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Modeling the Volatility of Returns on Commodities: An Application and Empirical Comparison of GARCH and SV Models

TESIS PARA OPTAR EL TÍTULO PROFESIONAL DE LICENCIADO EN ECONOMÍA

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Fuimos, somos y seremos series que convergieron en este plano llamado vida,
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Abstract

Seven GARCH and stochastic volatility (SV) models are compared to model empirically the volatility of returns on four commodities relevant for South America economies: gold, copper, oil, and natural gas. Our results show that SV models outperform GARCH models on average. We find that the best-performing return volatility models are: GARCH-t for gold, SV-t for copper and oil, and SV with leverage effects (SV-L) for natural gas. The inclusion of fat tails and jumps components largely raise the performance of GARCH models, while this contribution is less for SV models. Even, SV models with jumps are usually outperformed by the basic SV model. We also find evidence of a leverage effect in oil and copper, resulting from their dependence on world economic activity; and of an inverse leverage effect in gold and natural gas, consistent with the former's role as safe asset and with uncertainty about the latter's future supply. Additionally, in most cases there is no evidence of an impact of volatility on the mean or MA-type first order autocorrelation.

JEL Codes: C11, C52, G15.

Keywords: Returns, Volatility, GARCH, Stochastic Volatility, Commodities, Bayesian Estimation, Fat Tails, Jumps, Leverage.

Contents

1 Introduction

Over the last three decades commodity spot markets have experienced increasing levels of volatility (see Calvo-Gonzalez et al. (2010) and Jacks et al. (2011)) and also became more sensitive to macroeconomic and financial developments (see Chen (2010) and Batten et al. (2010)). This is relevant for South American countries that are commodity export dependent according to UNCTAD (2017). For example, as of 2015, Peru and Chile's copper exports amounted to 19% and 23% of total export value, respectively; while Colombia and Ecuador's oil exports reached 34% of total export value[1]. Moreover, gold amounted to 43%, 24%, and 20% of international reserves in Venezuela, Colombia and Bolivia, respectively during period 2000-2017 according to the World Gold Council (2017). These features reduce resilience agaisnt commodity volatility surges which can impair economic growth of primary commodity exporters through lower accumulation of capital, higher tax revenue volatility, currency devaluations, higher financial sector fragility, among other negative implications; see Cavalcanti et al. (2015), Ehrhart and Guerineau (2013), Jhonson and Soenen (2009), Kinda et al. (2016), and Fernández et al. (2016). Hence, obtaining accurate volatility estimates is an issue of interest for policy makers decisions.

This paper uses Chan and Grant's (2016a) approach to compare seven pairs of GARCH and SV models applied to daily return series for four important commodities in South American countries export basket: gold, copper, oil, and natural gas, by using Bayes Factors. The authors expect to fill a gap in studies on volatility applied to commodity markets that provide a formal comparison of the goodness of fit of both model families.

Volatility changes over time have been conventionally estimated using GARCH models, where the conditional variance is considered a deterministic process. For example, Akgiray et al.(1991) argue that political tensions impact gold and silver volatility by analyzing changes in GARCH coefficient estimates in four different sub-periods. Bracker and Smith (1999) find significant evidence of leverage effect on copper futures returns using a battery of asymetric GARCH models. Narayan and Narayan's (2007) estimation of asymmetric GARCH models applied to returns on oil in 1991-2006 point to the existence of regime changes. Additionally, Nomikos and Andriasopoulos (2012) study volatility in futures contracts for eight energy markets using an EGARCH model with jumps in the returns equation. The results show that a shock on oil volatility takes longer than natural gas one to revert (998 vs. 155 days, respectively), while the jump component takes longer to dissipate for natural gas than for oil (72 vs. 36 days, respectively). They also highlight that all products, except oil, show an inverse leverage effect; i.e., positive returns today announce an increase in volatility tomorrow. The presence of jumps in two natural gas series from different markets (UK and U.S.) is examined by Mason and Wilmot (2014), who find that jumps are more significant in the UK than in the U.S., reflecting structural dissimilarities between both markets. Moreover, Hammoudeh and Yuan (2008) find, after cotrolling by oil and interest rate shocks, evidence of leverage effects on copper while the contrary holds for gold and silver, which turns the latter two into safe assets in uncertainty scenarios[2].

Alternatively, other research works consider stochastic volatility (SV) models, where volatility is a latent variable governed by a stochastic process. For example, Vo (2009) suggest that the inclusion of regime changes into the SV model applied to WTI oil returns prevents the overestimation of the

[1] Data obtained from OEC (2015).

[2] For further references see Ramírez and Fadiga (2003), Lucey and Tully (2006), Watkins and McAleer (2008), Ewing and Malik (2010), Fong and See (2002), Choi and Hammoudeh (2010) and Charles and Darné (2014)

parameters for volatility persistence. Larsson and Nossman (2011) model WTI oil returns in 1989-2009 using an SV model with correlated jumps (SV-CJ) introduced by Duffie et al. (2000). They suggest that extreme changes in oil returns during the 1990s were governed by price jumps, while in the 21st century they were dominated by volatility jumps. Additionally, the authors argue that models that do not include SV or jumps do not provide a good representation of oil volatility during stress periods like the Gulf War and the 2009 crisis. Du et al. (2011) use an SV-J model to find that inventories and speculation are relevant to explain oil volatility. Brooks and Prokopczuk (2013) study volatility in three commodity market segments (metals, energy, and agriculture) and the S&P 500 index comparing[3] three SV models (SV, SV-J, and SV-CJ). They indicate that models with jumps perform better and confirm that the correlation between commodity and S&P 500 volatilities is low, suggesting that commodities can be instrumental in diversifying risk. Additionally, Liu et al. (2014) study the volatility of returns on copper and aluminum spot and futures markets using the SV-J and SV-CJ models; and find that models that include jumps provide a better measure of risk than the standard SV model[4].

Nonetheless, literature comparing the goodness of fit and inference capabilities of GARCH and SV model families still scarce. For instance, Taylor (1994), Ghysels et al. (1996), Andersson (2001), Carrasco and Chen (2002) and Bai et al. (2003) use a theoretical approach to compare both families based on the similarity of kurtosis and first-order autocorrelation of the squared returns generated by the models relative to the properties of the original series. Other studies like Garcia and Renault (1998), Lehar et al. (2002), Fleming and Kirby (2003) and Pederzolli (2011) compare both families using a value-at-risk approach, while works like Danielson (1994), Kim et al. (1998), Gerlach and Tuyl (2006), Nakajima (2012) and more recently Chan and Grant (2016a) compare both families using a Bayesian approach.

The main results of this paper shows that on average SV models perform better than GARCH family for modeling commodity returns volatility. For gold, GARCH-t model outperforms the rest of models in both families. For copper and oil, SV-t is selected, while for natural gas, SV-L (SV with leverage effect) performs best. Moreover, considering fat tails and jumps component substantially boots model performance in GARCH family, although this contribution is lower for the SV family. Even, SV model with jumps are usually outperformed by the basic SV model. Also, there is evidence of leverage effect in copper and oil, because of their relationship with the state of the global economy, while for gold and natural gas, there is evidence of inverse leverage effect, where future volatility is expected to be higher in response to current positive returns. In the case of gold, this effect is due to its role as safe asset in stress episodes, while in the case of natural gas, demand pressures increase uncertainty about future supply. Finally, there is no evidence of an impact of volatility on the mean in most products nor of MA-type first order autocorrelation in the returns.

