilon^2 \end{pmatrix} \right\},$$

where each state or regime occurs with probabilities α and $(1-\alpha)$. More details regarding the state-space representation and other cases for the matrix Q can be found in Wada and Perron (2006) and Guillén and Rodríguez (2014).

The estimation stages are: (i) use of the Kalman filter to obtain the best estimator of the state vector and its respective variance-covariance matrix; (ii) use of the filter of Hamilton (1989) to infer the probabilities associated with the vector of states and its variance-covariance matrix; (iii) collapse process. Since there is a dimensionality problem (4^t estimates), Perron and Wada (2009) use the collapsing algorithm following Harrison and Stevens (1976). Then there is the problem called label switching which consists in the fact that it is not possible to differentiate between two states $(i,\ i^*)$ without making a normalization which depends on each series under analysis; see Hamilton et al. (2007). Full details can be found in Wada and Perron (2006) and Perron and Wada (2009, 2016).

3.4 UCUR Models

Grant and Chan (2017a) carry out a Bayesian comparison of the trend-cycle decomposition models proposed by Clark (1987) and Morley et al. (2003), based on the following specification:

$$y_t = \tau_t + c_t, \tag{13}$$

$$\tau_t = \mu + \tau_{t-1} + \eta_t, \tag{14}$$

$$c_t = \varphi_1 c_t + \cdots + \varphi_p c_{t-p} + \varepsilon_t, \tag{15}$$

[10]The representation can be generalized to the other cases; see Guillén and Rodríguez (2014).

where y_t is the logarithm of the real GDP, τ_t is the trend component, c_t is the cycle and μ is the drift that can be interpreted as the average growth rate of the output. The non-stationary trend component τ_t is modeled as a random walk with drift, while the cyclic component c_t is modeled as a zero-mean AR(p) process. Furthermore, in this model the initial condition τ_0 can be modeled as a parameter to be estimated, while for simplicity it is assumed $c_{1-p} = \cdots = c_0 = 0$. The model (13)-(15) is denoted as UCUR model. Likewise, following Morley et al. (2003) is established $p = 2$ so that $c_t = \varphi_1 c_{t-1} + \varphi_2 c_{t-2} + \varepsilon_t$ in (15) and it is assumed that the distributions η_t and ε_t are jointly Normal:

$$\begin{pmatrix} \eta_t \\ \varepsilon_t \end{pmatrix} \sim N\left(\mathbf{0}, \begin{pmatrix} \sigma_\eta^2 & \rho\sigma_\eta\sigma_\varepsilon \\ \rho\sigma_\eta\sigma_\varepsilon & \sigma_\varepsilon^2 \end{pmatrix}\right), \tag{16}$$

where it is observed that the model allows a correlation ρ nonzero between disturbances η_t and ε_t, therefore, it encompasses Clark's (1987) model, as a special case when $\rho = 0$, which is denoted as the UC0 model.

Grant and Chan (2017a) also consider a specification in which a change in trend is modeled, allowing the existence of breaks in the UCUR model. In this way the equation (14) can be rewritten as $\tau_t = \mu_1\mathbf{1}(t < t_0) + \mu_2\mathbf{1}(t \geq t_0) + \tau_{t-1} + \eta_t$, where $\mathbf{1}(\cdot)$ determines that the stochastic trend τ_t has a growth rate of μ_1 before the date of the break t_0 and a growth rate of μ_2 after t_0. The model is denoted by UCUR$-t_0$ while when we have two breaks, the model is denoted by UCUR$-(t_0, t_1)$. On the other hand, the authors consider a set of models with a deterministic trend following the findings of Wada and Perron (2006) and Perron and Wada (2009, 2016); however, the estimate of the variance of trend innovation σ_η^2 is estimated to be zero, which is outside the parameter range because the variance should be positive. To avoid this problem, the following specification is considered: $\tau_t = \mu_1\mathbf{1}(t < t_0) + \mu_2\mathbf{1}(t \geq t_0) + \tau_{t-1}$, which is denoted as model DT$-t_0$ and as DT model when considering the model without a break.

For estimation, the authors use the Markov Chain Monte Carlo (MCMC) procedure for the posterior distribution of the UCUR model. The other UC models can be estimated in a similar way and technical details can be found in Appendix A of Grant and Chan (2017a). Markov sampling is used to obtain the posterior distributions based on sparse matrix algorithms developed in Chan and Jeliazkov (2009) and Chan (2013) which are more efficient than those based on the Kalman filter; see McCausland et al. (2011). In this way, the posteriors can be obtained sequentially from the sampling of the following densities: (i) $p(\tau|\mathbf{y},\varphi,\sigma^2_\eta,\sigma^2_\varepsilon,\rho,\mu_1,\tau_0)$; (ii) $p(\varphi|\mathbf{y},\tau,\sigma^2_\eta,\sigma^2_\varepsilon,\rho,\mu_1,\tau_0)$; (iii) $p(\sigma^2_\varepsilon|\mathbf{y},\tau,\varphi,\sigma^2_\eta,\rho,\mu_1,\tau_0)$; (iv) $p(\sigma^2_\eta|\mathbf{y},\tau,\varphi,\sigma^2_\varepsilon,\rho,\mu_1,\tau_0)$; (v) $p(\rho|\mathbf{y},\tau,\varphi,\sigma^2_\eta,\sigma^2_\varepsilon,\mu_1,\tau_0)$; and (vi) $p(\tau_0|\mathbf{y},\tau,\varphi,\sigma^2_\eta,\sigma^2_\varepsilon,\mu_1,\rho)$. The estimation of the models UCUR$-t_0$ and UCUR$-(t_0,\ t_1)$ follows a similar procedure, which are detailed in Appendix A of Grant and Chan (2017a); see also Section 1.2 of Grant and Chan (2017a).

For comparison purposes, assume we want to compare a set of possibly non-nested models $\{M_1, ..., M_K\}$. Each model M_k is formally defined by two components: a likelihood function $p(y|\theta_k, M_k)$ which depends on the vector of parameters specific to the model (θ_k) and a prior density ($p(\theta_k|M_k)$). The marginal likelihood function under the model M_k is defined as $p(y|M_k) = \int p(y|\theta_k,\ M_k)p(\theta_k|M_k)d\theta_k$. Thus, given two models M_i and M_j, if the marginal likelihood of the model M_i is greater than that associated with the model M_j -that is, the observed data are more likely to be generated by the model M_i compared to model M_j- then this is seen as evidence in favor of the model M_i. The weight of the evidence can be evaluated using the posterior odds ratio between two models which can be written as: $\frac{P(M_i|y)}{P(M_j|y)} = \frac{P(M_i)}{P(M_j)} \times \frac{p(y|M_i)}{p(y|M_j)}$ where $\frac{P(M_i)}{P(M_j)}$ is the prior odds ratio and the marginal likelihood ratio $\frac{p(y|M_i)}{p(y|M_j)}$ is called the Bayes factor (BF) in favor of the model M_i against M_j. If both models are equally likely a priori, that is, the prior odds ratio is equal to unity, then the posterior odds ratio between the two models is equal to the BF. So, for example, if $BF_{ij} = 50$, this implies that the model M_i is 50 times more likely than the model M_j given the data information. For a more detailed discussion of BF, see Koop (2003).

To calculate the marginal likelihood, the importance sampling method is used. Grant and Chan (2017a) use an improved version of the classical cross-entropy method that was originally developed by Rubinstein (1997, 1999); see also Rubinstein and Kroese (2004). Chan and Kroese (2012) showed that importance sampling density can be optimally obtained in a single stage of MCMC methods; see Chan and Eisenstat (2015).

The importance sampling density is the posterior density $p(\theta\ |y,\ M_k)$. This density cannot be used in practice but it provides a good reference point to obtain a plausible density. The idea is to locate a density that is close to the ideal density. In operational terms, the density is found within a convenient family of distributions such that the Kullback-Leibler divergence -the cross-entropy distance from the ideal density- is minimized. Once the optimum density is obtained, let $g(.)$, this is used to construct the importance sampling estimator: $p(\mathbf{y}) = \frac{1}{R}\sum_{r=1}^{R}\frac{p(\mathbf{y}|\theta^{(r)})p(\theta^{(r)})}{g(\theta^{(r)})}$, where the dependency of M_k has been removed to save notation and $\theta^{(1)}, \ldots, \theta^{(R)}$ are the draws of the sampling density $g(\theta)$. More complete details can be found in Appendix B of Grant and Chan (2017) as well as in Chan and Eisenstat (2015).

3.5 PSY (2017) Method

PSY (2017) propose a method to identify and estimate the presence of approximate flexible nonlinear trends via Fourier expansions. For this, they propose a statistic whose null hypothesis is the non-existence of nonlinearities versus the opposite, this statistic being robust to the presence of $I(0)/I(1)$ components in the noise function. The statistic is of the Wald type and generalizes the proposal of Perron and Yabu (2009a,b) who evaluate the presence of a single structural break in the trend of the series. Instead of evaluating the presence of one, two or more breaks, a generalization is to allow the existence of a nonlinear trend which is approximated by Fourier expansions. Although this technique has been applied to model trends in temperature series, we apply this technique to approximate the trend component of the output, while the cycle is obtained by residual. Perron et al. (2020) apply this technique to extract trends in the unemployment rate in G7 countries, obtaining results compatible with the idea that the unemployment rate converges to a long-term unemployment rate (consistent with an $I(0)$ process) and compatible with the idea of hysteresis (consistent with an $I(1)$ process).

The general model implies the possibility of more than one frequency in the Fourier expansion and a general serial correlation structure in the noise component[11]:

$$y_t = \sum_{i=0}^{p_d} \beta_i t^i + \sum_{j=1}^{n} \gamma_{1j} \sin(2\pi k_j t/T) + \sum_{j=1}^{n} \gamma_{2j} \cos(2\pi k_j t/T) + u_t, \quad (17)$$

$$u_t = \sum_{i=0}^{\infty} a_i u_{t-1} + e_t, \quad (18)$$

for $t = 1, 2, \ldots, T$, e_t is a martingale difference sequence with respect to the information set F_t $(e_{t-s}, s > 0)$, that is, $E(e_t | F_{t-1}) = 0$ with $E(e_t^2) = \sigma^2$, $E(e_t^4) > \infty$, $u_0 = O_p(1)$ and where the frequencies k_j are non-negative integers for $j = 1, \ldots, n$ and n is the total number of frequencies used in the Fourier expansion. The usual cases use $p_d = 0$ (without trend) and $p_d = 1$ (linear trend). The set of frequencies k_j can be a suitable subset of all integers between 1 and the maximum frequency k_n, such that k_n does not need to match the n-th frequency. Two possible structures are assumed for the noise component: (i) for the case $I(0)$, we have $u_t = C(L)e_t$, where $C(L) = \sum_{i=0}^{\infty} c_i L^i$, $\sum_{i=0}^{\infty} i|c_i| < \infty$, $0 < |C(1)| < \infty$; (ii) for the case $I(1)$, we have $\Delta u_t = D(L)e_t$, where $D(L) = \sum_{i=0}^{\infty} d_i L^i$, $\sum_{i=0}^{\infty} i|d_i| < \infty$, $0 < |D(i)| < \infty$. These conditions ensure that a functional central limit theorem can be applied to the partial sums of u_t in the case $I(0)$ and the partial sums of Δu_t in the case $I(1)$.

The objective is to statistically evaluate the null hypothesis of the absence of non-linear components and if this hypothesis is rejected, to estimate said trend. To better understand, assume for simplicity that there is only one frequency ($n = 1$) in (17) with $u_t = \alpha u_{t-1} + e_t$, i.e., the basic model. In this way we have $H_0 : \gamma_1 = \gamma_2 = 0$ against the alternative of the presence of a nonlinear component approximated by the Fourier expansion, i.e., $H_1 : \gamma_1 \neq 0$ and/or $\gamma_2 \neq 0$. If the coefficient α of the AR(1) process were known, the quasi-differentiation transformation $1 - \alpha L$ could be applied to the equation (17) and the statistical test would then be equivalent to using a standard Wald test based on the OLS estimate from the quasi-differentiated regression. Because the coefficient α is unknown, the authors use the robust FGLS procedure proposed by Perron and Yabu (2009a).

[11]The basic model is $y_t = \sum_{i=0}^{p_d} \beta_i t^i + \gamma_1 \sin(2\pi kt/T) + \gamma_2 \cos(2\pi kt/T) + u_t$, $u_t = \alpha u_{t-1} + e_t$, where it is assumed that $-1 < \alpha \leqslant 1$, so that both the stationary process, $I(0)$ with $|\alpha| < 1$, as the integrated process, $I(1)$ with $\alpha = 1$, are considered. In this case, the only frequency k in the Fourier expansion series it is assumed to be a fixed and known variable. The goal is to prove the absence of nonlinear components, i.e., $H_0 : \gamma_1 = \gamma_2 = 0$, against the alternative of the presence of a nonlinear component approximated by the Fourier expansion, i.e., $H_1 : \gamma_1 \neq 0$ and/or $\gamma_2 \neq 0$.

Perron and Yabu's (2009a) approach consists of two steps that aim to obtain a Wald statistic based on a FGLS regression such that the limiting distribution is χ^2 standard (or Normal) both in the case $I(0)$ as in the $I(1)$. The first step is to obtain an estimate of α which is $\sqrt{T}$ consistent in the case $I(0)$ and "super efficient" (α_S) in the case $I(1)$. The second step consists of calculating the Wald statistic based on the FGLS estimator using an estimate of α with the mentioned properties. To easily illustrate and see the implications of this procedure, we first consider the model with a single regressor $y_t = \gamma \sin(2\pi kt/T) + u_t$, with $u_t = \alpha u_{t-1} + e_t$. So, the OLS estimator of α is $\alpha = \sum_{t=2}^{T} u_t u_{t-1} / \sum_{t=2}^{T} u_{t-1}^2$. Applying the Cochrane and Orcutt (1949) transformation, the FGLS estimate is obtained by applying an OLS regression to the equation $y_t - \alpha y_{t-1} = \gamma\{\sin(2\pi kt/T) - \alpha \sin(2\pi k(t-1)/T)\} + u_t - \alpha u_{t-1}$ for $t = 2, \ldots, T$ with $y_1 = \gamma sin(2\pi k/T) + u_1$. This corresponds to the FGLS estimator assuming that $u_0 = 0$. When $|\alpha| < 1$, the FGLS estimator of γ is asymptotically efficient and $t_\gamma \Rightarrow N(0, 1)$ under the null hypothesis of $\gamma = 0$. However, the limiting distribution is different when $\alpha = 1$ and has the form of a unit root statistic as in Said and Dickey (1984); see details in Appendix of PSY (2017). So, in the case that $\alpha = 1$, in order to obtain a standard Normal limit distribution, Perron and Yabu (2009a) suggest replacing α by a "super efficient" estimator (α_S) which converges to unity at a rate greater than T defined by $\alpha_S = \alpha$ if $T^\delta\ |\alpha - 1| > d$ and 1 otherwise, for $\delta \in (0, 1)$ and $d > 0$. Therefore, when α is in the neighborhood $T^{-\delta}$ of 1, α_S takes the value of 1 and so then $t_\gamma \Rightarrow N(0, 1)$. We now consider the procedure with the model $y_t = \gamma \cos(2\pi kt/T) + u_t$, with $u_t = \alpha u_{t-1} + e_t$. Although the difference between the sine and cosine functions seems smaller, the same FGLS estimator combined with the estimator α_S using the Cochrane-Orcutt transformation, which results in $y_t - \alpha_S y_{t-1} = \gamma\ [\cos(2\pi kt/T) - \alpha_S \cos(2\pi kt(t-1)/T)] + u_t - \alpha_S u_{t-1}$ and $y_1 = \gamma \cos(2\pi kt/T) + u_1$ will not produce the same limit distribution. On the contrary, when $\alpha = 1$, $t_\gamma \Rightarrow \sigma^{-1} u_1 = \sigma^{-1}(u_0 + e_1)$ so that the limit distribution of the statistic t_γ is dominated by the initial condition and the first value of the disturbance. This problem can be remedied using the FGLS estimator proposed by Prais and Winsten (1954), which is obtained using the above Cochrane-Orcutt transformation together with $(1 - \alpha_S^2)^{1/2} y_1 = (1 - \alpha_S^2)^{1/2} \gamma \cos(2\pi kt/T) + (1 - \alpha_S^2)^{1/2} u_1$. This estimator differs from the Cochrane-Orcutt FGLS estimator only in how the initial observation is transformed. Thus, the null limit distribution of the statistic to test $\gamma = 0$ based on this alternative FGLS estimator is $t_\gamma \Rightarrow N(0, 1)$ when $\alpha = 1$. It can be shown that the use of the FGLS estimator of Prais and Winsten (1954) gives the same asymptotic distribution of the statistic t_γ when the sine function is used as a regressor. Therefore, when it comes to tests related to the nonlinear trends generated by the Fourier expansions, it is necessary to use the FGLS estimator of Prais and Winsten (1954) and the limiting distribution of the statistic is then Standard Normal in the cases $I(0)$ and $I(1)$[12]. See sections II and III and Appendices of PSY (2017) for further details.

Returning to the case with two regressors, the Wald test is obtained as follows. Be $y_t = x_t'\Psi + u_t$. In this case $x_t = (z_t', f_t')'$ with $z_t = (1, t, \ldots, t^{p_d})'$, $f_t = (\sin(2\pi kt/T), \cos(2\pi kt/T))'$, the parameters are $\Psi = (\beta', \gamma')'$, $\beta = (\beta_0, \beta_1, \ldots, \beta_{p_d})'$ and $\gamma = (\gamma_1, \gamma_2)'$, $H_0 : R\Psi = 0$ where $R = [0 : I_2]$ is a matrix $2 \times (p_d + 3)$ of restrictions. Therefore, $\Psi = (X'X)^{-}(X'y)$ is the Prais-Winsten FGLS estimator where X is a matrix $T \times (p_d + 3)$ of transformed data whose t^{th} row is given by $x_t' = (1 - \alpha_S L)x_t'$ except for $x_1' = (1 - \alpha_S^2 L)^{1/2}x_1'$. The vector $T \times 1$ y is similarly defined as $y_t = (1 - \alpha_S L)y_t$ for $t = 2\ldots, T$ and $y_1 = (1 - \alpha_S^2 L)^{1/2}y_1$. Defining $s^2 = T^{-1}\sum_{t=1}^{T} e_t^2$, the statistic is $W_\gamma = \Psi'R'[s^2R(X'X)^{-}R]^{-1}R\Psi \Rightarrow \chi^2_{(2)}$ for $I(0)/I(1)$ processes.

3.6 Hamilton (2018) Method

Hamilton (2018) criticizes the HP filter for the following reasons: (i) the HP filter introduces spurious dynamic relationships that are not supported by the underlying data generating process; (ii) the values at the end of the sample are very different from those at the middle of the sample and there is spurious dynamics; (iii) a statistical formulation of the filter typically produces values of the smoothing parameter largely different from common practice. The alternative proposed by Hamilton (2018) is a one-variable regression at date t on the four most recent values at date $t - h$ which would allow to obtain all the desired objectives of the users of the HP filter with none of its deficiencies.

Thus, Hamilton (2018) suggests an alternative concept of what we can interpret as

[12]This contrasts with the cases of a linear trend model considered in Perron and Yabu (2009a) and the break model considered in Perron and Yabu (2009b) since the asymptotic results for these models do not depend on the choice of the FGLS estimator.

the cyclical component of a possibly non-stationary series: how different is the value at date $t + h$ of the values that we could have expected observing the behavior in date t? This concept of cyclical component has several attractive features. First, as noted by Den Haan (2000), the forecast error is stationary for a wide class of non-stationary processes. Second, the primary reason why we might fail to predict the value of most macroeconomic and financial variables over a horizon of $h = 8$ quarters ahead are cyclical factors such as whether a recession occurs in the next two years and the date or timing of an economic recovery. Computing this cyclical component concept does not require knowing the stationary/non-stationary nature in order to have a correct model to predict the series. Instead, a simple prediction can be made within a restricted class: the population linear projection y_{t+h} on a constant and the four recent values of y on date t. This process exists and can be consistently estimated for a wide range of non-stationary processes.

Assume that the difference d^{th} of y_t is stationary for some d. For example, $d = 2$ would mean that the growth rate is non-stationary but the change in the growth rate is stationary. Notice that the difference d^{th} is also stationary for any series with a deterministic trend characterized by a polynomial of order d^{th}. For such a process, we can write the value of the process y_{t+h} as a linear function of initial conditions in the period t plus a stationary process. For example when $d = 1$, assuming $u_t = \Delta y_t$ we can write $y_{t+h} = y_t + w_t^{(h)}$ where the stationary component is given by $w_t^{(h)} = u_{t+1} + \cdots + u_{t+h}$. For $d = 2$ and $u_t = \Delta^2 y_t$ we have $y_{t+h} = y_t + h\Delta y_t + w_t^{(h)}$, where now $w_t^{(h)} = u_{t+h} + 2u_{t+h-1} + \cdots + hu_{t+1}$.

Proposition 4 of Hamilton (2018) states that if we estimate an OLS regression of y_{t+h} over a constant and $p = 4$ latest values of y to the date t: $y_{t+h} = \beta_0 + \beta_1 y_t + \beta_2 y_{t-1} + \beta_3 y_{t-2} + \beta_4 y_{t-3} + c_{t+h}$, the residuals, $c_{t+h} = y_{t+h} - \beta_0 - \beta_1 y_t - \beta_2 y_{t-1} - \beta_3 y_{t-2} - \beta_4 y_{t-3}$ provide a reasonable way to build the cyclical component of a large class of underlying processes. The series is stationary since the fourth differences of y_t are stationary, a goal that the HP filter attempts to achieve but is generally not achieved. Thus, while the HP filter imposes four roots, the specified regression uses only the four differences if this is the characteristic of the data. The Hamilton (2018) procedure has other advantages over the HP filter. First, any evidence that c_{t+h} predicts some other variable x_{t+h+j} represents a real skill of y to predict x more than just an artifact of the way y_t is detrended. Second, unlike the cyclic component of the HP filter, the value of c_{t+h} will be, by construction, difficult to predict from some variable in time t or before. If we find such predictability, this would mean something about the actual data-generating process, for example, that x causes (in the Granger sense) y. Third, the value of c_{t+h} is free of models and assumptions about the data. Regardless of how the data was generated, the important thing is that $(1-L)^d y_t$ is covariance stationary for some $d \leq 4$, then there is a population linear projection of y_{t+h} on $(1, y_t, y_{t-1}, y_{t-2}, y_{t-3})'$. This projection is a characteristic of the data-generating process that can be used to define what is considered the cyclical component of the process and can be consistently estimated from the data[13].

A related aspect is the choice of h. For any fixed value of h, there is a sample size T for which the results of Proposition 4 of Hamilton (2018) will hold. If we are interested in economic cycles, a horizon of two years should be the standard or reference value. In summary, for quarterly data, Hamilton (2018) suggests $p = 4$ and $h = 8$.

[13]According to Hamilton (2018), we might be tempted to use a richer model to predict y_{t+h}, like using a vector of variables instead of just the four lags or even a nonlinear relationship. However, the author argues that such refinements are unnecessary for the purpose of extracting the stationary component and have the disadvantage that when more parameters are used it can cause results in small samples that will differ from the asymptotic predictions.

4 Empirical Results

This Section is divided into two parts. The first part presents and discusses the results of the BN decomposition, the UCMN model by Perron and Wada (2009, 2016), the UCUR model by Grant and Chan (2017a), and the flexible nonlinear approximation via Fourier expansions by PSY (2017). The second part compares the business cycle estimates obtained from the 10 procedures presented in Section 3.

In the case of the G7 countries, the sample covers the period 1960Q1-2019Q4, with the exception of Canada and France, for which the sample begins in 1961Q1 and 1970Q1, respectively. In all cases, the source of information is the OECD database. In the case of the LA countries, the availability of information is scarce, limited and varies according to the country. Thus, for the cases of Mexico and Peru the sample is 1980Q1-2019Q4, for Argentina it is 1993Q1-2019Q4, for Brazil it is 1996Q1-2019Q4 and for Chile it is 1995Q1-2019Q4. In the cases of Argentina, Brazil and Peru, the source of information comes from the respective Central Banks, while for Chile and Mexico it comes from the OECD database.

4.1 Methodologies Evaluation and Selection

4.1.1 BN Decomposition

For each country, eight ARIMA(p,1,q) models are estimated with $p,\ q = 0,\ 1,\ 2$. Table 1 presents the results of the selected models using three criteria: greatest log-likelihood (LL), statistically significant coefficients and a business cycle behavior consistent with the stylized facts. The latter is discussed in further detail in Section 4.2 along with the rest of the methodologies. For most countries an ARIMA(p,1,0) model with $p = 1,\ 2$ is selected; these findings are consistent with the literature; see Nelson and Plosser (1982), Campbell and Mankiw (1987a,b, 1988) and Stock and Watson (1988). Exceptions are found for the UK and Germany, where the selected models also present a moving average (MA) component, that is, $q = 1$.

According to Table 1, the estimates of the autoregressive parameters (φ_1 and φ_2)

are statistically significant. These results show different specifications between the two groups of countries, we find that for most of the G7 countries an AR(2) model is necessary, while in the case of all LA countries an AR(1) model is selected. The sum of the autoregressive coefficients oscillates between 0.40 and 0.75 for the G7 and between 0.22 and 0.50 for LA. Values far from one imply a reduced persistence in the cycles and, consequently, a short duration. In other words, it is expected to observe high volatility and low depth in the cycles estimated under this procedure, especially for LA countries. Likewise, in no case are the characteristic roots complex, indicating the absence of pseudocyclic behavior.

Low persistence or duration can also be inferred by calculating a long-run persistence measure, denoted by $\psi(1)$[14]. For all countries, the value of $\psi(1)$ is greater than one, suggesting that a one percent shock to the trend component changes the long-run path of output by more than one percent. These results show that shocks to the trend component are more important than shocks to the cyclical component and that shocks to the trend component may have permanent effects on the output; see Nelson and Plosser (1982) and Campbell and Mankiw (1987a,b, 1988).

The results of $\psi(1)$ for the G7 countries are mostly consistent with the results of Campbell and Mankiw (1987a, 1988). We find that there are some differences in the calculated values for $\psi(1)$ for both groups of countries. While for the G7 countries the average value is 1.694, in LA countries the average value is 1.576. Although this suggests similarities in the impacts of the shock on the trend component in both groups of countries, it is important to note that the average value of LA is being raised by the estimates for Argentina and Peru, which show $\psi(1) = 1.999$ and $\psi(1) = 1.618$, respectively. For these two countries, it could be argued that given the magnitude of the recessions that both economies have experienced, specifically the crisis generated by the failure of the Convertibility plan in Argentina[15] and hyperinflation and the internal conflict in Peru[16], the effect of the shocks of

[14]Given that $\varphi\ (L)c_t = \theta\ (L)\varepsilon_t$ and given the stationarity condition, we have that $c_t = \varphi\ (L)^{-1}\theta\ (L)\varepsilon_t = \psi(L)\varepsilon_t$ which corresponds to the MA(∞) representation of c_t. Hence, $\psi(1) = \varphi\ (1)^{-1}\theta\ (1)$.

[15]This was a plan by the Argentine Currency Board that pegged the Argentine peso to the US dollar between 1991 and 2002 in an attempt to eliminate hyperinflation and stimulate economic growth. When the recession and the massive bank withdrawals started in 2000, the plan began to collapse what was given in 2002.

[16]The 1980s were characterized by the beginning and rise of terrorism that caused great destruction to

the trend component on the long-term output is greater than in the rest of the LA countries.

4.1.2 UCMN Models

Seven UCMN models are estimated for each country with one, two and three mixtures of normals in each of the components: cycle, trend and slope. Thus, we define the following models: (i) UC-C: a model with a mixture of normals in the cyclical component (ε_t); (ii) UC-N: a model with a mixture in the disturbances of the trend level (η_t); (iii) UC-P: a model with a mixture in the disturbances of the trend slope (υ_t); (iv) UC-CN: a model with mixtures in the disturbances of the cyclical component and the trend level (ε_t, η_t); (v) UC-CP: a model with mixtures in the disturbances of the cyclical component and the trend slope (ε_t, υ_t); (vi) UC-NP: a model with mixtures in the disturbances of the trend level and trend slope (η_t , v_t); and (vii) UC-CNP: a model with mixtures in the disturbances of the cyclical component, the trend level and the trend slope (ε_t, η_t, υ_t).

Furthermore, in order to increase the possibility of obtaining a global maximum in the likelihood function, each model is estimated 900 times with different starting values that are obtained from the normal distributions $N(0, 1)$, $N(0, 2)$ and $N(0, 3)$. Additionally, the recursive method of the Kalman filter requires starting values for the state vector $x_{0|0}$. Perron and Wada (2009, 2016) consider $x_{0|0} = [\tau_0, 0, 0, \beta_0]$, where the starting value of the trend, τ_0, is the first observation of the GDP and the starting value of the slope, β_0, is the first value of the output growth rate. However, since Peru shows an irregular behavior on the output at the beginning of the sample, Guillén and Rodríguez (2014) suggest that β_0 should be estimated as the simple average of the growth rate of the first four quarters of the output. We consider this suggestion for all LA countries.

Table 2 shows the estimates of the best models for the G7 and LA countries, obtained using the three criteria already mentioned in 4.1.1. The selected models are: the UC-N model for Germany and Japan; the UC-CP model for the US, Italy and Brazil; the UC-NP model for Canada and Chile; and the UC-CN model for France, the UK, Argentina,

the country, coupled with hyperinflation that began in 1989. The new government applied a stabilization program (monetary shock) in the mid-1990s to stop hyperinflation. Furthermore, in 1992 the capture of the terrorist leader took place, which implied a great blow for the defeat of terrorism.

Mexico and Peru. For the US and Italy, the selected models coincide with those chosen by Perron and Wada (2016), while for the case of Peru the model coincides with Guillén and Rodríguez (2014). In no case is a UCMN model with three mixtures selected (UC-CNP).