The rest of the paper is organized as follows. Section 2 describes the seven pairs of GARCH and SV models, as well as the Bayesian estimation and comparison methods. Section 3 presents the data and estimation results for the four commodities; and a justification is provided for the selection of the models based on historical events and the nature of each good. Section 4 presents

[3] The authors use the Deviance Information Criterion (DIC) which is a Bayesian criterion proposed by Spiegelhalter et al. (2002) and used to compare SV models using the conditional data likelihoods. This criterion is also used and discussed by Chan and Grant (2016b).

[4] Schmitz et al. (2014) arrived to the same conclusion for the soy and wheat markets (and for agricultural markets in general).

the conclusions.

2 Methodology

This section briefly presents the two classes of models with time-changing volatilities used in the empirical section. The first class are GARCH models developed by Bollerslev (1986) as an extension of the seminal work by Engle (1982). The second group are SV models originally developed by Taylor (1986, 1994).

2.1 GARCH Models

Following the notation used by Chan and Grant (2016a), Bollerslev's GARCH (1,1) model is defined as:

$$\begin{aligned} y_t &= \mu + \epsilon_t, \\ \sigma_t^2 &= \alpha_o + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \\ \epsilon_t &\sim N(0, \sigma_t^2), \end{aligned} \tag{1}$$

where $\epsilon_0 = 0$, $\sigma_0^2 = var(y_t)$, $\alpha_0 > 0$, $\alpha_1 \geq 0$, $\beta_1 \geq 0$ and $\alpha_1 + \beta_1 < 1$. This model is named GARCH-1. A GARCH (2,1) model is considered next:

$$\sigma_t^2 = \alpha_o + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 \sigma_{t-2}^2, \tag{2}$$

where $\sigma_{-1}^2 = \epsilon_0 = 0$, $\sigma_0^2 = var(y_t)$ is a constant and the same restrictions are applied to the coefficients of σ_t^2 to ensure non-negativity and stationarity. This model is named GARCH-2. The third model preserves the same form as (1), but with t-student innovations; i.e., $\epsilon_t \sim t_\nu(0, \sigma_t^2)$. The selection of this distribution can capture extreme events that may be omitted by the GARCH-1 and GARCH-2 models. This model is named GARCH-t. The fourth model, named GARCH-J, modifies (1) by including a random jump component that allows adjustment to infrequent data changes:

$$y_t = \mu + k_t q_t + \epsilon_t, \tag{3}$$

where q_t is a jump that follows a Bernoulli distribution with a success probability of $prob(q_t = 1) = \kappa$. When $q_t = 1$ a jump takes place in period t with a magnitude determined by $k_t \sim \mathcal{N}(\mu_k, \sigma_k^2)$. The fifth model includes σ_t^2 in the equation for the conditional mean; i.e., the returns are dependent on volatility. This model is named GARCH-M:

$$y_t = \mu + \lambda \sigma_t^2 + \epsilon_t, \tag{4}$$

where the parameter λ may be understood as the risk premium. The sixth is a GARCH model that includes a dynamic element in the errors of process y_t via a first-order moving average component (MA(1)); i.e., (1) is:

$$\begin{aligned} y_t &= \mu + \epsilon_t, \\ \epsilon_t &= u_t + \psi u_{t-1}, \end{aligned} \tag{5}$$

where the invertibility of the MA(1) component is ensured by assuming $|\psi| < 1$ and $u_t \sim \mathcal{N}(0, \sigma_t^2)$. This model is named GARCH-MA. The last specification uses the asymmetric GARCH structure proposed by Glosten et al. (1993); i.e., introduces an additional impact from the excess negative returns on the variance in (1):

$$\sigma_t^2 = \alpha_o + [\alpha_1 + \delta \mathbf{1}(\epsilon_{t-1} < 0)]\, \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \tag{6}$$

where $\mathbf{1}(.)$ is an indicative function that is triggered when $\epsilon_{t-1} < 0$. In this scenario, the asymmetric effect is represented by $\delta > 0$[5], which measures the additional impact from a lagged negative shock on current volatility. This model is named GARCH-L.

2.2 SV Models

Following Chan and Grant's (2016a) notation, the model named SV-1 is the canonical SV model:

$$\begin{aligned} y_t &= \mu + \epsilon_t^y, \\ h_t &= \mu_h + \phi_{h_1}(h_{t-1} - \mu_h) + \epsilon_t^h, \\ \epsilon_t^y &\sim N(0, e^{h_t}), \\ \epsilon_t^h &\sim N(0, \omega_h^2), \end{aligned} \tag{7}$$

where h_t is the log-volatility following an $AR(1)$ stationary process, as $|\phi_{h_1}| < 1$, disturbances ϵ_t^h and $\epsilon_t^y$are uncorrelated, and the process is initialized with $h_1 \sim \mathcal{N}(\mu_h, \frac{\omega_h^2}{(1-\phi_{h_1}^2)})$. The second model, named SV-2, has the same equation for the mean as (7), but includes an additional lag in the log-volatility equation:

$$h_t = \mu_h + \phi_{h_1}(h_{t-1} - \mu_h) + \phi_{h_2}(h_{t-2} - \mu_h) + \epsilon_t^h,$$

where ϕ_{h_1}, ϕ_{h_2} are assumed to lie inside the unit circle and h_1 and h_2 are initialized with: $h_1, h_2 \sim \mathcal{N}(\mu_h, \frac{(1-\phi_{h_2})\omega_h^2}{(1+\phi_{h_2})((1-\phi_{h_2})^2 - \phi_{h_1}^2)})$. The model named SV-t is similar to SV-1 but with:

$$\epsilon_t^y \sim t_\nu(0, e^{h_t}). \tag{8}$$

The fourth SV model admits the existence of random jumps and is named SV-J. The equation for the mean in (7) is:

$$y_t = \mu + k_t q_t + \epsilon_t^y, \tag{9}$$

where the jump indicator q_t and the jump size k_t have the same characteristics of a GARCH-J model. The fifth model, Koopman and Uspensky's (2002) SV-M model, where the stochastic volatility is included in the equation for the mean, is:

$$y_t = \mu + \lambda e^{h_t} + \epsilon_t^y. \tag{10}$$

[5] A value of $\delta < 0$ indicates that in response to a positive return scenario today, volatility tomorrow is higher. This is evidence of an inverse leverage effect.

An SV model is also specified to include the dynamics of innovations in y_t, represented by an MA(1) process named SV-MA:

$$\begin{aligned} y_t &= \mu + \epsilon_t^y, \\ \epsilon_t^y &= u_t + \psi u_{t-1}, \\ u_t &\sim \mathcal{N}(0, e^{h_t}), \end{aligned} \tag{11}$$

where $u_0 = 0$, $|\psi| < 1$. Finally, this paper uses an SV model with leverage effect, named SV-L. In this specification, the mean and log-volatility equations are the same as in (7), but disturbances ϵ_t^h and ϵ_t^y are allowed to be correlated:

$$\begin{pmatrix} \epsilon_t^y \\ \epsilon_t^h \end{pmatrix} \sim \mathcal{N}\left[0, \begin{pmatrix} e^{h_t} & \rho e^{\frac{1}{2}h_t}\omega_h \\ \rho e^{\frac{1}{2}h_t}\omega_h & \omega_h^2 \end{pmatrix}\right], \tag{12}$$

where ρ is the parameter indicating a correlation between the shocks. When $\rho < 0$, there is a negative correlation in the returns and their volatility[6].