Table 2 indicates that almost all countries, with the exception of the US, Italy and Brazil, present a mixture of normals in η_t. Regarding the estimate σ_{η_1}, we find that it is not significant for the cases of the UK (UC-CN model), France (UC-CN) and Mexico (UC-CN model), which translates into a smooth behavior of τ_t. For the UK and France we find $\sigma_{\eta_1} = 0.001$, while for Mexico we find $\sigma_{\eta_1} = 0.000$. Since σ_{η_1} takes values close to or equal to zero, a deterministic component τ_t is obtained with occasional changes in its level, which translates into the cycle shocks (ε_t) being more important than trend shocks (η_t). Likewise, the probability of occurrence of σ_{η_1} (α_1 for Canada, Germany, Japan and Chile, and α_2 for the UK, France, Argentina, Mexico and Peru) is significant in all cases, with these probabilities being greater than 50%, which means that economies are most of the time in "normal" periods. For example, for the UK and France (both under the UC-CN model) $\alpha_2 = 91\%$ and 80%, respectively, are obtained. Another case is that of Chile (UC-NP model) which presents $\alpha_1 = 95\%$.

On the other hand, for all the countries that have a mixture of normals in η_t, the standard deviation of the high volatility regime (σ_{η_2}) is significant, as well as its probability of occurrence (α_1 for Canada, Germany, Japan and Chile, and α_2 for the UK, France, Argentina, Mexico and Peru). For example, for the case of Germany and Japan (both under the UC-N model) we obtain $\sigma_{\eta_2} = 1.893$ and 2.013 with probabilities $(1 - \alpha_1) = 17\%$ and 9%, respectively. Other examples are Argentina and Peru (both under the UC-CN model) which have $\sigma_{\eta_2} = 2.238$ and 5.733 with probabilities $(1 - \alpha_2) = 5\%$ and 11%, respectively. Therefore, in these cases, σ_{η_2} generates a stochastic trend meaning that these economies are in "atypical" periods and where shocks to the trend component are relevant compared to shocks to the cyclical component. However, the probability of occurrence of this regime is relatively low.

Table 2 indicates that for all the countries that have a mixture of normals in υ_t (US, Canada, Italy, Brazil and Chile) the standard deviations associated with a low variance regime (σ_{υ_1}) are small in magnitude, which would indicate that a relatively stable growth rate is obtained in the long run. This is consistent with the findings of Perron and Wada (2009, 2016) and Guillén and Rodríguez (2014), who indicate that, in general, σ_{υ_1} is small for most of the time and only takes large values in "atypical" periods. Table 2 shows that the probability (α_2 for all cases) of σ_{υ_1} is significant. Thus, for the US (UC-CP model), Canada (UC-NP model) and Italy (UC-CP model) we have that $\sigma_{\upsilon_1} = 0.008,\ 0.050$ and 0.000 with probabilities $\alpha_2 = 83\%$, 95% and 93%, respectively. On the other hand, for Brazil (UC-CP model) and Chile (UC-NP model) we have that σ_{υ_1} is equal to 0.184 and 0.048 with probabilities $\alpha_2 = 82\%$ and 80%, respectively. For its part, the standard deviation of υ_t associated with a high variance regime (σ_{υ_2}) is only significant for Chile (UC-NP model) which shows $\sigma_{\upsilon_2} = 0.059$ with probability of occurrence $(1 - \alpha_2) = 20\%$.

According to Table 2, most of the G7 and LA countries, with the exception of Canada, Germany, Japan and Chile, present a mixture of normals in ε_t. Thus, we obtain that σ_{ε_1} is significant and small for all countries, which suggests the existence of asymmetries in the cycles, which is consistent with the findings of Neftçi (1984), Diebold et al. (1993) and Friedman (1993). Furthermore, the probabilities associated with σ_{ε_1} (α_1 for all cases) are also significant. For example, for the US (UC-CP model) and France (UC-CN model) we have σ_{ε_1} equal to 0.143 and 0.243 with probabilities α_1 equal to 67% and 97%, respectively. Other examples are the cases of Argentina and Peru (both under the UC-CN model) that have values of σ_{ε_1} equal to 0.614 and 0.558 with probabilities α_1 of 86% and 89%, respectively. For its part, σ_{ε_2}, as well as its probability of occurrence ($1 - \alpha_1$ for all cases), are also significant for all countries. For example, for Italy (UC-CP) we obtain $\sigma_{\varepsilon_2} = 1.988$ with probability $(1 - \alpha_1) = 9\%$, while for Mexico (UC-CN) we obtain $\sigma_{\varepsilon_2} = 5.211$ with probability $(1 - \alpha_1) = 2\%$. In general, it is observed that for all the countries $\sigma_{\varepsilon_2} > \sigma_{\varepsilon_1}$, which implies that the economies are most of the time in normal periods with alterations or exceptional disturbances of great magnitude that would have a strong impact on the product in the short term and at the series level.

Table 2 also shows the estimates of the autoregressive coefficients (φ_1 and φ_2), which are statistically significant for all countries. The sum of both coefficients ranges between 0.86 and 0.98 for the G7 countries, the lowest value being for Japan and the highest for the UK. In the case of the LA countries, the sum of the coefficients is between 0.83 and

0.96, which are obtained in Peru and Argentina, respectively. The values obtained clearly contrast with the ones obtained in the BN decomposition. In the present case, the sum is higher and closer to unity, which indicates that the cycles obtained are characterized by high persistence (longer duration, lower volatility) and greater depth.

Table 2 also shows the characteristic roots implied by the AR model estimated for each country. For the cases of Italy and Japan, complex roots are obtained that indicate an average duration of the cycles of 4.97 and 4.89 years, respectively. For the cases of Mexico and Peru we obtain an average duration of 8.14 and 3.98 years, respectively[17]. Argentina shows a duration of 13.9 years which seems overestimated.

4.1.3 UCUR Models

For each country, the following models were estimated: DT, DT$-t_0$, UC0, UCUR, UCUR$-t_0$ and UCUR$-(t_0,\ t_1)$. Each model is estimated using 110,000 posterior draws followed by a burning of 10,000 draws, while 50,000 draws are performed using the importance sampling to calculate the log of the marginal likelihood (logML). Additionally, for the estimation of each model it is necessary to calculate the mean priors of τ_0, μ_0 and φ_0. The mean of the parameter τ_0 is obtained from the natural logarithm of the first value of the GDP for each country multiplied by 100. The prior value of μ_0 is obtained from the annualized quarterly growth rate of each country. Finally, to obtain the priors of φ_0, the cycle of the GDP of each country is estimated through the BK filter. In the case of specifications with breaks, the DT$-t_0$, UCUR$-t_0$ and UCUR$-(t_0,\ t_1)$ models were estimated with eight different break dates, which were obtained using two statistics. The first is Perron and Yabu (2009a,b) structural change statistic which is robust to disturbances that are $I(0)/I(1)$. The second is the endogenous two-break LM-type unit root statistic proposed by Lee and Strazicich (2003).

Table 3 shows the posterior means of the parameters φ_1, φ_2, σ_ε^2, σ_η^2 and ρ, the value of the logML, the characteristic roots associated with the autoregressive polynomial corresponding to the cycle component as well as the average duration of the respective cycles,

[17]Using the results of Guillén and Rodríguez (2014) for Peru where $\varphi_1 = 1.416$ and $\varphi_2 = -0.519$, the average duration of the cycles is of 8.45 years. Our sample includes an ad ditional 9 years (36 quarters).

for the cases in which complex roots are obtained. For the model selection process, the BF is used. For each country, the columns of Table 3 indicate, respectively, the following: the best DT$-t_0$ model, the UC0 model as a reference as it is a UC model without correlation, the UCUR model (without break), the best UCUR$-t_0$ model and best UCUR$-(t_0,\ t_1)$ model. The countries that have a higher BF in favor of the UCUR$-t_0$ and UCUR$-(t_0,\ t_1)$ models (in relation to the DT$-t_0$ model) are Peru and Japan with values of 3.2×10^8 and 4.6×10^6 respectively, while the countries with a lower BF are Canada, Chile and Mexico, where the results show values of 2.03, 3.05 and 3.10, respectively. BF values close to one may suggest that the trend estimated by the DT$-t_0$ model is similar to that estimated by the UCUR$-t_0$ models.

The results indicate that in no case is the DT model selected. In the case of the DT$-t_0$ model and for G7 countries, it is found that the best models are selected using a break around 1973. Such are the cases of the US, Canada, Germany, France, Italy and Japan. Only the UK indicates a break in 2008. In the case of the LA countries, the breaks appear on different dates associated more with domestic events: 2012 (Argentina), 2013 (Brazil and Chile), 1982 (Mexico) and 1992 (Peru). However, the results support the extensive empirical evidence in favor of break-correlated UC models (UCUR$-t_0$); see Morley et al. (2003), Morley and Piger (2012), Luo and Startz (2014), and Grant and Chan (2017a). This model outperforms the DT$-t_0$, UC0 and UCUR models although there are some nuances. According to the BF, the selected models (for G7 countries) are: UCUR-73 (US), UCUR-75 (Canada) and UCUR-73 (UK). A model with two breaks is selected for Germany (UCUR-(73, 91)), France (UCUR-(73, 08)), Italy (UCUR-(74, 07)) and Japan (UCUR-(73, 89)). In the cases of US, Canada and UK, the break date is related to the oil price shock. In the cases of France and Italy the break dates suggest the oil shock and the financial crisis of 2008 which seems reasonable. In the case of Germany, dates breaks dates are associated with the oil shock and the unification of Germany are also identified. In the case of the LA countries, the selected models are: UCUR-01 (Argentina), UCUR-82 (Mexico) and UCUR-92 (Peru). In the cases of Brazil and Chile, the BF suggests a model with two breaks: UCUR-(09, 13) and UCUR-(98, 15), respectively. Again, the selected break points are related to domestic event in the region. In the case of Argentina, it coincides with the 2001 crisis as a result of the failure of the Convertibility plan; Brazil with the political crisis of 2014 together with the drop in metal prices; Chile with the Asian crisis in 1998; Mexico with the external debt crisis in 1982; and Peru with the period of hyperinflation and internal conflict in 1992.

According to Table 3, the sum of the posterior means of the autoregressive coefficients (φ_1 and φ_2) is between 0.46 and 0.69 for Chile, Mexico, Brazil and all the G7 countries. This contrasts with the results found using the Perron and Wada (2009, 2016) procedure and is comparable to the results obtained using the BN decomposition. This finding translates into a high volatility of the economic cycles estimated under this procedure. There are two exceptional cases: Argentina and Peru with models UCUR-01 and UCUR-92, respectively. In these cases, it is found that the sum of the posterior means of φ_1 and φ_2 are 0.96 and 0.87 which is more in line with the findings obtained using the Perron and Wada (2009, 2016) method where valuesof 0.96 and 0.83 were obtained for Argentina and Peru, respectively. Consequently, in these cases the cyclical component shows high persistence compared to the rest of the countries. The difference in the findings and behavior of the cases of Argentina and Peru can be explained by the extreme and prolonged fluctuations of the output of both countries as a consequence of crises that occurred in the first years of the samples of said countries. Due to the scarce and varied availability of data, the samples of Brazil, Chile and Mexico do not present recessive periods at the beginning of their samples that are comparable with those found in Argentina and Peru[18]. These countries show recessive periods (Tequila, financial crises, among others) but they do not have the magnitude of the events that occurred in Argentina and Peru. Our hypothesis at this point is that if all the major crises in LA countries were included, we would observe behaviors and parameters very similar to those found for Argentina and Peru.

The results of the estimated posterior means of the variances (see Table 3) for Brazil, Chile, Mexico and all the G7 countries, reveal that the trend shocks (η_t) are more important than the shocks that affect the cycle (ε_t) in explaining the variation of the output. This implies that during recessive periods, shocks η_t affect the long-term level of GDP in

[18]Recall that the samples of Brazil and Chile begin in 1996Q1 and 1995Q1, respectively. These countries went through severe domestic crises in the 1980s and 1970s, respectively.

these countries, an opposite result to that found in the UCMN model by Perron and Wada (2009, 2016). This is confirmed in the posterior densities of the variance ratio $\sigma_\eta^2/\sigma_\varepsilon^2$, which are found in a region greater than one[19]. In the cases of Argentina and Peru, the results are opposite in the sense that the shocks ε_t are more important than the shocks η_t in explaining output fluctuations. In this case, the region of posterior densities of the ratio of variances $\sigma_\eta^2/\sigma_\varepsilon^2$ are below one. The greater importance of the shocks ε_t in Argentina and Peru can be explained by the characteristics of the crises in these countries, which, although they generated permanent changes in productivity, had a greater effect on the cycle. In the case of Argentina we find the economic crisis that occurred between 1998 and 2002, associated with the loss of competitiveness as a result of the fixed exchange rate regime between the peso and the US dollar, known as the Convertibility plan, as well as its collapse. In the case of Peru, recessions are associated with the economic crisis in the late 1980s and early 1990s, characterized by hyperinflation. Likewise, it is important to mention the disaster that occurred at the beginning of the 1980s related to the destruction of the north of the country due to the phenomenon of the coastal named El Niño.

The estimated posterior means of the parameter ρ (Table 3) for Brazil, Chile, Mexico and all the G7 countries are negative and large in magnitude, which is close to the $\rho = -1$ correlation imposed by the BN decomposition. These results would indicate that the shocks that affect the trend are not the same shocks that affect the cycle and that, in addition, there is an empirical relevance of allowing a non-zero correlation between the trend and cyclical shocks, which reaffirms what was found in Morley et al. (2003) and Grant and Chan (2017a). Likewise, it is possible to observe that the posterior densities of ρ are close to -1, which reinforces the importance of trend shocks in economies. Stock and Watson (1988) consider that these shocks related to the trend generate long-term variations in GDP, since they can change its trajectory and generate fluctuations in the short term, where the cycle constantly adjusts to said changes. An interesting aspect is the fact that the estimated posterior mean of ρ decreases for all countries as one or two breaks in the trend are introduced for the UCUR model, while its statistical significance

[19] The posterior densities calculated for the variance ratio $\sigma_\eta^2/\sigma_\varepsilon^2$ are available upon request.

also decreases. This implies that the use of more breaks reduces the correlation between shocks to the trend and to the cycle, making transitory shocks more important compared to permanent shocks. This coincides with the arguments of Perron (1989) in opposition to what was proposed by Nelson and Plosser (1982).

In the cases of Argentina and Peru, opposite results are found where the estimated posterior means of ρ take values of 0.62 and 0.43, respectively, while the posterior densities of this parameter are close to one, having little concentration close to zero. Our results contradict the conclusions of Morley et al. (2003), because these authors consider that when the non-correlation restriction between cycle and the trend disturbances is relaxed, the UC model leads to a decomposition and an univariate representation similar to that of the BN decomposition, that is, $\rho \approx -1$. In contrast, our results for Argentina and Peru indicate that when this restriction is relaxed, a positive and large magnitude correlation can be found between both disturbances[20].

From the complex characteristic roots of the best UCUR models, it is found that, with the exception of the US, UK and Peru, business cycles have an average duration of 2.31 years, with the lowest value being that of Mexico (1.91 years) and the highest value is that of Germany (4.29 years). These values are low and are consistent with the low persistence found using the sum of the posterior means of the autoregressive coefficients. However, in the case of Peru, the duration obtained is 6.02 years.

4.1.4 PSY (2017)

Prior to estimating the cycle, we calculate the Wald-type statistical test robust to the presence of $I(0)/I(1)$ components in the noise function, in which the null hypothesis is the non-existence of nonlinearities. The Wald test is presented in Table 4 and the results indicate rejection of the null hypothesis for all countries; that is, there is evidence of non-linear trends in all cases. The results indicate that the LA countries present a greater rejection of the null hypothesis compared to the G7 countries, with the exception of Japan. This seems to suggest that, on average, the LA countries present stronger evidence of nonlinearities in the trend than the G7 countries, which is related to the greater incidence and

[20]The posterior densities calculated for ρ are available upon request.

depth of recessive processes in these countries. The exceptional case of Japan can be explained by the higher incidence of crisis in the periods after the 2008 financial crisis, where three recessive episodes were recorded between 2010 and 2015 as a result of lower population growth. Table 4 also shows three values of the α coefficient of the estimated AR process for the noise function: OLS, Median Unbiased, and Super Unbiased. In all cases, the truncation is done with $\alpha = 1$, which allows equal critical values for the Wald statistic for processes $I(0)$ and $I(1)$ to be obtained.

In the case of the G7 countries, with the exception of Canada, the UK and, to a lesser extent, Japan, the statistical significance of the coefficients indicates that in order to estimate the non-linear trend of these economies, it is only necessary to include parameters with a lower number of frequencies such as $\cos(2\pi t/T)$ and $\cos(4\pi t/T)$; see Table 4. These frequencies imply less recurrent undulation, which results in a smoother estimate of the trend. This behavior is found in the US, Germany, France and Italy, where only two frequencies are necessary to estimate the non-linear trend component. This is in line with the stylized facts, since the observed GDP in these countries presents a stable periodic behavior without a greater incidence of crisis.

Unlike the above, for the LA countries, with the exception of Brazil, to estimate the non-linear trend, parameters with a greater number of frequencies must be included; see Table 4. This greater number of frequencies translates into more recurrent undulations, which generates an estimate of the trend with a greater number of nonlinearities. We find statistical significance for frequencies such as $\cos(10\pi t/T)$ in Chile, Mexico and Peru, and for $\cos(6\pi t/T)$ in Argentina, Chile and Peru.

4.2 Estimated Business Cycle Evaluation and Comparison

We have organized the results into four blocks: (i) the BN decomposition and the UCUR model of Grant and Chan (2017a); (ii) the statistical filters of HP, BK, CF and KMW; (iii) the UCMN model of Perron and Wada (2009, 2016), the method of flexible nonlinear trends via Fourier expansions of PSY (2017) and the Hamilton (2018) method; and (iv) the DT$-t_0$ model from Grant and Chan (2017a). This division corresponds to the different behavior of the cycles in the four groups mentioned. The estimates are illustrated in Figure 1, where gray bars denote periods of recession. These periods have been previously identified by the Economic Cycle Research Institute (ECRI) for the cases of Brazil, Mexico and the G7, with the exception of Canada, which follows the dates established by the Central Bank of Canada. For Argentina, Chile and Peru, recessions were identified as those in which negative GDP growth rates are observed for two or more consecutive quarters.

4.2.1 BN Decomposition and UCUR Models

The business cycle estimates under these two methods are shown in the first and second panels of Figure 1. The simple correlation coefficient between both methods ranges between 0.61 and 0.79 for the cases of the US, UK and Canada. For France, Italy, Germany and Japan this relationship ranges between -0.42 and 0.45, respectively. For the cases of LA, the simple correlation coefficient ranges between -0.16 and 0.27, being the lowest in the case of Argentina and the highest in the case of Chile. This low correlation between methods for both groups of countries is consistent with what was found in 4.1.1 and 4.1.3 and is also a consequence of the far from unity sum of the estimates of the autoregressive parameters, especially in the case of the BN decomposition. Despite the low correlation between the cycles from both methods, all the countries show volatile business cycles that are small in magnitude (compared to the other methods; see below). This does not allow for an adequate identification of recession and expansion periods.

In the case of the G7 countries, an important aspect found is that both models are not consistent with the gray bars that denote recessions. In some cases, the methods suggest an expansion when the gray bars specify a drop in output. For example, for the US, both models estimate an expansion that reaches a level of 0.9% during the 2001 crisis, characterized by the increase in unemployment. The same happens during the financial crisis where both methods indicate a growth of 2.9%. Another example is that of the UK, which in 1980 experienced a crisis as a result of the depreciation of its currency, high interest rates and a contractionary fiscal policy, where both models estimate an expansion that reaches a level of 1.9%. Another case is that of Italy during the financial crisis, where the BN decomposition estimates an expansion that reaches a level of 5%. Also, in some cases the models estimate downturns in the cycle starting between one and four quarters after what the gray bars indicate. For example, Canada experienced a fall in output in 1981 characterized by a reduction in domestic demand. In this case, the economic cycles estimated under the UCUR model and the BN decomposition show a fall in the cycle that occurs three and four quarters after what the Central Bank of Canada identifies. In the case of France and Italy during the 1973 oil crisis, the fall in the cycle under the BN decomposition occurs one and three quarters after what the ECRI identifies, respectively. It is important to mention that although in some recessions and expansions the UCUR model seems to better approximate recession periods, when taking into account the confidence bands (not reported)[21] said identification loses consistency, implying a null or insignificant output gap.

Our results are similar to the findings of Marczak and Beissinger (2012) for Germany, who uses the BN decomposition and finds volatile business cycles. In the case of France, we found differences in the depth of the cycles with respect to the findings of Doz et al. (1995), who estimate an ARIMA(2,1,0) model. In the case of Canada, we find different results from those of Cayen and van Norden (2005), who use the BN decomposition and the UC0 model; however, this difference is due to the fact that these authors use the output in real time and a small sample. On the other hand, for the US, our results under the UCUR model are similar to the findings of Morley and Piger (2012) and Grant and Chan (2017a). For the case of Italy, we obtain cycles (under the UCUR model) that are small in magnitude compared to Zizza (2006), who uses annual data.

For the LA countries, it is also not possible to clearly identify the recession periods under both methods. Once again, business cycles are volatile and in some cases, expansions are found instead of recessions; however, a slightly higher persistence is observed in the UCUR model. For example, Brazil during 2014 and 2016 experienced a crisis characterized by high levels of unemployment, high inflation and contractions in most of the productive sectors; nonetheless, the BN decomposition and the UCUR model estimate an expansion that reaches a level of 0.9% and 0.8%, respectively. Another case is that of Mexico during the 2008 financial crisis, where the BN decomposition estimates a down-

[21]Since there are no confidence bands for the other methods, these bands are not reported for the UCUR model in order to standardize the results in the Figures. Results of confidence bands are available upon request.

turn in the cycle that occurs two quarters after what the gray bar specifies. It is found that the UCUR model obtains better results identifying the Tequila crisis (1994) in Mexico, the Asian and Russian crisis in Chile and the financial crisis in both countries. It is important to mention that these results weaken when considering the confidence bands in the UCUR model.

For the case of Mexico, Galindo et al. (2019) use the BN decomposition and find persistent cycles that are different from our results, which may be due to the fact that the authors use annual data. On the other hand, under the UCUR model for Mexico, our results are consistent with those estimated by Faal (2005), who also obtains volatile and small cycles. On the other hand, for Brazil, Tiecher et al. (2010) use a UC0 model for the period between 1996 and 2008, and obtain a cycle that, unlike our results, is characterized by being recessive most of the time.

For the cases of Argentina and Peru, the cycles estimated by both methods contradict each other, as well as with respect to the other countries. On one hand, it is found that in some periods the UCUR model suggests a recession in the economic cycle, while the BN decomposition suggests an expansion. In the case of Peru during the 1990 crisis, the UCUR model estimates a recession that reaches a level of -13.8% in 1990Q3, while the BN decomposition estimates an expansion of 8.5% in 1990Q3. This is clearly contradictory and in favor of the UCUR model given that in that quarter Peru adopted a monetary and fiscal stabilization plan that meant one of the largest drops in output in the entire history of the country. In the case of Argentina, during the 2001 crisis as a result of the failure of the Convertibility Plan, the UCUR model estimates a recession of -13.9% in 2001Q4, while the BN decomposition estimates an expansion of 6.3% on the same date. Likewise, it is important to remember that for these two countries, under the UCUR model, we find greater depth and persistence in the economic cycles, which contrasts with the other LA countries and the G7.

Our results for Argentina under the BN decomposition are different from the findings of Trajtenberg (2004) and Rabanal and Baronio (2010), who use an ARIMA(2,1,2) model and an ARMA(1,1) model in first differences, respectively. However, this difference is due to the fact that these authors estimate the cycle for a much shorter sample and that

Rabanal and Baronio (2010) use annual data. With respect to the UCUR model, we find very similar results in terms of the depth of the economic cycles, with those estimated by Florián and Martínez (2019) for the Peruvian case.

The opposite behavior of the economic cycles estimated between both methods is explained by the correlation parameter ρ. In the UCUR model, the posterior value of ρ can take values between -1 and 1, while the BN decomposition imposes $\rho = -1$. Contrary to other countries, in the cases of Argentina and Peru our estimates suggest a positive posterior value of ρ. Thus, these two countries show opposite results to Morley et al. (2003). But in the rest of the countries (G7 and LA) we find a negative correlation between the shocks of η_t and ε_t , which is consistent with Morley et al. (2003) arguments.

Except for the cases of Argentina and Peru, the results mentioned show cycles of low persistence which are difficult to reconcile with the stylized facts and the gray bars in Figure 1. In the case of the G7 countries, the sum of coefficients of the autoregressive values are between 0.41 (Italy and Japan) and 0.74 (Germany), which are small values far from one. In the case of the LA countries, these values are between 0.41 and 0.45, which implies less persistent economic cycles.

4.2.2 HP, BK, CF and KMW Filters

The estimates of the economic cycles under these filters are presented between the third and sixth panels of Figure 1. The estimates of the cycles are characterized by a greater persistence and depth, allowing for a better identification of the cycles compared to the BN decomposition and the UCUR models from Grant and Chan (2017a). The HP, BK and CF filters show a simple correlation that ranges between 0.91 and 0.99 for the G7 countries and between 0.92 and 0.98 for the LA countries. This indicates a high similarity between the estimates of these filters. However, the correlation of these three filters with the KMW filter ranges between 0.43 and 0.69 for the G7 countries, with the lowest correlation in the case of Japan between the BK and KMW filters and the highest correlation in the case of Italy between HP and KMW filters. In the case of the LA countries, the correlation between the three filters and the KMW filter ranges between 0.39 and 0.68, with the lowest correlation for Brazil between the BK and KMW filters and the highest correlation between the CF and KMW filters for the case of Mexico.

In the case of the G7 countries, it is observed that the identification of recessions is generally consistent with the gray bars. In addition, we find a similar behavior in the depth of recessions. For example, all filters identify the recessive period in the early 1980s in the UK, showing an (average) fall in output of -2.2% in 1981Q1. Another case is that of Italy during the 1980 crisis, where an (average) level of -1.9% was reached in 1983Q4. However, in most cases under the KMW filter, downturns in the cycle occur three to four quarters earlier than the other filters estimate. For example, this is the case for all the G7 countries during the 1973 oil crisis and the 2008 financial crisis. Likewise, we find evidence that the expansionary periods estimated by the KMW filter are smaller than those estimated by the other filters. For example, in the case of the US, prior to the oil crisis, there is an expansion that reaches a level of 1.23% in 1973Q2 with the KMW filter, while the other filters reached a level of 3.4% at the same time. This can also be observed in the case of France, prior to the financial crisis of 2008, where the KMW filter indicates a drop of -0.12% in 2008Q1, contrasting with the 2.3% expansion estimated by the other filters.

With respect to the HP filter, we find economic cycles similar to those of Blackburn and Ravn (1992) for the case of the UK and to those of Brandner and Neusser (1992) and Marczak and Beissinger (2012) for the case of Germany. For the case of Italy, our results are very similar to those of Busetti and Caivano (2013), although we find deeper cycles in magnitude compared to Zizza (2006), who uses annual data. We obtain great similarity with the findings of Gallego and Johnson (2005) using the HP and CF filters for all G7 countries, despite the fact that these authors use a smaller sample. On the other hand, we obtain different results from Rünstler and Vlekke (2017) using the CF filter, who estimate the economic cycle for the G7 countries, with the exception of Canada and Japan. This difference is due to the fact that these authors use annual data.

In the case of LA, consistency of the estimated business cycles with recession periods is also observed. For example, in the case of Mexico, all the filters manage to identify the recessive periods associated with the Tequila crisis (1994) and in Peru during the economic crisis at the beginning of the 1990s. However, as in the case of the G7 countries, there are several cases in which the KMW filter estimates drops in the cycle that occur earlier than estimated by the other filters. For example, in the case of Mexico during the Tequila crisis (1994) and in Argentina during the financial crisis of 2008. Likewise, we find that the expansive periods estimated by the KMW filter are of lesser amplitude and in some cases are at levels close to zero, which suggests that the output is close to the level of trend output. This is observed in Brazil prior to the 2013-2016 crisis, where the cycle is at a level of -0.5% in 2014Q1 under the KMW filter and at a level of 3% under the other filters in the same period. Another example is that of Chile prior to the 2008 financial crisis, where the economic cycle estimated by the KMW filter reaches a level of 1.1% in 2008Q1, while the other filters reach a level of 3.4% in the same period.

Regarding the HP filter, our results are consistent with the findings of Trajtenberg (2004) and Krysa and Lanteri (2018) for the case of Argentina, although they differ with the estimates of Rabanal and Baronio (2010), who estimate very persistent cycles; however, this difference may be due to the fact that these authors use annual data. For the case of Brazil, we find consistency with the results of Tiecher et al. (2010) and less persistent cycles compared to Araujo et al. (2008), who use annual data. For Chile, we find similarities with the results of Chumacero and Gallego (2002) despite the fact that these authors use a very short sample. For Peru, our results differ from those of Jiménez (1997) and Dancourt and Jiménez (2009) who use information in annual frequency for a shorter period. A similar aspect occurs with Jiménez (2016) who estimates cycles that are much more persistent and deeper in magnitude, possibly due to the use of annual data; see also Jimenez (2011). With respect to the BK filter, we obtain the same results as Catalán and Romero (2018) for the case of Mexico, who also find a great similarity with the estimates under the HP filter. For the case of Peru, using the BK filter, we find consistency with the results of Castillo et al. (2006).

Finally, it is important to note that the KMW filter allows cycles to be obtained that are quite different from those obtained using the BN decomposition. The evidence suggests that the KMW filter, although it does not adequately estimate expansions in most cases, is capable of estimating recessions that mostly coincide with the gray bars and that are similar to the recessions estimated by the other traditional filters (HP, BK and CF) both in depth and duration. This is because, as Kamber et al. (2008), the BN decomposition imposes a high signal-to-noise ratio (the sum of the autoregressive coefficients is small and far from unity), obtaining a cycle with low persistence. The simple solution of Kamber et al. (2008), which is to use a smaller signal-to-noise ratio, shows that the result is adequate since we can find more persistent and deeper cycles consistent with the recession episodes indicated by the gray bars.