2.3 Bayesian Estimation and Comparison Models

Briefly, the Bayesian approach follows Bayes' theorem: $\pi(\theta|y) \propto f(y|\theta)\pi(\theta)$, where $\pi(\theta|y)$ is the posterior distribution of a set of parameters represented by θ, conditional on the data; $f(y|\theta)$ is the likelihood function; and $\pi(\theta)$ is the prior or ex-ante criterion for the behavior of θ. Along these lines, Markov Chain Monte Carlo (MCMC) methods are used to sample the posterior distributions of interest, $\pi(\theta|y)$. Some details of the model estimation and comparison process are discussed next.

2.3.1 The Priors

The same priors as Chan and Grant (2016a) are used to perform inferences. The selected priors share three characteristics: (i) they are the same for the GARCH and SV models; (ii) they are functions whose densities are independent and can be integrated into the unit; and (iii) they are relatively non-informative, and therefore play an unimportant role in using the data to derive a posterior distribution function for the parameters.

First, the assumptions for the GARCH-1 model are $\mu \sim \mathcal{N}(\mu_0, V_u)$ and $\log \boldsymbol{\gamma} \sim \mathcal{N}(\boldsymbol{\gamma}_0, V_\gamma)\mathbf{1}(\alpha_1 + \beta_1 < 1)$, where $\boldsymbol{\gamma} = (\alpha_0, \alpha_1, \beta_1)'$ follows a truncated log-normal distribution that admits certain parameter space to ensure stationarity. The hyper-parameters have the following values: $\mu_0 = 0$, $V_u = 10$, $\boldsymbol{\gamma}_0 = (1, \log 0.1, \log 0.8)'$ and $V_\gamma = diag(10, 1, 1)$. In GARCH-2, $\widetilde{\boldsymbol{\gamma}} = (\alpha_0, \alpha_1, \beta_1, \beta_2)$, and therefore $\log \widetilde{\boldsymbol{\gamma}} \sim \mathcal{N}(\widetilde{\boldsymbol{\gamma}}_0, V_{\widetilde{\gamma}})\mathbf{1}(\alpha_1 + \beta_1 + \beta_2 < 1)$, $\widetilde{\boldsymbol{\gamma}}_0 = (1, \log 0.1, \log 0.8, \log 0.1)$ and $V_{\widetilde{\boldsymbol{\gamma}}} = diag(10, 1, 1, 1)$. In the remaining GARCH models, the priors for μ and $\boldsymbol{\gamma}$ are the same as in GARCH-1. In GARCH-J, the intensity of jumps is assumed to follow a uniform distribution, whereas the average jump and the jump variance, $\boldsymbol{\delta} = (\mu_k, \log \sigma_k^2)'$, behave like a normal bivariate distribution: $\kappa \sim \mathcal{U}(0, 0.1)$ and $\boldsymbol{\delta} \sim \mathcal{N}(\boldsymbol{\delta_0}, V_{\boldsymbol{\delta}})$. The hyper-parameters are $\boldsymbol{\delta}_0 = (0, \log 10)'$ and $V_{\boldsymbol{\delta}} = diag(10, 1)$. In the case of GARCH-M, $\lambda \sim \mathcal{N}(\lambda_0, V_\lambda)$, where $\lambda_0 = 0$ and $V_\lambda = 100$. In the

[6] A value of $\rho > 0$ indicates a positive correlation between the returns and their volatility and, therefore, provides evidence of an inverse leverage effect.

case of GARCH-MA, $\psi \sim \mathcal{N}(\psi_0, V_\psi)\mathbf{1}(|\psi| < 1)$, where $\psi_0 = 0$ and $V_\psi = 1$. In GARCH-t, $\nu > 2$ and $\nu \sim \mathcal{U}(2, 100)$. Finally, the distribution of the δ parameter in GARCH-L is assumed to be $(\delta|\boldsymbol{\gamma}) \sim \mathcal{U}(-\alpha_1, 1 - \alpha_1 - \beta_1)$.

Regarding the SV models, the priors for parameters μ, μ_h, ϕ_{h_1} and ω_h^2 in SV-1 are $\mu \sim \mathcal{N}(\mu_0, V_\mu)$, $\mu_h \sim \mathcal{N}(\mu_{h_0}, V_{\mu_h})$, $\phi_{h_1} \sim \mathcal{N}(\phi_{h_0}, V_{\phi_{h_1}})\mathbf{1}(|\phi_{h_1}| < 1)$ and $\omega_h^2 \sim \mathcal{IG}(\nu_h, S_h)$, respectively. The hyper-parameters are $\mu_0 = 0, \mu_{h_0} = 1, V_\mu = V_{\mu_h} = 10, \phi_{h_0} = 0.97, V_{\phi_{h_1}} = 0.1^2, \nu_h = 5$ and $S_h = 0.16$. The same priors are maintained for SV-2, and $\boldsymbol{\theta}_h = (\phi_{h_1}, \phi_{h_2})'$, such that: $\boldsymbol{\theta}_h \sim N(\boldsymbol{\theta}_{h_0}, V_{\boldsymbol{\theta}_h})\mathbf{1}(\boldsymbol{\theta}_h \in A)$, where A is a space where stationarity is ensured. The hyper-parameters are $\boldsymbol{\theta}_{h_0} = (0.97, 0)'$, $V_{\boldsymbol{\theta}_h} = diag(0.1^2, 1)$. Concerning the remaining models: (i) the same priors are assumed as in the SV-1 model for parameters μ, μ_h, $\phi_{h_1}$and ω_h^2; (ii) the priors for the additional parameters are the same as the prior for their GARCH counterparts, and (iii) the ρ parameter is distributed as $\mathcal{U}(-1, 1)$.

2.3.2 The Algorithm

Both kinds of models are estimated using the MCMC methods proposed by Chan and Grant (2016a, b). The purpose is taking samples from the posterior distributions of the models by performing Markov-type sampling and using posterior draws to calculate several magnitudes of interest such as the posterior means and marginal likelihoods[7].

Metropolis-Hastings algorithms are used in the GARCH models to sample from the posterior distributions. The parameters of the models are grouped to perform the estimations in different blocks. In this way, it is possible to begin sampling from the complete conditional densities of the parameters. It is important to mention that, in contrast with Chan and Grant (2016a), who use a normal distribution to sample the parameters of the volatility equation, this paper uses a $\mathcal{B}eta$ distribution. The reason for this change is that the MCMC chains of these parameters in most GARCH models presented a "sticky chain problem." Following Rosenthal (2011) and Junker et al. (2016), these papers set out to solve it by raising the level of acceptance of the parameters, thereby avoiding stagnation of the chain by diminishing the variance of the generating density. If the problem persists, it is possible to change the generating density of the algorithm. In our case, while diminishing the variance level increases the level of the acceptance ratio, the chains continued to show protracted stagnation during several simulation periods. In response, a $\mathcal{B}eta$ distribution is substituted for the density of the Metropolis-Hastings algorithm, thereby obtaining well-behaved chains with higher acceptance ratios.

In the SV models it is necessary to simulate the conditional density of the vector of non-observable log-volatilities, $p(\mathbf{h}|\mathbf{y}, \Phi_i)$, where Φ_i represents the parameter vector for each model $i = 1, \ldots, n$. For this purpose, this paper uses the adaptive Metropolis-Hastings algorithm proposed by Chan (2015) and based on Chan and Jeliazkov (2009)[8]. Carrying out the simulation of marginal likelihoods requires a generating (or importance) density, from which the target density draws, $p(y|M_i)$, are done, where M_i represents the i model; see Koop (2003). Once all draws are done, a weighted average is calculated, where the weights are assigned according to the contribution of each draw to decrease the bias of the proposed density relative to the target density, so as to obtain an unbiased and consistent estimator for the latter. The importance sampling estimator for $p(y|M_i)$

[7] Complete details about the algorithm may be found in the appendix of Chan and Grant (2016a).

[8] For details about the sampling for SV-MA and SV-t, see Chan (2013) and Chan and Hsiao (2014), respectively.

is defined as $\hat{p}_{IS} = \frac{1}{N} \sum_{n=1}^{N} \frac{p(\mathbf{y}|\boldsymbol{\theta}_n)p(\boldsymbol{\theta}_n)}{g(\boldsymbol{\theta}_n)}$, where $\boldsymbol{\theta}_1 \ldots \boldsymbol{\theta}_N$ are independent draws obtained from the importance density. Along these lines, the selection of this density is based on Chan and Eisenstat (2015), who propose approximating $g(\boldsymbol{\theta})$ through the cross-entropy method, which is instrumental in deriving an optimal density that minimizes the Kullback-Leibler divergence relative to the ideal density[9]. Additionally, the ideal density should be in the same parametric class as the densities for the priors.

2.3.3 The Bayes Factor

Once the marginal likelihoods are obtained, a formal method for comparing the models is the Bayes factor (BF). The goal is comparing a group of models $\{M_1, \ldots, M_i\}$, where $i = 1, \ldots, n$ and each M_k model is made up of two separate components: a likelihood function $p(y|\boldsymbol{\theta}_k, M_k)$ and a prior density $p(\boldsymbol{\theta}_k|M_k)$. The BF provides a criterion for comparing models M_i and M_j, defined as $BF_{ij} = \frac{p(\mathbf{y}|M_i)}{p(\mathbf{y}|M_j)}$, where $p(\mathbf{y}|M_k)$ is the marginal likelihood of model M_k $(k = i, j)$ and models are *a priori* equally likely. A possible interpretation is that the marginal likelihood is a predictive density under M_k across the observed data $\mathbf{y}$. Therefore, $BF_{ij} > 1$ will indicate that the observed data are more likely to be obtained using M_i rather than M_j, which can be considered as evidence in favor of M_i. The strength of this evidence is proportional to the value obtained by the BF[10].

3 Empirical Analysis

3.1 The Data

This paper uses four S&P GSCI spot price index time series. The returns are expressed as $y_t = 100 \times (\log P_t - \log P_{t-1})$, where P_t is the closing price in period t. The commodities analyzed are gold and copper (metal items), as well as oil and natural gas (energy items). The daily data are drawn from Bloomberg: January-1983-April 2017 for gold and copper; January 1987-April 2017 for oil; and January 1994-April 2017 for natural gas.

A disaggregated analysis is warranted by the fact that the dynamics of the volatility of returns is governed by peculiar characteristics in each market, and therefore shows a different behavior over time; see Batten et al. (2010). Figure 1 shows the four return series, which show both turbulence and stability episodes. This behavior is similar to that of financial return series, which show clusters originated by episodes of macroeconomic, financial, or geopolitical stress.

Table 1 shows a summary of statistics for the returns and square returns. Panel A shows that: (i) the average return of the four series is close to zero; (ii) all series except natural gas show negative returns on average; (iii) the series show high kurtosis (above six on average); and (iv) gold and gas show the narrowest and widest distance between the maximum and minimum returns, respectively (which is consistent with the standard deviation of each series). Panel B shows that the empirical distribution of the square returns on oil have greater symmetry and kurtosis than the other goods, mainly due to the extreme event created by the Gulf War[11].

[9] For further details, see appendix B in Chan and Grant (2016a) and appendices A and B in Chan and Grant (2016b).

[10] For a further discussion about the BF, see Koop (2003).

[11] If the Gulf War is excluded from the sample, the kurtosis diminishes by two-thirds of its current value.

3.2 Estimation Results

One hundred thousand simulations were performed for each parameter with a burn-in of 50 thousand to ensure convergence[12]. It is important to note that there is a group of common parameters (i.e., $\alpha_0, \alpha_1, \beta_1, \mu, \mu_h, \phi_{h_1}$, ϕ_{h_2}, and ω_h^2) belonging to the seven GARCH and SV models for each good; and that none of them show a zero value in their credibility intervals. The following sections describe the estimation results for the remaining models taking GARCH-1 and SV-1 as reference[13].

The results are presented for each commodity, where the even- (2-8) and odd-numbered (3-9) Tables show the results for the GARCH and SV families, respectively.[14]

In Figure 2, the estimated implicit volatilities for the best model in each family (for each commodity) are presented and compared with the square returns. The models capture accurately the dates of the events that caused volatility surges; and the volatilities estimated for both families are highly correlated during their periods of analysis.

3.2.1 Gold

Table 2 shows the results for the GARCH family. The estimation of GARCH-t yields $\nu = 3.95$, suggesting the occurrence of extreme events affecting gold returns (Figure 1). Additionally, in GARCH-J, κ indicates that the jump probability is 0.10; i.e., 25 jumps per year on average,[15] with an average magnitude of $\mu_k = -0.144$, implying that the series experienced more negative jumps over time. Regarding GARCH-M, no evidence is found that market participants demand a risk premium to invest in gold. Similarly, GARCH-MA shows no evidence of first-order serial correlation in the returns; i.e., the persistence of unexpected shocks in the process governing the returns is no greater than one day. The results are consistent with Lucey and Tully (2006), who also find no evidence of ARCH-M and/or first-order serial correlation effects in gold returns. The GARCH-L model shows evidence of an inverse leverage effect ($\delta < 0$) implying that positive returns today generate higher volatility tomorrow. Lucey and Tully (2006) and Hammoudeh and Yuan (2008) find similar results and suggest that gold can play a role as safe asset in the face of adverse events. Finally, based on the marginal log-likelihoods, GARCH-t provides the best fit, followed by GARCH-J and GARCH-2. The $Q(20)$ and $Q_2(20)$ statistics suggest that only GARCH-t, GARCH-J do not reject the null hypothesis of no autocorrelation in the standardized residuals and the square standardized residuals.

Table 3 shows the results for the SV models. The SV-t model shows that $\nu = 12.60$, suggesting evidence of extreme events, although to a lesser extent than the GARCH-t model. Regarding SV-J, $\kappa = 0.01$, i.e., there are 2.5 jumps per year with an average magnitude of $\mu_k = -0.35$, implying that returns experienced a greater number of falls over time. In this regard, Brook and Prokopzuk (2013) estimate an SV-J and find a jump intensity of 0.0532; i.e., 13 jumps per year, while for an SV-CJ model they find a jump intensity of 0.0172; i.e., 2.52 jumps per year, similar to our results. Additionally, SV-M and SV-MA do not show evidence of a risk premium or serial correlation in the

[12] Trace plots, histograms and autocorrelation figures were generated for each parameters in all GARCH and SV models to confirm the existence of convergence. All figures are available upon request.

[13] Information from the U.S. Geological Survey (USGS) annual reports was used to identify the historic events that may have affected gold and copper volatility directly.

[14] The Tables include the Ljung-Box Q and McLeod-Li Q_2 statistics applied to the standardized residuals and their squares, respectively. In both cases the null hypothesis is the absence of autocorrelation.