The estimation of an AR(2) model for the cyclical components extracted by the four filters indicates the following: (i) in the case of the HP filter, the sum of the estimated autoregressive coefficients is between 0.754 (Italy) and 0.841 (France). These are higher values than those found in the previous group of procedures. The average cycle lengths are 10.95 years (Canada), 10.89 years (France), and 6.54 years (Italy). In the case of the LA countries, the persistence of the cycles is between 0.701 (Brazil) and 0.83 (Argentina), values that are also higher than those obtained by the BN decomposition and the UCUR models. The average duration is 4.69 years (Argentina), 5.73 years (Brazil), 5.55 years (Chile), 3.89 years (Mexico) and 3.38 years (Peru); (ii) in the case of the BK filter, the persistence is between 0.81 (Italy) and 0.87 (US), with the average durations being 3.83 years for Japan and 4.31 years for the US and the UK. In the case of LA, we find persistences values of 0.79 for Peru and 0.85 for Argentina. Some average durations are 3.34 years for Peru and 4.09 years for Argentina; (iii) in the case of the CF filter, the inertia of the cycles is between 0.781 (Italy) and 0.841 (US and UK). Thus, we have average durations of 3.26 years for Italy and 3.84 and 3.85 years for the US and UK, respectively. In the case of LA, the persistence values range between 0.731 (Peru) and 0.82 (Argentina). The average duration is 2.91 years for Peru and 3.55 years for Argentina; values relatively lower than those implied by the HP and BK filters; (iv) for the KMW filter, the persistence values are between 0.787 (Germany) and 0.881-0.900 (US and UK). Thus, we have average durations of 21.6 years for Canada and 5.6 years for Italy, the first case being a clear overestimation. In the case of LA, the persistence of the cycles is between 0.74 (Peru) and 0.86 (Argentina), whose average durations are 3.01 years and 4.31 years, respectively.

In general, it can be seen that the persistence values are very similar between the four filters and higher than those obtained by the BN decomposition and the UCUR models. This explains the longer duration of the cycles and their more adequate consistency with the gray bars and the stylized facts of the different countries.

4.2.3 UCMN Model and PSY (2017) and Hamilton (2018) Methods

The estimates of the business cycle under these three methods are between the seventh and ninth panel of Figure 1. According to the results, we find business cycles with high persistence (low volatility) and depth, which allows for a better identification of recessions and expansions. These features are attributed to the greater importance of cycle shocks on output compared to trend shocks. This allows us to differentiate the results from the estimates of the BN decomposition and UCUR models, although maintaining a high similarity with the statistical filters. Although the three methods share these characteristics, differences are found in the levels of correlation. The simple correlation between the estimated output gaps for the G7 countries ranges between 0.39 and 0.84, with the lowest coefficient found in the case of France between the UCMN model and the PSY (2017) method and the highest relationship for the case of France and Italy between the UCMN model and the Hamilton (2018) method. Furthermore, for this group of countries we obtain an average correlation of 0.60. For LA countries, the simple correlation ranges between 0.38 and 0.92, with the lowest correlation for the case of Chile between the UCMN model and the PSY (2017) method and the highest correlation for the case of Argentina between the UCMN model and the PSY (2017) method. For this group of countries, we obtain an average correlation of 0.57.

In the case of the G7 countries, an important feature is that the estimates are generally consistent with the gray bars denoting recessions. For example, for the US, the methods identify the recessive period at the beginning of the 1980s, where the cycle reaches a level between -6% and -8% in 1982Q3 as a consequence of the increase in inflation. Other cases are those of Japan at the beginning of the 1990s during the crisis generated by the financial and real estate bubble and France at the beginning of the 2000s as a result of the bursting of the *dotcom* bubble generated by the speculation around internet companies. In the estimates of the G7 countries, the methods allow identifying the oil crisis of 1973 and the financial crisis of 2008, showing a drop in the cycle that ranges between -3% and -7% depending on the country considered. However, there are cases such as Germany where except for the Hamilton (2018) method, the methods estimate an expansion in the period prior to unification in which there was a recession in East Germany as a result of the collapse of the Soviet Union and in Japan where, except for the UCMN model, none of the methods estimates a recession during the economic crisis of 2014, characterized by a drop in consumption and productivity. Likewise, the fluctuations in the cycle under the method of Hamilton (2018) occur earlier than what is estimated by the other models. This can be seen, for example, in the case of the UK and Canada in the early 1990s and in France during the financial crisis of 2008.

With the exception of the UK and Germany, the persistence (amplitude) of the economic cycles estimated by the UCMN models are consistent with the findings of Perron and Wada (2009, 2016), despite the fact that we used a larger number of observations and that the best models for each country are different from those identified by the authors. However, it is important to mention that we coincide with the selection of models for the cases of the US (UC-CP) and Italy (UC-CP).

In the case of the LA countries, it is found that the economic cycles are also consistent with respect to the gray bars that denote recessions. As in the G7 countries, the cycles show high persistence. For example, in Brazil the methods adequately identify the recessive period during 2014 and 2016. Another case is that of Mexico during the Tequila crisis (1994), which was characterized by a contraction in the output generated by the devaluation of the Mexican currency (peso). In addition, there is a correct estimate of the recession in the case of Argentina during the 1998-2002 crisis, characterized by the loss of competitiveness and the collapse of the Convertibility plan; and for Peru during the economic crisis at the beginning of the 1990s, as a consequence of hyperinflation and the internal conflict. Other examples are those of Brazil, Chile and Mexico during the financial crisis. However, there are cases where recessions are not adequately identified under these methods. For example, for Argentina, the UCMN model estimates an expansion instead of a recession during the 2008 financial crisis, while the PSY (2017) method presents a greater depth in recessions and expansions throughout the entire sample, compared to the other methods.

For the LA countries, our observations regarding the UCMN model are consistent with what was found by Guillén and Rodríguez (2014) for the Peruvian case, who also select the UC-CN model and conclude that there is a similarity with the HP filter throughout the entire sample. In the case of the other LA countries, there is no literature to compare with since the UCMN models, the PSY (2017) method and the Hamilton (2018) method are recent methodologies, for which there are currently no empirical applications.

Although the Hamilton (2018) method manages to identify recessive periods, it is observed that in most cases the estimated business cycle has a greater depth than the UCMN model and the PSY (2017) method. For example, in the cases of the UK and Canada at the beginning of the 1990s, the Hamilton (2018) method estimates cycles of greater depth than the other methods, finding levels of -6.5% in the period 1992Q2 with respect to values around -2.3% for the UK and -8.36% in the period 1992Q1 with respect to values around -3% for Canada. This is also observed in the LA countries, as is the case of Peru, where a recession of up to -30% is estimated in 1989Q3 during the hyperinflation crisis and the internal conflict, contrasting with the other methods that reach a rate between -13% and -15%. The results found using the Hamilton (2018) method are consistent with the arguments of Schüler (2021) and Donayre (2021), who find that this methodology has certain disadvantages, highlighting the following: (i) introduction of false positive deviations in the trend; (ii) overestimation of the amplitude of recessions and expansions; and (iii) overestimation of the average duration of expansions and recessions. In short, the authors consider that this method can fail in the decomposition of series that have a non-linear behavior, as is the case in all the countries in the sample and in particular the LA countries; see Table 4 with the results of non-linear trends from PSY (2017)[22].

When estimating an AR(2) model for the cycle estimated by PSY (2017) for the G7 countries, the persistence values are between 0.84 (Canada) and 0.94 (US and France).

[22]In addition to the OLS method, Hamilton (2018) proposes a method that assumes that the series follows a random walk behavior. Despite the positive arguments of Hamilton (2018), we find shortcomings in the estimated business cycles regarding the identification of recessions using this alternative. The results of this method are available upon request.

In the case of the LA countries, these values are between 0.64 (Chile) and 0.98 (Argentina). The average durations are 2.83 years for Peru and 3.21 years for Mexico. In the case of the Hamilton (2018) method, the persistence values are between 0.86 (Germany) and 0.891 (Japan), while for LA, these values are between 0.86 (Peru) and 0.88 (Argentina). It is important to remember that in the case of the UCMN model (see Section 1.1.2) the persistence is between 0.85 (Japan) and 0.98 (UK). In the case of LA countries, these values are between 0.83 (Peru) and 0.96 (Argentina). Likewise, the average durations are 4.97 years for Italy and 4.89 years for Japan. In the case of Argentina, Mexico and Peru, the average durations are 13.9, 8.14 and 3.98 years, respectively.

In general, it can be seen that the persistence values are similar and slightly higher than those obtained in the four statistical filters and it is evident that they are higher than those obtained by the BN decomposition and the UCUR model. This explains the longer duration of the cycles and their more adequate consistency with the gray bars and the stylized facts of the different countries.

4.2.4 DT Model with Break: Back to Simplicity

According to Table 3 and the BF, the DT$-t_0$ models are not selected. Despite this, we consider this model due to the simplicity of its estimation; recall that before the influence of Nelson and Plosser (1982), and still today, it is simple to estimate the cycle by fitting a linear trend with a break at some point in time. Another argument is Perron (1989) where the existence of breaks is the only source of permanent shocks in the output, all the rest being transitory shocks; that is, the cyclical component acquires greater importance compared to the trend component.

The DT$-t_0$ models selected for both groups of countries share the breaks chosen for the UCUR$-t_0$ models, with the exception of the UK, Argentina and Chile; see Table 3 and Section 4.1.3. In the case of the G7 countries, the breaks coincide with global recessive processes such as the 1973 oil crisis and the 2008 financial crisis. In the case of the LA countries, the breaks are mainly associated with local crisis episodes. In the case of Argentina, the break chosen coincides with the crisis of 2012 generated by the increase of inflation and restrictions in the foreign exchange market, Brazil with the crisis of 2014 generated by an internal political crisis and by the fall in prices of metals, Chile with the drop in metal prices that began in 2012, Mexico with the foreign debt crisis in 1982, and Peru with the period of hyperinflation and internal conflict in 1992.

Table 3 shows that the sum of the posterior means of the estimates of the autoregressive parameters (φ_1 and φ_2) ranges between 0.97 and 0.99 for the G7 countries, being the lowest sum for the UK and Germany, and the highest for Japan. For LA countries, this sum ranges between 0.91 and 0.96, being the lowest in the case of Peru and the highest in the case of Argentina. These values are the highest compared to any of the other nine procedures. This sum close to unity for both groups of countries implies that the cycles are highly persistent, which in several cases allows a good identification of expansions and recessions.

The simple correlation coefficient between the DT$-t_0$ model and the procedures of the first group of methods (Section 4.2.1) is only high for the cases of Argentina (0.87) and Peru (0.89) under the UCUR model, while for the rest of the cases there is a low correlation. Regarding the simple correlation between the DT$-t_0$ model and the statistical filters, it is only high in the cases of Argentina (0.78), Brazil (0.74) and Chile (0.83) under the BK filter and Chile (0.79) under the HP filter, while for the rest of the cases this relationship is less than 0.74. The simple correlation coefficient between the DT$-t_0$ model and the UCMN model is relatively high for the cases of the UK (0.82), Canada (0.73), Argentina (0.81) and Chile (0.71), while for the rest of the countries and for the rest of the methods this relationship is around 0.67.

Although a high correlation is not obtained for all countries between the DT$-t_0$ model and the procedures of the third group, we make a comparison with these methods, given that they adjust better to periods of recession. In the case of the G7 countries, it is found that the estimates are consistent with some recessive periods. For example, in Canada the method adequately identifies the crisis at the beginning of 1980 caused by the energy crisis of 1979, where a level of -6.12% is reached, which is similar to that found by the UCMN model and the PSY (2017) method. Other cases are those of the US during the 1973 oil crisis and that of Germany during the 2008 financial crisis. However, we find cases where the DT$-t_0$ model does not coincide with the gray bars that indicate recession and in some cases, it estimates expansions instead of recessions. For example, in the US during the 2008 financial crisis and the UK during the 1973 oil crisis, the DT$-t_0$ model estimates drop in the cycle that do not reach negative values. In the cases of France, Italy and Japan starting from the oil crisis of 1973, a period in which a break in the trend is introduced for these countries, the method estimates a large-scale expansion that extends until periods after the financial crisis of 2008 and reaches maximum levels of 9.44%, 16.43% and 29.97%, respectively.

In the case of the LA countries, an important characteristic is that, unlike the G7 countries, in some cases recessions are better identified. For example, in Mexico the Tequila crisis of 1994 is identified, which reaches a level of -9.78%, similar to that obtained under the UCMN model and Hamilton (2018) method. Other cases are those of Argentina during the 2001 crisis generated by the failure of the Convertibility plan and Chile during the 2008 financial crisis. However, we also find cases in which the DT$-t_0$ model fails to identify recessive periods. This is observed, for example, in Mexico during the financial crisis, where the DT$-t_0$ model estimates an expansion, as well as in Peru during the crisis generated by the 1983 El Niño phenomenon, where the model fails to identify the recessive period. A potential explanation for these inconsistencies is that they may be caused by the choice of the break or the presence of more breaks. In this case, the PSY (2017) method is advisable.

5 Conclusions

There is a wide literature on empirical applications of trend-cycle output decomposition methods. However, few works make a comparison between several of these methods as well as for a large sample of countries. We have also found that the literature has proposed new and recent methodological alternatives to decompose the output. To our knowledge, these procedures have not yet been applied empirically or compared with approaches already established in the literature. This document contributes to this discussion using ten methods and a group of heterogeneous countries (G7 and LA).

The questions raised in the Introduction Section have been answered throughout this document and that we summarize below. In general, we find a difference in the depth of the business cycles estimated for Argentina and Peru compared to the rest of the countries. While expansions and recessions reach in general levels between 5% and -5% for most countries, in the cases of Argentina and Peru the economic cycles estimated by almost all procedures, with the exception of the BN decomposition, show high persistence and depth reaching levels between 20% and -30%. This may be due to the severe domestic economic crises that took place during the first years of the samples, which is not possible to observe in the other LA countries due to the limitations on the availability of the information. On one hand, Argentina had a crisis generated by the failure of the Convertibility plan and, on the other hand, Peru had a prolonged period of hyperinflation and internal conflict. Although the rest of the countries had local recessive periods, such as the Tequila crisis in Mexico, the bursting of the technological bubble in Japan and the dissolution of the Soviet Union affecting East Germany, and international crisis processes such as the oil crisis of 1973, the Asian crisis of 1997, the Russian crisis of 1998, and the financial crisis of 2008, these were episodes of little duration and depth.

On the other hand, our results indicate that the BN decomposition and the UCUR model estimate cycles of low persistence that do not allow rigorous and adequate identification of recession periods. Statistical filters better identify expansions and recessions, although band-pass filters estimate cycles with smooth behavior, while the KMW filter overestimates expansion periods. Therefore, the best trend-cycle decomposition procedures are the following: the UCMN model by Perron and Wada (2009, 2016), the PSY (2017) method and the Hamilton (2018) method. These methods estimate high persistence business cycles compared to the other procedures, and better approximate the recession periods identified by ECRI. All of this implies that most of the variation in output are attributed to the shocks that affect the cyclical component rather than to the shocks that affect the trend. In the BN decomposition and the UCUR model, the opposite occurs. Additionally, despite the fact that the DT$-t_0$ model by Grant and Chan (2017a) was not selected by means of the BF, we have considered this procedure given that it estimates highly persistent cycles and that in some cases it adequately identifies periods of recession. This is a simple output decomposition method where breaks are the only source of permanent shocks. Despite its simplicity, it seems to produce good results, since in several periods the cycles are similar to those obtained using the best methods; see Guillén and Rodríguez (2014) for more arguments.

Current research can be extended in several directions. First, other economic indicators such as consumption, unemployment and inflation can be used to carry out the same comparison exercise and analysis of the different decomposition procedures. Second, the comparison can be extended to include the other LA countries and Southeast Asian countries. Third, the work can be extended to a multivariate level using and expanding the literature that it proposes to incorporate, for example, the Phillips curve or some relationship with unemployment through Okun’s law.