[15] The average number of jumps is denoted as $j = \frac{\kappa \times n}{N}$, where κ is the jump probability, n is the number of observations, and N is the number of years in the sample.

returns. SV-L yields $\rho > 0$, implying a positive relationship between shocks on the returns process and on volatility; i.e., there is evidence of an inverse leverage effect as in GARCH-L. In this respect, Brooks and Prokopzuk (2013) also find that $\rho > 0$ and suggest that this kind of asymmetry, which is the opposite of what is normally found in stock markets, is created by changes in the share of market participants that perform hedging and speculative operations. They argue that when the returns are positive, the number of speculators increases more than the number of hedgers, and therefore future volatility will be higher when today's returns are positive. Thus, positive returns on gold may indicate the beginning of a new stress episode. Finally, SV-t provides the best goodness of fit, followed by SV-L and SV-1.The $Q(20)$ and $Q_2(20)$ statistics suggest that SV-2, and SV-t do not reject the null hypothesis of no autocorrelation in the standardized residuals and the square standardized residuals.

From an economic perspective, it is arguable that models with fat tails in both GARCH and SV families provide a better fit for gold. Unlike other commodities, gold is considered highly liquid in financial markets and plays a role as store of value, portfolio diversifier, and safe asset in adverse macroeconomic scenarios. Therefore, events that trigger increases in gold volatility are often associated with international instability episodes, which may explain the existence of fat tails. In regard to the evidence of inverse leverage effect, this feature indicates that events related to abrupt falls in gold returns[16] are more relevant in explaining volatility that those that triggered its role as safe asset[17]. Moreover, the role of safe asset is not always triggered as, for example, in the 1990s when the strength of the dollar and central bank gold sales deteriorated the role of gold as an investment asset, although events like the Asian and Russian crises took place in that period[18].

Figure 2 shows relevant episodes in the history of gold volatility fluctuations. The beginning of the sample in 1983 shows a growing trend in volatility, created by the launching of gold futures trading in the London and Tokyo stock exchanges and by the run to safe gold investments in the wake of the 1980s crisis. A jump in volatility took place in 1985, probably triggered by banks' adoption of a net seller position and the weakening of the dollar against European currencies and the yen. Later the collapse of stock exchanges around the world on October 19th, 1987 (Black Monday) caused a surge in the demand for gold[19].

Between August 1990 and February 1991, the Gulf War provoked an increasing trend in volatility. Next, the bursting of the Dotcom bubble in 2000 and the 9/11 attacks[20] brought volatility to new peaks and triggered gold's safe asset role in response to a weakening dollar. Other jumps were associated with bank sales increasing by 40% and 80% in 2005 and 2007, and with investment flows towards ETFs[21]. Later, the Global Financial Crisis (GFC) (September 2008) seriously affected

[16] Dollar appreciation episodes, unexpected gold sales by central banks, and appearance of substitute investment instruments, among others.

[17] The 1980s crisis, the Savings & Loan crisis, the 2001 and 2008 financial crises, geopolitical tensions between the U.S. and other countries, among other factors.

[18] Developments like the 1994 Mexican Crisis, the 1997 Asian Crisis, and the 1998 Russian Crisis created financial panic but did not drive investors to use gold massively as safe asset. On the contrary, in this period (1992-1999) gold returns experienced negative jumps.

[19] In that year, futures trading operations increased 20% relative to the months in the run-up to Black Monday (1986).

[20] It should be noted that the consolidation of the mining industry reduced the number of firms, limited the gold supply, and made the supply/demand balance more sensitive during the first decade of the new millennium.

[21] Additionally, the 2006 U.N. Security Council sanctions to curb Iran's nuclear program created considerable geopolitical tensions.

economic activity in most developed economies and provoked a massive run towards gold, in turn causing a surge in volatility until end-2009. An important development was the greater frequency of clusters in contrast to the ones occurred in the 1980s, which may be associated with the weakening of the dollar during the U.S. recessions from 2001 to a few years after the 2008 crisis. This is consistent with Batten et al. (2010), who indicate that gold responds to monetary variables like inflation and money supply growth.

Another abrupt increase in volatility in 2011 was caused by growing speculative demand for gold in the run-up to the European Union debt crisis. Next, a new peak in volatility occurred in 2013, associated with the upcoming change in the Fed's monetary stance, which would result in future interest rate increases and the phasing out of Quantitative Easing. This prompted a 50% fall in global gold investment.

3.2.2 Copper

Table 4 shows the results for the GARCH family. The GARCH-t model yields $\nu = 6.38$, suggesting the occurrence of extreme events. The average jump in GARCH-J is $\mu_k = -0.33$ (i.e., the fall in returns is almost twice as large as for gold), indicating that the series for returns on copper experienced more falls than jumps across the sample. Additionally, $\kappa = 0.10$, which translates into 25 jumps per year. GARCH-M shows evidence that market participants demand a risk premium. At the same time, there is no evidence of first-order serial correlation in the disturbances of the returns on copper. GARCH-L yields $\delta > 0$; i.e., there is a leverage effect where, given current negative returns, volatility will be greater in the future. Along these lines, Hammoudeh and Yuan (2008) find that $\delta > 0$ and argue that the returns on copper are asymmetrical because they are linked to global economic activity[22]. Finally, based on the values of the marginal log-likelihoods, GARCH-t provides the best fit, followed by GARCH-J and GARCH-2. The $Q(20)$ and $Q_2(20)$ statistics suggest that none of the models in the GARCH family reject the null hypothesis.

Table 5 shows the results for the SV family. SV-t yields $\nu = 10.52$ and at the same time, SV-J yields $\kappa = 0.04$ and $\mu_k = -0.07$, indicating 10 jumps per year on average and a lower negative jump than for gold. For example, Liu et al. (2014) find a jump probability of 0.05 and $\mu_k < 0$, similar to the results found in this paper. Additionally, there is no evidence of a risk premium and serial correlation in SV-M and SV-MA, respectively. SV-L shows a negative correlation between shocks on copper returns and their volatility, $\rho = -0.12$, similar to the findings by Liu et al. (2014). Finally, SV-t provides the best fit, followed by SV-2 and SV-L. The $Q(20)$ and $Q_2(20)$ statistics suggest that none of the models in the SV family, but only SV-2 and SV-J reject the null hypotheses.

The history of copper returns can be instrumental in explaining the findings in this paper. As copper is used mainly in manufacturing,[23] its volatility is governed by supply and demand movements. Gerwe (2016) points out that inventories, economic activity and the dollar exchange rate are copper price fundamentals[24]. Figure 2 shows a volatility peak in 1987, when COMEX and LME (London Metal Exchange) copper inventories experienced a fall caused by developments like the highest copper consumption in eight years, supply disruptions created by miners' strikes

[22] In contrast, Bracker and Smith (1999) find an inverse leverage effect, like in the case of gold, for three asymmetric GARCH models.

[23] Gerwe (2016) indicates that, in 2014, 39% of final copper consumption was used to produce electric and electronic goods, while 30% was used in construction.

[24] Gerwe (2016) also indicates that wars cause an increase in the demand for copper; however, the war periods mentioned in the section about gold were not accompanied by considerable volatility surges.

in Canada, operative problems in African ports, and concerns about COMEX's ability to fulfill contracts for physical delivery of copper[25]. Next, high volatility persisted between 1988 and 1989 because of strike announcements in Peru and Papua New Guinea and the closure of foundries in Chile and the U.S.

Later, in June 1996 the scandal around the Sumimoto Corporation (which reported USD 1.8 billion losses resulting from illicit copper trading operations) brought volatility to levels similar to the 1987 peak. Moreover, volatility remained high in response to concerns about LME regulatory agency practices.

Subsequently, in 2004 the market became sensitive to industry announcements, especially concerns about growth in China and other Emerging Market Economies. In 2005 volatility reached 2004 levels, reflecting developments like miners' strikes in Chile and the U.S. and market speculation (third and fourth quarters). Volatility jumped again in 2006 in response to falling COMEX and LME inventories and a sustained increase in Asian demand for copper.

Volatility in 2008 was associated with a price rally triggered by a drop in world copper inventories. The abrupt drop in returns at the beginning of the GFC was partially offset by a 38% increase in Chinese consumption, but volatility remained high until end-2009. Eventually, a new peak in volatility appeared at the end of 2011 as a consequence of Chinese imports and an inventory buildup in metal exchanges. At last, in the next years, volatility showed a downward trend in despite of changes in China's consumption pattern and other industry announcements.

In sum, evidence suggests that volatility in copper returns was governed by changes in world inventories, global consumption, market speculation, and industry announcements. The presence of fat tails and jumps in copper returns is founded on elements such as temporary adverse sentiment in the market for copper securities in response to price manipulation or increased uncertainty about potential production cuts. The results also suggest the presence of leverage effects in copper's volatility, as during global recession episodes world copper consumption falls relatively more than in a normal macroeconomic environment. Preference for SV-t over SV-L seems to be explained by the fact that the structure of SV-L does not generate sufficient variability to capture the impact of more transitory events compared with crisis events, which have a longer effect on financial market sentiment.

3.2.3 Oil

Table 6 shows the results for the GARCH family. GARCH-t estimation yields $\nu = 7.63$, suggesting the existence of extreme values. The average jump in GARCH-J is -0.55, indicating that, on average, jumps in returns were negative. Additionally, the jump probability is 0.09, which translates into 23 jumps per year. Compared with copper and gold, falls in oil returns are more abrupt. GARCH-M shows evidence of a risk premium demanded by market participants to invest in the oil market. Again, there is no evidence of first-order serial correlation in GARCH-MA. Regarding GARCH-L, $\delta > 0$; i.e., there is evidence of asymmetry in oil returns. This is consistent with the literature; see for example Nomikos and Andriasopoulos (2012) and Chan and Grant (2016a). Regarding goodness of fit, GARCH-t is preferred, followed by GARCH-J and GARCH-L. Also, the $Q(20)$ and $Q_2(20)$ statistics show that none of the models reject the no autocorrelation null hypothesis. Moreover, oil results in Chan and Grant(2016a) are qualitatively similar to the ones obtained in this research.

[25] In October 1987, the CFTC (Commodity Future Trading Commission) unsuccessfully looked for evidence of copper price manipulation.

For example, their GARCH-J model yields $\mu_k < 0$ and a jump intensity of 0.05, while GARCH-L yields $\delta > 0$. The only exception is GARCH-MA, which indicates the presence of first-order serial correlation and provides the best fit in the GARCH family. This last feature may be a consequence of using weekly series where first-order autocorrelation exist, in contrast with using daily data.

Table 7 shows the results for the SV family. The SV-t model yields $\nu = 12.58$, which is much higher that for gold and copper; and indicates that oil returns show innovations that further depart from the t-Student distribution. This is also present in the results obtained by Chant and Grant (2016a), where the value of ν is higher than 40. In SV-J, the jump probability is 0.05 and $\mu_k < 0$. In the same line, Larsson and Nossman (2011) and Brooks and Prokopczuk (2013) find that the jump probability in SV-J is 0.01 and 0.025, respectively; i.e., six jumps per year on average. Additionally, both calculate a negative jump value for the returns. In contrast, neither SV-M nor SV-MA provide an additional contribution in modeling oil volatility. The SV-L model shows a negative correlation between shocks on oil returns and their volatility because $\rho = -0.25$, which is similar to the results found by Brooks and Prokopczuk (2013). Based on the values for the marginal log-likelihood, the selected model is SV-t, followed by SV-L and SV-2. Comparing the results in this paper for the SV family with those obtained by Chan and Grant (2016a), the jump value for the returns in SV-J is positive, $\mu_k > 0$. Additionally, SV-t tails are similar to those in a normal distribution, as $\nu = 56.13$. At the same time, they present SV-MA as the best model, while this paper finds a low goodness of fit like in SV-1. Again, these differences reflect the fact that the authors use weekly data beginning in 1994; i.e., they do not include the impact of the Gulf War and the 1990s crises on oil returns. Moreover, using weekly data implies smoothing out the extreme data that may appear in a daily basis. The $Q(20)$ and $Q_2(20)$ statistics show that none of the models reject the no autocorrelation null hypothesis.

The results suggest that introducing fat tails and a leverage effect is beneficial in the case of oil. While oil depends on supply/demand fundamentals like short-run supply inelasticity in the face of price changes, history shows that geopolitical conflicts, as well as announcements by the Organization of Petroleum Exporting Countries (OPEC) cartel, have a substantial impact on expectations about the global availability of oil inventories, and therefore about the process governing oil volatility. Figure 2 shows that the first volatility peak (also the largest in the whole sample) occurred during the Gulf War in 1990-1991. Another peak occurred, on May 23, 1998, when OPEC agreed to cut production to mitigate the decline in returns created by weak global consumption in the wake of the Asian crisis. Moreover, Operation Desert Fox (a massive bombing campaign against Iraq) was launched in December of the same year. Later, the bursting of the Dotcom bubble at end-2000 triggered a recession, which in turn caused a fall in U.S. oil consumption and negative returns in the oil market. Next, during the 2001-2002 period, the 9/11 attacks, the subsequent U.S. invasion of Iraq and an oil strike in Venezuela further exacerbated volatility.

In 2005, damages in the U.S. Gulf of Mexico oil facilities caused by the Rita and Katrina hurricanes and sustained Asian demand drove global oil inventories down and triggered widespread uncertainty. In the run-up to the GFC, oil volatility escalated as a result of a speculative commodity boom. When the GFC put an end to it, oil volatility climbed to its highest levels since the Gulf War. In February 2011, the Libyan civil war affected oil exports and created considerable uncertainty in the crude oil market. At last, Chinese slowdown contributed to increasing volatility since 2014; and economic and political instability took the market to considerable peaks in 2016.

In sum, the presence of fat tails originates in unexpected changes in the output of strategic crude producers and U.S.-OPEC political tensions. The results also suggest the presence of leverage effects, implying that negative oil returns today indicate higher volatility tomorrow. This usually takes place in the wake of economic crises, where a downward trend in oil returns prompts massive inventory selloffs and volatility surges as a consequence.

3.2.4 Natural Gas

Table 8 shows the results for the GARCH family. The GARCH-t model yields $\nu = 10.55$, similar to oil. At the same time, GARCH-J yields $\mu_k = 0.45$ and a jump probability of 0.09; i.e., 23 jumps per year. In contrast with previous series, only gas yields $\mu_k > 0$. The results for GARCH-J are consistent with the literature; e.g., Mason and Wilmot (2014) find a jump intensity of 0.015, with a positive jump value. Additionally, Nomikos and Andriosopoulos (2012) find a positive average jump with an intensity of occurrence of 0.058. At the same time, there is no evidence of a risk premium as a requirement to invest in natural gas market; and GARCH-MA does not make a significant contribution. In parallel, GARCH-L results suggest $\delta < 0$, pointing evidence of an inverse leverage effect like in the case of gold. Nomikos and Andriosopoulos (2012) argue this feature is explained by the fact that positive demand shocks dominated supply shocks, and therefore natural gas price increases become an indicator of volatility-enhancing scarcity. Moreover, based on the marginal log-likelihoods, the preferred model is GARCH-t, followed by GARCH-J and GARCH-L.