6 References

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7 Tables

7.1 Table 1. Maximum Likelihood Estimates for ARIMA(p,1,q) for the Decomposition of Beveridge and Nelson (1981)

	Canada	France	Germany	Italy	Japan	UK	US	Argentina	Brazil	Chile	Mexico	Peru
LL	-273.434	-128.368	-354.180	-312.285	-377.778	-314.023	-270.044	-214.103	-150.035	-149.743	-262.053	-369.737
c	0.775 (0.09)	0.524 (0.07)	0.596 (0.11)	0.577 (0.11)	0.892 (0.13)	0.576 (0.10)	0.734 (0.09)	0.521 (0.39)	0.552 (0.16)	0.870 (0.19)	0.566 (0.17)	0.776 (0.41)
φ_1	0.358 (0.05)	0.491 (0.05)	0.732 (0.36)	0.403 (0.04)	0.406 (0.04)	0.400 (0.24)	0.259 (0.06)	0.500 (0.08)	0.218 (0.09)	0.353 (0.13)	0.307 (0.05)	0.382 (0.04)
φ_2	0.088 (0.06)	-	0.015 (0.08)	-	-	0.154 (0.06)	0.200 (0.05)	-	-	-	-	-
θ_1	-	-	-0.648 (0.36)	-	-	-0.333 (0.25)	-	-	-	-	-	-
se	0.600 (0.05)	0.219 (0.01)	1.134 (0.06)	0.831 (0.04)	1.381 (0.09)	0.810 (0.04)	0.561 (0.04)	3.194 (0.34)	1.377 (0.15)	1.204 (0.11)	1.581 (0.10)	6.122 (0.40)
$roots$	0.52, -0.17	0.49	0.75, -0.02	0.40	0.41	0.64, -0.24	0.60, -0.34	0.50	0.22	0.35	0.31	0.38
$\psi(1)$	1.804	1.965	1.388	1.675	1.683	1.496	1.850	1.999	1.278	1.545	1.442	1.618

Parentheses denote standard errors for coefficients.

Source: Own elaboration.

7.2 Table 2. Maximum Likelihood Estimates using Perron and Wada (2009, 2016)

	Canada	France	Germany	Italy	Japan	UK	US	Argentina	Brazil	Chile	Mexico	Peru
	UC-NP	UC-CN	UC-N	UC-CP	UC-N	UC-CN	UC-CP	UC-CN	UC-CP	UC-NP	UC-CN	UC-CN
LL	-279.782	-123.715	-349.498	-285.764	-356.432	-280.747	-262.414	-211.232	-144.725	-151.766	-241.966	-336.205
σ_{η_1}	0.356 (0.04)	0.001 (0.00)	0.360 (0.02)	0.000 (0.00)	0.017 (0.00)	0.001 (0.05)	0.001 (0.01)	0.639 (0.24)	0.000 (0.03)	0.001 (0.00)	0.000 (0.06)	0.641 (0.21)
σ_{η_2}	0.357 (0.09)	0.434 (0.23)	1.893 (0.13)	-	2.013 (0.09)	2.059 (0.15)	-	2.238 (0.81)	-	2.869 (1.52)	1.126 (0.19)	5.733 (1.63)
σ_{υ_1}	0.050 (0.01)	0.061 (0.03)	0.034 (0.00)	0.000 (0.00)	0.109 (0.00)	0.021 (0.02)	0.008 (0.00)	0.049 (0.05)	0.184 (0.20)	0.048 (0.01)	0.017 (0.03)	0.132 (0.06)
σ_{υ_2}	0.060 (0.05)	-	-	0.408 (0.28)	-	-	0.144 (0.21)	-	0.185 (0.21)	0.059 (0.03)	-	-
σ_{ε_1}	0.497 (0.04)	0.243 (0.08)	0.352 (0.02)	0.220 (0.07)	0.498 (0.02)	0.144 (0.01)	0.143 (0.02)	0.614 (0.25)	0.497 (0.09)	0.842 (0.03)	0.521 (0.08)	0.558 (0.25)
σ_{ε_2}	-	1.278 (0.80)	-	1.988 (0.43)	-	0.838 (0.65)	1.042 (0.13)	2.923 (0.93)	2.005 (0.42)	-	5.211 (2.74)	3.961 (1.42)
σ_{ω}	0.000 (0.09)	0.113 (0.03)	0.262 (0.01)	0.314 (0.03)	0.360 (0.01)	0.131 (0.07)	0.236 (0.03)	0.000 (0.02)	0.000 (0.01)	0.004 (0.00)	0.000 (0.02)	0.000 (0.03)
φ_1	1.478 (0.06)	1.552 (0.07)	1.691 (0.03)	1.556 (0.05)	1.399 (0.04)	1.370 (0.06)	1.395 (0.07)	1.674 (0.08)	1.120 (0.10)	1.267 (0.10)	1.407 (0.07)	1.446 (0.09)
φ_2	-0.529 (0.05)	-0.594 (0.09)	-0.712 (0.03)	-0.670 (0.05)	-0.544 (0.03)	-0.387 (0.07)	-0.852 (0.46)	-0.710 (0.08)	-0.239 (0.10)	-0.323 (0.05)	-0.514 (0.07)	-0.613 (0.10)
α_1	0.533 (σ_η) (0.22)	0.973 (σ_ε) (0.04)	0.826 (σ_η) (0.06)	0.911 (σ_ε) (0.03)	0.908 (σ_η) (0.03)	0.500 (σ_ε) (0.03)	0.672 (σ_ε) (0.07)	0.863 (σ_ε) (0.10)	0.747 (σ_ε) (0.10)	0.949 (σ_η) (0.05)	0.976 (σ_ε) (0.02)	0.885 (σ_ε) (0.07)
α_2	0.947 (σ_υ) (0.09)	0.797 (σ_η) (0.42)	-	0.932 (σ_υ) (0.09)	-	0.905 (σ_η) (0.08)	0.826 (σ_υ) (0.46)	0.950 (σ_η) (0.16)	0.824 (σ_υ) (0.23)	0.803 (σ_υ) (0.24)	0.510 (σ_η) (0.17)	0.892 (σ_η) (0.05)
roots	0.87, 0.61	0.87, 0.69	0.90, 0.80	0.78 ± 0.25i (4.97)	0.70 ± 0.23i (4.89)	0.97, 0.40	0.81, 0.59	0.84 ± 0.10i (13.43)	0.83, 0.29	0.91, 0.35	0.70 ± 0.14i (8.14)	0.72 ± 0.30i (3.98)

Parentheses denote standard errors for coefficients and mean duration in years for roots.

Source: Own elaboration.

7.3 Table 3. Estimated Posterior Means using Grant and Chan (2017a)

	Canada					France				
	DT-75	UC0	UCUR	UCUR−75	UCUR-(75,08)	DT-73	UC0	UCUR	UCUR-73	UCUR-(73,08)
log *ML*	-283.196	-299.137	-287.838	-282.486	-282.921	-146.161	-155.412	-145.709	-141.407	-140.024
	(0.01)	(0.04)	(0.11)	(0.10)	(0.17)	(0.00)	(0.01)	(0.14)	(0.05)	(0.32)
φ_1	1.31	1.52	0.58	0.75	0.80	1.44	1.65	0.75	0.95	1.02
	(0.06)	(0.10)	(0.14)	(0.37)	(0.40)	(0.07)	(0.09)	(0.11)	(0.24)	(0.32)
φ_2	-0.33	-0.53	0.03	-0.12	-0.15	-0.46	-0.66	-0.06	-0.26	-0.34
	(0.06)	(0.10)	(0.11)	(0.21)	(0.23)	(0.07)	(0.09)	(0.10)	(0.18)	(0.22)
σ_ε^2	0.57	0.40	1.11	0.99	0.88	0.22	0.13	0.96	0.67	0.44
	(0.05)	(0.13)	(0.54)	(0.63)	(0.59)	(0.02)	(0.04)	(0.53)	(0.43)	(0.32)
σ_η^2	-	0.22	2.10	1.43	1.34	-	0.08	1.37	0.91	0.63
		(0.11)	(0.47)	(0.58)	(0.63)		(0.03)	(0.53)	(0.40)	(0.33)
ρ	-	-	-0.92	-0.83	-0.77	-	-	-0.95	-0.94	-0.85
			(0.04)	(0.24)	(0.32)			(0.02)	(0.03)	(0.26)
roots	0.97, 0.34	0.98, 0.54	0.63, -0.05	0.52, 0.23	0.50, 0.30	0.96, 0.48	0.97, 0.68	0.66, 0.09	0.48 ± 0.19i	0.51 ± 0.28i
									(4.22)	(3.10)

	Germany					Italy				
	DT-73	UC0	UCUR	UCUR-73	UCUR-(73,91)	DT-74	UC0	UCUR	UCUR-74	UCUR-(74,07)
log*ML*	-364.651	-368.996	-366.202	-361.419	-361.535	-325.056	-346.000	-326.644	-317.501	-314.996
	(0.01)	(0.01)	(0.10)	(0.10)	(0.05)	(0.01)	(0.05)	(0.13)	(0.17)	(0.06)
φ_1	1.03	0.78	0.78	0.77	0.81	1.29	1.56	0.78	0.83	0.98
	(0.07)	(0.54)	(0.35)	(0.45)	(0.46)	(0.06)	(0.09)	(0.18)	(0.26)	(0.33)
φ_2	-0.06	-0.29	-0.09	-0.17	-0.19	-0.31	-0.57	-0.22	-0.34	-0.46
	(0.07)	(0.32)	(0.29)	(0.28)	(0.29)	(0.06)	(0.09)	(0.15)	(0.18)	(0.18)
σ_ε^2	1.10	0.09	0.71	0.52	0.54	0.78	0.51	1.26	0.82	0.56
	(0.10)	(0.09)	(0.62)	(0.58)	(0.63)	(0.07)	(0.16)	(0.55)	(0.54)	(0.40)
σ_η^2	-	1.08	1.97	1.28	1.25	-	0.35	2.48	1.77	1.31
		(0.13)	(0.62)	(0.48)	(0.50)		(0.14)	(0.35)	(0.44)	(0.43)
ρ	-	-	-0.73	-0.45	-0.40	-	-	-0.91	-0.87	-0.77
			(0.16)	(0.36)	(0.42)			(0.03)	(0.06)	(0.21)
roots	0.97, 0.06	0.39 ± 0.37i	0.64, 0.14	0.39 ± 0.15i	0.41 ± 0.16i	0.97, 0.32	0.98, 0.58	0.39 ± 0.26i	0.42 ± 0.41i	0.49 ± 0.47i
		(2.06)		(4.29)	(4.15)			(2.67)	(2.02)	(2.06)

	Japan					UK				
	DT-73	UC0	UCUR	UCUR-73	UCUR-(73,89)	DT-08	UC0	UCUR	UCUR-73	UCUR-(73,08)
log *ML*	-1412.927	-1447.715	-1436.000	-1403.314	-1397.577	-331.774	-335.624	-331.792	-331.178	-331.468
	(0.15)	(0.06)	(0.36)	(0.09)	(0.03)	(0.01)	(0.02)	(0.07)	(0.06)	(0.08)
φ_1	1.09	1.07	0.84	0.87	0.89	1.07	1.51	0.95	0.84	0.91
	(0.06)	(0.49)	(0.18)	(0.26)	(0.34)	(0.07)	(0.30)	(0.32)	(0.30)	(0.31)
φ_2	-0.10	-0.31	-0.36	-0.31	-0.32	-0.10	-0.59	-0.27	-0.19	-0.23
	(0.06)	(0.31)	(0.19)	(0.26)	(0.29)	(0.07)	(0.24)	(0.24)	(0.26)	(0.26)
σ_ε^2	0.15	0.20	0.24	0.24	0.18	0.83	0.13	0.73	0.82	0.69
	(0.05)	(0.26)	(0.16)	(0.23)	(0.20)	(0.08)	(0.11)	(0.58)	(0.69)	(0.73)
σ_η^2	-	1.67	1.94	1.70	1.42	-	0.67	1.78	1.76	1.67
		(0.33)	(0.05)	(0.21)	(0.31)		(0.12)	(0.51)	(0.50)	(0.53)
ρ	-	-	-0.80	-0.66	-0.47	-	-	-0.81	-0.83	-0.79
			(0.10)	(0.18)	(0.36)			(0.16)	(0.13)	(0.20)
roots	0.99, 0.10	0.54 ± 0.15i	0.42 ± 0.43i	0.44 ± 0.35i	0.45 ± 0.35i	0.97, 0.10	0.76 ± 0.14i	0.48 ± 0.21i	0.42 ± 0.12i	0.46 ± 0.15i
		(5.60)	(1.97)	(2.33)	(2.36)		(8.49)	(3.76)	(5.80)	(4.88)

	US					Argentina				
	DT-73	UC0	UCUR	UCUR-73	UCUR-(73,07)	DT-12	UC0	UCUR	UCUR-01	UCUR-(01,12)
log *ML*	-290.009	-291.547	-285.489	-285.335	-286.078	-687.156	-685.412	-684.212	-682.886	-683.215
	(0.01)	(0.02)	(0.11)	(0.06)	(0.11)	(0.10)	(0.08)	(0.20)	(0.27)	(0.19)
φ_1	1.29	1.61	0.79	0.76	1.12	1.44	1.55	1.54	1.54	1.53
	(0.06)	(0.11)	(0.22)	(0.24)	(0.45)	(0.08)	(0.11)	(0.13)	(0.13)	(0.13)
φ_2	-0.31	-0.63	-0.18	-0.16	-0.37	-0.48	-0.59	-0.58	-0.58	-0.56
	(0.06)	(0.11)	(0.16)	(0.17)	(0.26)	(0.08)	(0.11)	(0.13)	(0.13)	(0.13)
σ_ε^2	0.58	0.21	0.95	0.84	0.56	2.85	2.08	2.08	2.13	2.09
	(0.05)	(0.09)	(0.57)	(0.52)	(0.50)	(0.14)	(0.47)	(0.61)	(0.58)	(0.61)
σ_η^2	-	0.33	1.79	1.67	1.04	-	1.84	1.69	1.75	1.80
		(0.09)	(0.49)	(0.50)	(0.69)		(0.67)	(0.76)	(0.76)	(0.76)
ρ	-	-	-0.89	-0.88	-0.53	-	-	0.59	0.62	0.60
			(0.08)	(0.09)	(0.53)			(0.30)	(0.29)	(0.29)
roots	0.97, 0.32	0.94, 0.67	0.40 ± 0.15i	0.38 ± 0.12i	0.56 ± 0.24i	0.92, 0.52	1.11, 0.44	0.88, 0.66	0.88, 0.66	0.92, 0.61
			(4.20)	(4.95)	(3.92)					