Table 9 shows the results for the SV family. While there is evidence of extreme events in the return series, innovations tend to a normal distribution, as SV-t yields $\nu = 48.95$. SV-J yields a jump probability of 0.05; i.e., 13 jumps per year on average, with a magnitude of 0.44. Again, the risk premium coefficient in SV-M and the serial correlation coefficient in SV-MA include zero in their respective credibility intervals. However, SV-L shows a positive asymmetric relationship between returns and volatility, implying an inverse leverage effect, as explained above. Finally, the marginal likelihoods indicate that the preferred model is SV-L, followed by SV-2 and SV-t.

The results suggest that the importance of the inverse leverage effect dominates the fat tail component. An inspection of high-volatility episodes in this market (Figure 2) shows that the first peak (February 2, 1996) was caused by scarce gas inventories and triggered a substantial price increase. Later, according to Roesser (2009), the Federal Energy Regulatory Commission (FERC) detected price manipulation to drive up the price of Western U.S. gas, which explains the volatility peak between end-2000 and the beginning of 2001. On February 2003, volatility surge was once more the result of supply/demand mismatches caused by severe winters, which affected U.S. gas distribution facilities and led to a peak in demand. The Katrina and Rita hurricanes (August and September 2005, respectively) caused serious damage to the Gulf of Mexico facilities and created considerable volatility in natural gas returns. The 2008 volatility peaks were caused by a mix of supply/demand fundamentals and financial market speculation. In April 2008, a gas leak in the Gulf of Mexico facilities caused concerns about a possible supply reduction in the context of the GFC. Figure 2 shows a growing trend in volatility until mid-2010 caused by global recovery.

The evidence of inverse leverage effect is consistent with the nature of the natural gas market, where production is inelastic to price. Therefore, the volatility of gas returns is highly sensitive to available supply, which may vary due to climatic developments and infrastructure restrictions. Positive returns indicate excess demand and therefore higher future uncertainty about natural gas supply conditions. This is consistent with Mu (2007) and Roesser (2009), who argue that an important fundamental in gas price is the supply/demand balance. More specifically, Mu (2007) finds that unexpected weekly inventory changes and temperature levels may cause uncertainty about future supply conditions. Moreover, Geman (2005) underscores that storage is costlier for gas than for oil, and therefore gas trade takes place in local markets, thereby reducing the industry's capacity to meet demand pressures.

The $Q(20)$ and $Q_2(20)$ statistics indicate that all models in both families reject the no autocorrelation null hypothesis[26].

3.2.5 Comparative Analysis

Tables 2-9 above show the marginal (log) likelihoods for each commodity and each of the seven GARCH and SV models. This allows calculation of the Bayes Factors (BFs hereinafter) to identify the best-fitting models.

First, GARCH-t (M_j following the notation in Section 2) is selected within the GARCH family. In this case, $BF_{ij} < 1$ indicates preference for M_j over M_i. For GARCH family, the results indicate that GARCH-t is better than all other models for all commodities. At the same time, GARCH-J is better than the other models except GARCH-t, for all commodities. Also, BF's show that not preferred models provide similar levels of goodness of fit.

On the other hand, SV-t (M_j) is selected from the SV family. This model is preferred for all commodities with exception of natural gas where SV-L is dominant. The BF is equal to one when comparing an SV-t and SV-2 for natural gas. Also, SV-L shows better performance than SV-J for all commodities. At last, as in GARCH family, BF's show that goodness of fit levels are similar between not preferred models.

Futhermore, when both families (GARCH and SV) are compared. The following models are selected: GARCH-t for gold; SV-t for copper and oil; and SV-L for natural gas[27].

According to the results, three SV models and one GARCH model were selected. This is explained by the structure of SV models, which includes a stochastic disturbance in the volatility process, while its GARCH counterparts assume that volatility is governed by a deterministic process. This makes the SV volatility process more flexible, as it incorporates information on events affecting the volatility process in real time. Moreover, our results are in line with Danielson (1994), Kim et al. (1998), Nakajima (2012) and Chant and Grant (2016a) that use a Bayesian approach to compare GARCH and SV models and suggest that SV family outperforms GARCH one on average. The only exception is GARCH-t for gold which outperforms every model in SV family. This can be possible as demostrated in Kim et al. (1998) and Nakajima (2012) when incorporating fat tails to GARCH model structure.

In relation to the above, the jump and fat tail extensions contribute to a lesser extent to SV family performance against its counterpart GARCH. This is reflected in the parameter estimates. Regarding the jump component, parameters κ and μ_k are higher on average in GARCH models, while parameter σ_k^2 is lower. An explanation for this feature, is that the jump component is capturing excess of kurtosis that GARCH structure cannot do by itself, which might be information not necessarily related to a jump event. Therefore, including "non-jump events information" in the jump component might introduce bias in parameter values. Jumps are supposed to be rare events that appear in the sample. On the contrary, SV models are more compatible with this assumption

[26] An option for ensuring no autocorrelation in the residuals is including more lags in the return or volatility equations. However, this warrants careful assessment in future research, as it would involve models different from the ones used in this paper.

[27] Tables showing the BFs are available upon request.

because their structure allows them to capture a greater proportion of the excess of kurtosis related to "non-jump events" than GARCH models do. As a consequence, the jump component in SV-J models is more likely to admit only extreme values which in turn produce more unfrequent(lower κ), larger (higher μ_k) and less disperse (lower σ_k^2) jumps in the sample. Our results are in line with Nakajima (2012) that finds lower kappas for SV-J models than for GARCH-J models, and suggests that when an excess shock appears, the volatility process of the SV-J model will capture the shock, while the GARCH model will capture it by the jump component. Nevertheless, in terms of performance, we find that SV-J models are usually outperformed by the basic SV model. Similar results are found in Nakajima (2009,2012) where SV and SV-J models showed similar performance when applied to model S&P 500, NASDAQ AND TOPIX indexes.

Regarding the fat tail component, the value of ν is lower in GARCH models than in SV models. The lower the value of ν, the larger the mass of probability allocated at the tails of the returns distribution. Therefore, this implies a higher probability of ocurrence of extreme events. As in the jump component, the fat tail component greatly increases GARCH models performance, not only by capturing extreme event information, but also by capturing excess of kurtosis GARCH structure is not able to do by itself. On the opposite, SV models capture enough variability, therefore, increasing the value of ν as a result. This last feature explains why ν parameter in SV models are usually higher which indicates that perturbances of the model are more likely to aproximate to a normal distribution.

Besides, we find evidence of leverage effect in all the return series, nonetheless, there are differences in terms of performance. For instance, for all products as except for natural gas, GARCH-L model is outperform by basic GARCH model on average. However, in SV family, SV-L model usually outperforms the basic SV model and also the SV-t model for the case of natural gas. This feature can be explain by the volatility process structure of SV models which can capture more excess kurtosis than GARCH models with no fat tail or jumps components incorporated. This is supported by Nakajima (2009,2012) results where the inclusion of leverage effect in SV models or GARCH models with fat tails or jumps components incorporated raise the level of goodness of fit. Additionally, this make sense with the levels of kurtosis of the products. We infer that the fat tail component is usually more beneficial for series with high kurtosis, whereas, the leverage component becomes more beneficial when kurtosis is lower as in natural gas. Regardless of the level of kurtosis, we can conclude that leverage effect is important for modeling commodity volatility dynamics.