	Brazil					Chile				
	DT-13	UC0	UCUR	UCUR-13	UCUR-(09,13)	DT-13	UC0	UCUR	UCUR-98	UCUR-(98,15)
log *ML*	-158.838	-162.479	-159.427	-157.003	-157.814	-158.902	-163.238	-160.331	-158.369	-157.786
	(0.01)	(0.02)	(0.05)	(0.04)	(0.08)	(0.01)	(0.02)	(0.07)	(0.09)	(0.09)
φ_1	1.10	0.99	0.60	0.71	0.70	1.27	1.38	0.64	0.69	0.75
	(0.11)	(0.44)	(0.38)	(0.43)	(0.41)	(0.11)	(0.24)	(0.49)	(0.50)	(0.52)
φ_2	-0.16	-0.27	-0.02	-0.13	-0.14	-0.35	-0.48	-0.20	-0.23	-0.25
	(0.11)	(0.29)	(0.29)	(0.31)	(0.29)	(0.11)	(0.18)	(0.27)	(0.27)	(0.26)
σ_ε^2	1.33	0.38	0.91	0.74	0.79	1.16	0.65	1.10	1.17	1.12
	(0.20)	(0.38)	(0.60)	(0.58)	(0.59)	(0.17)	(0.36)	(0.76)	(0.80)	(0.80)
σ_η^2	-	1.13	2.69	1.82	1.81	-	0.63	2.03	1.87	1.62
		(0.43)	(0.92)	(0.88)	(0.80)		(0.38)	(0.66)	(0.67)	(0.72)
ρ	-	-	-0.77	-0.55	-0.56	-	-	-0.77	-0.76	-0.70
			(0.19)	(0.36)	(0.35)			(0.24)	(0.25)	(0.31)
roots	0.93, 0.17	0.50 ± 0.16i	0.56, 0.04	0.36 ± 0.06i	0.35 ± 0.13i	0.87, 0.40	0.69 ± 0.06i	0.32 ± 0.31i	0.35 ± 0.33i	0.38 ± 0.33i
		(5.08)		(8.94)	(4.35)		(17.40)	(2.03)	(2.03)	(2.17)

	Mexico					Peru				
	DT-82	UC0	UCUR	UCUR-82	UCUR-(82,97)	DT-92	UC0	UCUR	UCUR-92	UCUR-(83,92)
log *ML*	-274.385	-274.212	-273.520	-273.253	-274.804	-397.354	-383.976	-383.659	-377.757	-378.499
	(0.01)	(0.02)	(0.07)	(0.01)	(0.09)	(0.15)	(0.08)	(0.08)	(0.27)	(0.16)
φ_1	1.28	1.38	0.93	0.87	0.88	1.31	1.45	1.46	1.38	1.38
	(0.08)	(0.11)	(0.45)	(0.45)	(0.45)	(0.05)	(0.10)	(0.10)	(0.10)	(0.10)
φ_2	-0.34	-0.47	-0.41	-0.41	-0.41	-0.40	-0.52	-0.49	-0.51	-0.51
	(0.08)	(0.12)	(0.15)	(0.15)	(0.15)	(0.05)	(0.09)	(0.09)	(0.09)	(0.09)
σ_ε^2	1.57	0.98	1.31	1.29	1.32	2.97	2.69	2.40	2.39	2.42
	(0.18)	(0.33)	(0.72)	(0.73)	(0.74)	(0.04)	(0.27)	(0.48)	(0.48)	(0.47)
σ_η^2	-	0.54	1.89	2.10	2.16	-	2.61	1.81	1.73	1.66
		(0.30)	(1.10)	(0.94)	(0.95)		(0.31)	(0.69)	(0.70)	(0.70)
ρ	-	-	-0.58	-0.66	-0.66	-	-	0.52	0.43	0.46
			(0.41)	(0.30)	(0.32)			(0.29)	(0.33)	(0.32)
roots	0.90, 0.38	0.77, 0.61	0.47 ± 0.44i	0.44 ± 0.47i	0.44 ± 0.47i	0.83, 0.49	0.80, 0.65	0.84, 0.62	0.69 ± 0.18i	0.69 ± 0.18i
			(2.07)	(1.91)	(1.93)				(6.02)	(6.02)

Parentheses denote standard errors for coefficients and mean duration in years for roots.

Source: Own elaboration.

7.4 Table 4. Estimates of the Nonlinear Trend Functions - Perron, Shintani and Yabu (2017) Method

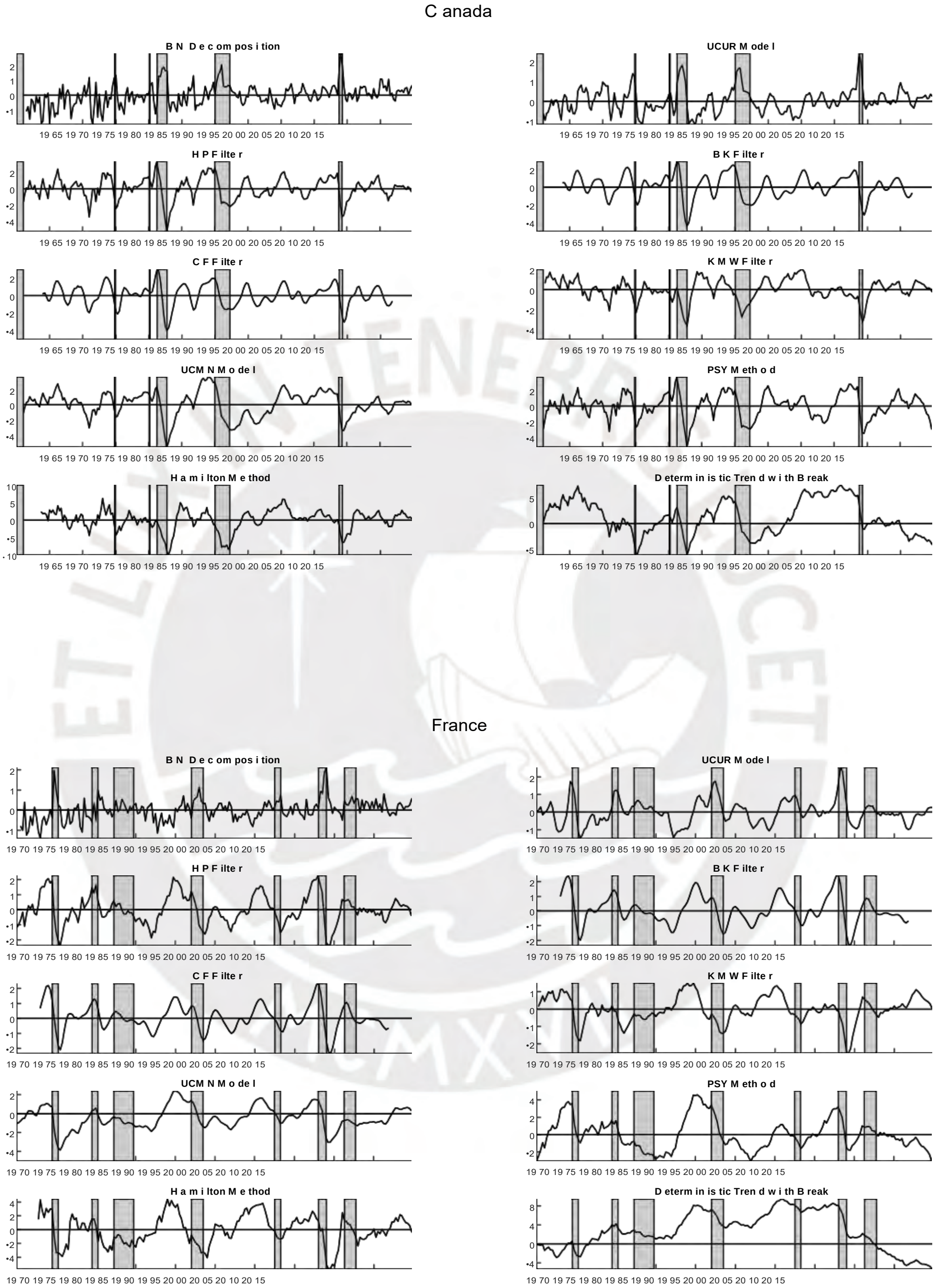
	Canada	France	Germany	Italy	Japan	UK	US	Argentina	Brazil	Chile	Mexico	Peru
Frequencies	5	2	2	2	3	4	2	3	1	5	5	5
Wald H_0	6.9498^{b}	5.7028^{c}	5.1490^{c}	5.3748^{c}	22.3536^{a}	6.6999^{b}	5.5161^{c}	7.8024^{b}	8.5278^{b}	6.7906^{b}	8.6183^{b}	5.8791^{c}
α(OLS)	0.844	0.931	0.888	0.903	0.894	0.888	0.931	0.855	0.823	0.631	0.793	0.756
α(Median Unbiased)	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	0.996	1.000	1.000	1.000
α(Supper Efficient)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Constant	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
Trend	0.008^{a} (0.001)	0.05^{a} (0.001)	0.06^{a} (0.001)	0.006^{a} (0.001)	0.009^{a} (0.001)	0.06^{a} (0.001)	0.07^{a} (0.001)	0.005^{a} (0.002)	0.006^{a} (0.001)	0.009^{a} (0.001)	0.006^{a} (0.001)	0.008^{a} (0.002)
$\sin(2\pi t/T)$	0.059^{c} (0.034)	0.004 (0.026)	0.044 (0.037)	0.031 (0.041)	0.106^{a} (0.036)	-0.021 (0.031)	-0.008 (0.038)	0.081^{c} (0.044)	-0.068^{b} (0.028)	-0.035 (0.024)	-0.029 (0.036)	-0.077 (0.072)
$\cos(2\pi t/T)$	-0.081^{b} (0.034)	-0.051^{b} (0.026)	-0.100^{a} (0.036)	-0.191^{a} (0.040)	-0.301^{a} (0.036)	-0.023 (0.031)	-0.055 (0.038)	0.007 (0.044)	-0.043 (0.027)	-0.019 (0.024)	-0.008 (0.036)	0.176^{b} (0.072)
$\sin(4\pi t/T)$	-0.004 (0.017)	-0.020 (0.013)	-0.025 (0.018)	0.013 (0.020)	0.040^{b} (0.018)	0.011 (0.016)	0.020 (0.019)	0.055^{b} (0.022)	-	0.006 (0.012)	0.003 (0.018)	0.001 (0.036)
$\cos(4\pi t/T)$	-0.053^{a} (0.017)	-0.023^{c} (0.013)	-0.033^{c} (0.018)	-0.068^{a} (0.020)	-0.079^{a} (0.018)	-0.034^{b} (0.016)	-0.040^{b} (0.019)	-0.033 (0.022)	-	0.003 (0.012)	0.028 (0.018)	0.023 (0.036)
$\sin(6\pi t/T)$	0.017 (0.011)	-	-	-	-0.021^{c} (0.012)	0.005 (0.010)	-	-0.030^{b} (0.015)	-	0.015^{c} (0.008)	0.001 (0.012)	0.028 (0.024)
$\cos(6\pi t/T)$	-0.012 (0.011)	-	-	-	-0.053^{a} (0.012)	-0.014 (0.010)	-	-0.035^{b} (0.015)	-	-0.018^{b} (0.008)	0.008 (0.012)	-0.032 (0.024)
$\sin(8\pi t/T)$	-0.004 (0.008)	-	-	-	-	-0.008 (0.008)	-	-	-	-0.007 (0.006)	0.009 (0.009)	-0.036^{b} (0.018)
$\cos(8\pi t/T)$	0.003 (0.008)	-	-	-	-	0.019^{b} (0.008)	-	-	-	0.004 (0.006)	0.002 (0.009)	0.004 (0.018)
$\sin(10\pi t/T)$	0.007 (0.007)	-	-	-	-	-	-	-	-	_0.003 (0.005)	0.006 (0.007)	0.008 (0.014)
$\cos(10\pi t/T)$	-0.016^{b} (0.007)	-	-	-	-	-	-	-	-	-0.012^{b} (0.005)	-0.020^{a} (0.007)	0.034^{b} (0.014)

a, b and c denote statistic significance at the 1%, 5% and 10% level, respectively. Wald test critical values: 90%=4.605, 95%=5.992, 99%=9.210.

Source: Own elaboration.

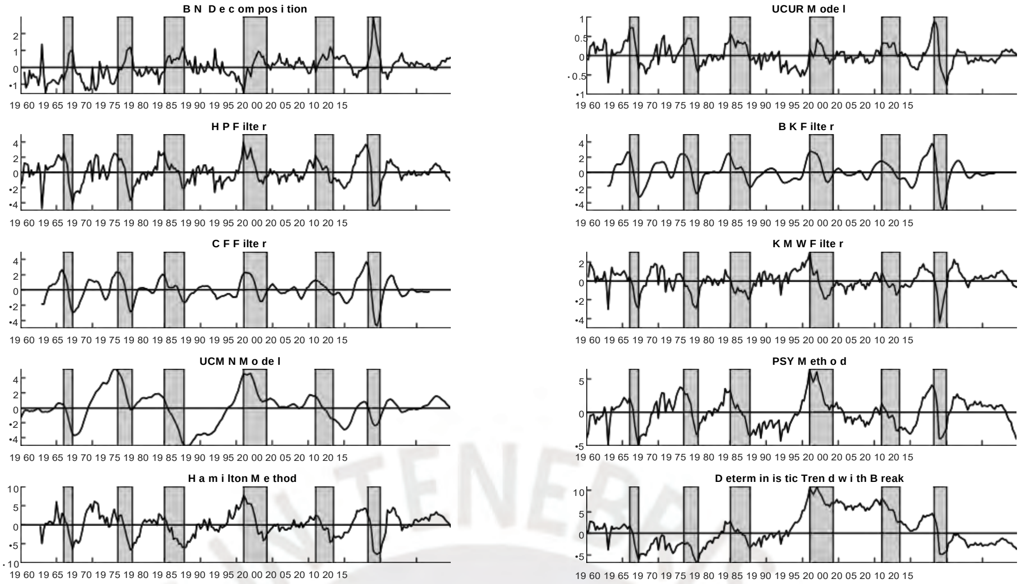
8 Figures

8.1 Figure 1. Business Cycles under different approaches

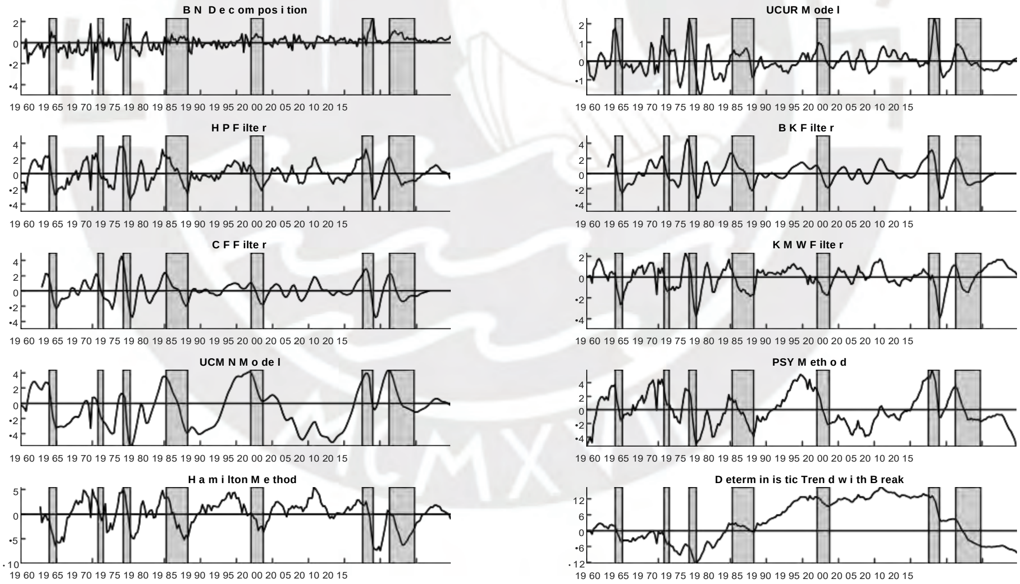


Source: Own elaboration.

Germany

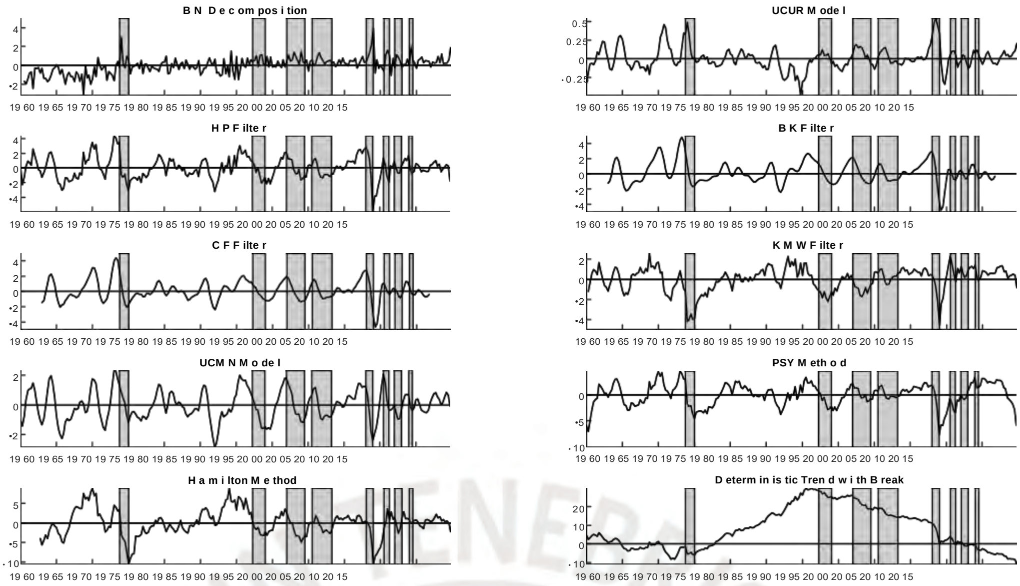


Italy

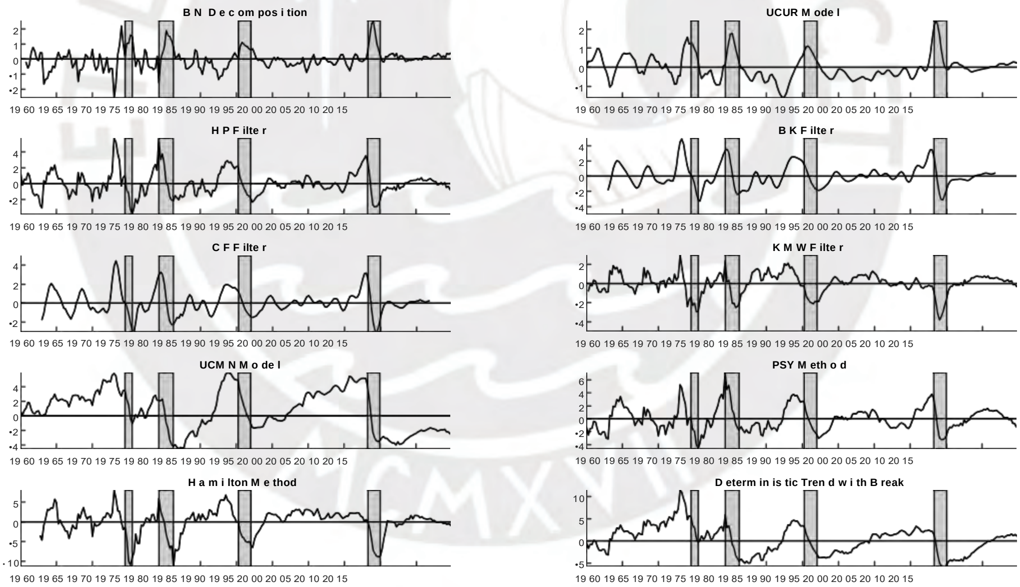


Source: Own elaboration.

Japan

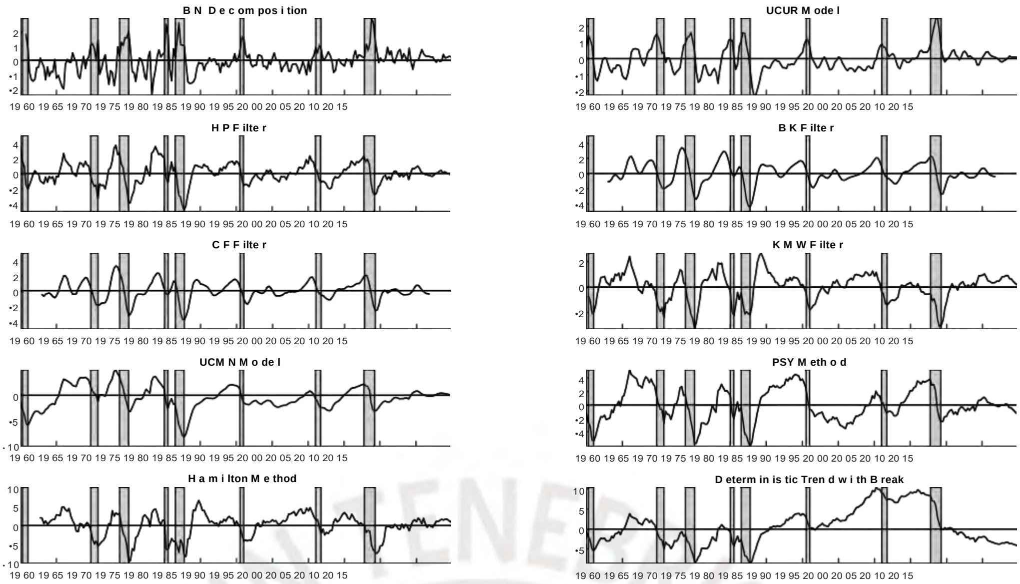


United Kingdom

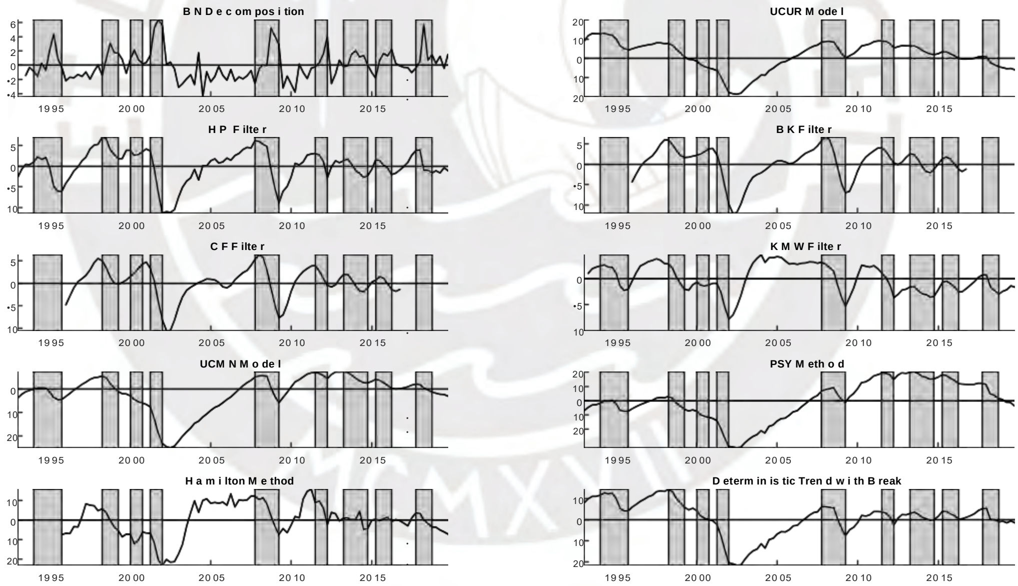


Source: Own elaboration.

United States

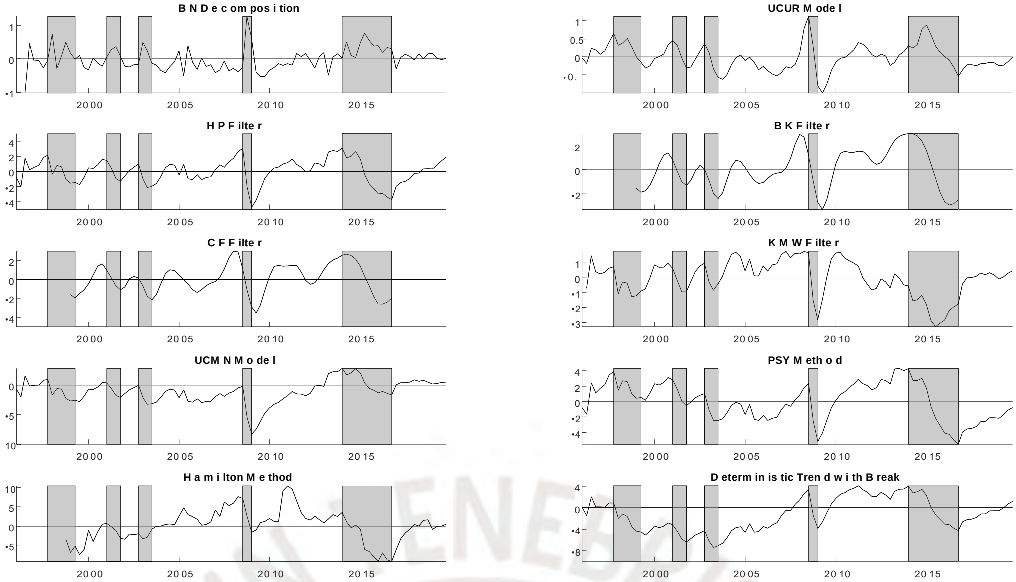


Argentina

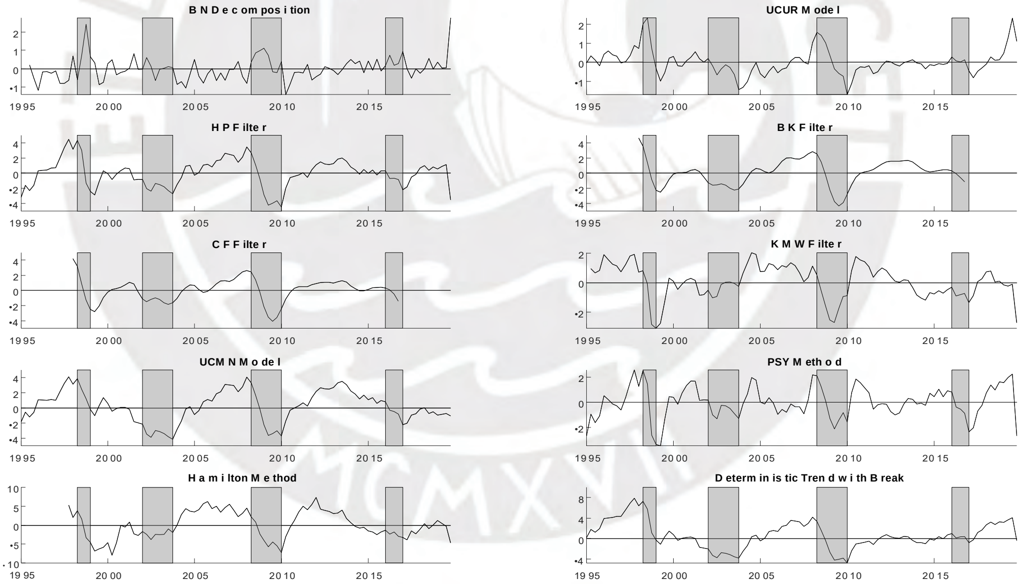


Source: Own elaboration.

Brazil

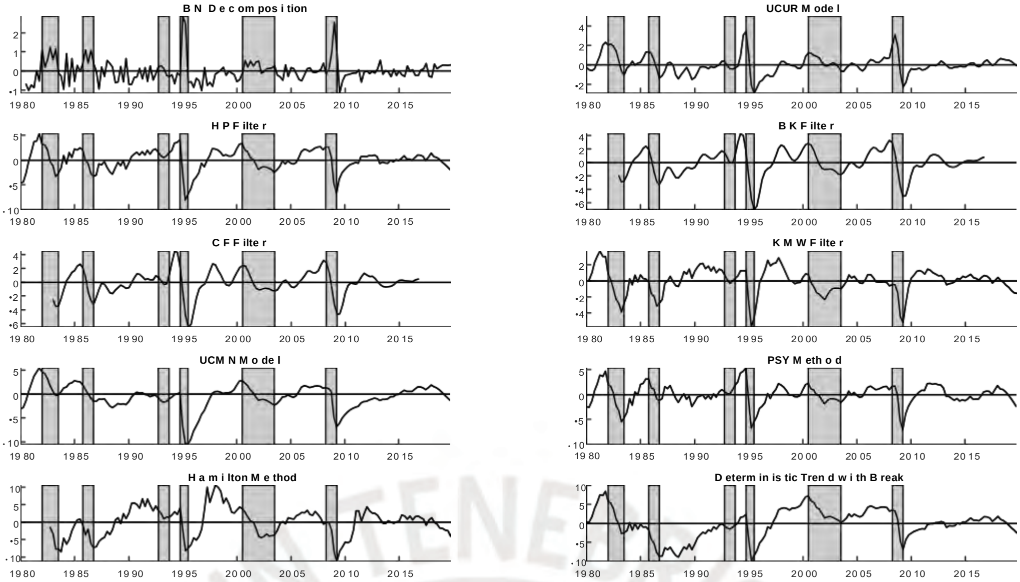


Chile

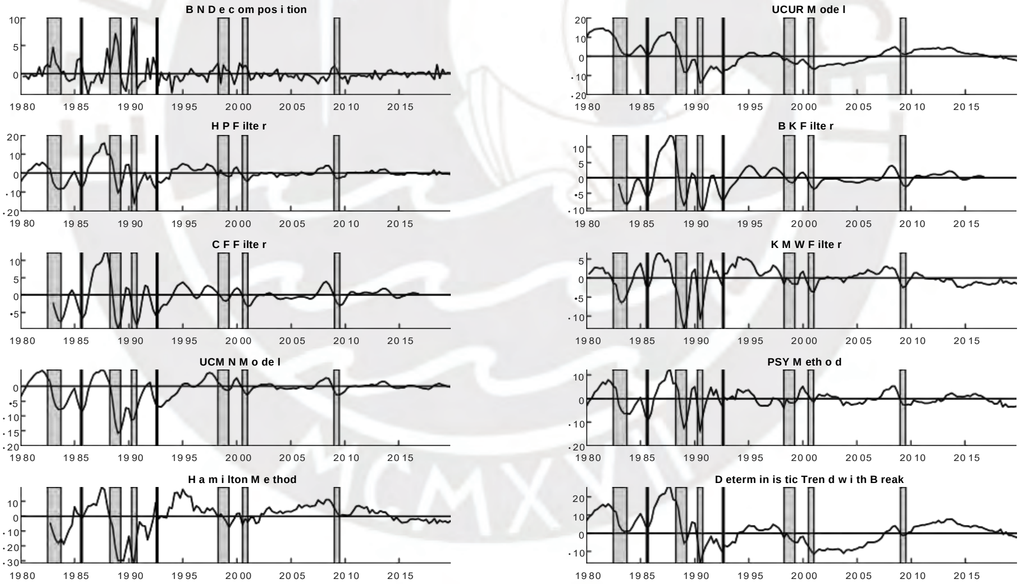


Source: Own elaboration.

Mexico



Peru



Source: Own elaboration.