3.3 Robustness Analysis

A sensitivity analysis is performed on the change in priors. For the sake of simplicity and brevity, only the best models in each family are re-estimated for each commodity. Robustness of the value and sign of the estimated coefficient is tested, as well as the marginal log-likelihood in a scenario of non-informative priors[28]. Therefore, the following hyper-parameters are assumed: $V_{\mu_0} = 100$, $V_{\gamma_0} = diag(100, 100, 100)$, $V_{\delta_0} = diag(100, 100)$ for GARCH-t and GARCH-J; and $V_{\mu_0} = 100$, $V_{\phi_{h_1}} = 100$, $V_{\mu} = 100$, $V_{\mu_h} = 100$, $\nu_h = 2.5$, $V_{\rho} = 100$ for SV-t and SV-L.

Table 10 shows the results for four commodities and for GARCH-t and GARCH-J. They suggest a slight goodness-of-fit decrease in GARCH-J for oil, equal to 1.4% relative to the initial value. Table 11 shows the same results for SV-t and SV-L. Evidence suggests similar conclusions. For example,

[28] A scenario using informative priors was also run. The results are available upon request.

marginal log-likelihood in SV-t for gold decreases by 0.21%. Parameters ν and ρ decrease for copper and natural gas in SV-t and SV-L, respectively.

It is possible to re-calculate the BFs using the new marginal log-likelihoods, but the order of preference of models for each commodity remains unchanged. GARCH-t continues to dominate the SV family in modeling gold volatility. SV-t continues to be preferred over GARCH-t for copper and oil, while again SV-L dominates SV-L and GARCH-t in modeling natural gas.

In sum, using of non-informative priors in both families does not introduce relevant changes in the results. The previously selected models (GARCH-t, SV-t, and SV-L) continue to provide the best fit for their respective commodity. No significant changes were identified in the value of the parameters estimated for both families relative to the baseline scenario. Additionally, there are no changes in the sign of the estimated coefficients.

4 Conclusions

Seven GARCH and SV models are compared to model empirically the volatility of returns on four commodities relevant for South America economies. We find that SV models outperform GARCH models on average. Overall, GARCH-t is the best model in both families for modeling gold volatility. However, SV-t outperforms the rest of models for copper and oil, while SV-L performs best for natural gas. The inclusion of fat tails and jumps components largely raise the performance of GARCH models, while this contribution is less for SV models. Even, SV-J models are usually outperform by the basic SV model. This findings are compatible with the results presented in Chan and Grant (2016a) and Nakajima (2009,2012). Moreover, fat tails are preferred in gold, cooper and oil which have historically shown sensitivity to global macroeconomic instability, geopolitical tensions, market speculation, as except for natural gas where short-term supply/demand mismatches are more relevant to explain its volatility dynamics.

Futhermore, there is evidence of leverage effect for all the products. Oil and copper show a standard leverage effect, where negative returns today indicate greater future uncertainty. In contrast, the results also suggest evidence of inverse leverage effects gold and gas show; i.e., positive returns today increase volatility tomorrow. Additionally, most products do not show evidence of a risk premium or MA-type first-order serial correlation.

A long-term view of volatility dynamics under different stress episodes that impacted commodity markets allow us to have better knowledge of what components contribute more to its modeling. Our results suggest that a development strategy based on primary commodity trade may involve greater volatility in macroeconomic aggregates, in turn translating into higher welfare costs because of the evidence of extreme events. Also, policy makers must focus on countercyclical economic policy because of the existence of leverage effect in commodities such as copper or oil. To cope with this issues fiscal programs such as stabilization funds or stronger promotion of derivative markets must be in the agenda.

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5 Tables

Table 1. Summary Statistics for Returns and Squared Returns

	Mean	Std. Deviation	Skewness	Kurtosis	Min.	Max.	Obs.
Panel A: Returns							
Gold	0.011	1.055	-0.214	9.910	-9.811	8.830	8653
Copper	0.015	1.619	-0.234	7.408	-12.516	11.902	8653
Oil	0.013	2.211	-0.825	18.385	-38.404	13.572	7643
Natural Gas	0.007	3.131	0.035	5.185	-16.698	18.765	5872
Panel B: Squared Returns							
Gold	1.100	3.300	11.900	221.500	0.000	96.300	8651
Copper	2.600	6.600	8.500	121.400	0.000	156.600	8651
Oil	4.900	20.400	51.300	3609.100	0.000	1474.900	7641
Natural Gas	9.800	20.000	6.000	59.300	0.000	352.100	5871

Table 2. Gold: Parameter Posterior Means, Standard Deviations (in parentheses) and 95% Credibility Intervals (in brackets) for the GARCH Models

	GARCH	GARCH-2	GARCH-t	GARCH-J	GARCH-M	GARCH-MA	GARCH-L
μ	0.007 (0.010) [−0.012,0.026]	0.007 (0.010) [−0.012,0.026]	0.007 (0.008) [−0.008,0.022]	0.012 (0.008) [−0.005,0.028]	−0.011 (0.014) [−0.038,0.017]	0.007 (0.009) [−0.012,0.026]	0.020 (0.010) [0.001,0.040]
α_o	0.034 (0.000) [0.034,0.034]	0.035 (0.000) [0.035,0.036]	0.002 (0.000) [0.002,0.002]	0.003 (0.000) [0.002,0.003]	0.034 (0.000) [0.034,0.035]	0.034 (0.000) [0.034,0.034]	0.042 (0.000) [0.042,0.042]
α_1	0.077 (0.000) [0.076,0.078]	0.080 (0.000) [0.079,0.081]	0.024 (0.000) [0.023,0.025]	0.031 (0.000) [0.031,0.032]	0.077 (0.000) [0.076,0.078]	0.077 (0.000) [0.076,0.078]	0.109 (0.001) [0.106,0.111]
β_1	0.896 (0.000) [0.895,0.896]	0.838 (0.001) [0.836,0.839]	0.954 (0.001) [0.952,0.955]	0.949 (0.000) [0.948,0.949]	0.896 (0.000) [0.895,0.896]]	0.896 (0.000) [0.895,0.896]	0.883 (0.000) [0.882,0.884]
β_2	-	0.054 (0.001) [0.052,0.056]	-	-	-	-	-
κ	-	-	-	0.100 (0.000) [0.100,0.100]	-	-	-
μ_k	-	-	-	−0.144 (0.086) [−0.314,0.024]	-	-	-
σ_k^2	-	-	-	3.663 (0.237) [3.208,4.149]	-	-	-
λ	-	-	-	-	0.021 (0.013) [−0.004,0.046]	-	-
ψ	-	-	-	-	-	−0.001 (0.012) [−0.025,0.024]	-
ν	-	-	3.947 (0.020) [3.908,3.983]	-	-	-	-
δ	-	-	-	-	-	-	−0.050 (0.002) [−0.054,−0.046]
$LogL$	−11930.3 (0.09)	−11928.4 (0.18)	−11154.7 (0.20)	−11245.5 (0.10)	11936.5 (0.19)	−11933.5 (0.12)	−11935.1 (0.15)
$Q(20)$	0.4339	0.4319	0.3502	0.3719	0.4294	0.3726	0.4070
$Q_2(20)$	0.9999	0.9998	0.9902	0.9992	0.9999	0.9999	1.0000

Notes: $Q(20)$ and $Q_2(20)$ are the p-values of the Ljung-Box and McLeod-Li statistics where the null hypotheses are no autocorrelation in the standarized residuals and standarized squared residuals, respectively.

Table 3. Gold: Parameter Posterior Means, Standard Deviations (in parentheses) and 95% Credibility Intervals (in brackets) for the SV Models

	SV	SV-2	SV-t	SV-J	SV-M	SV-MA	SV-L
μ	0.00 (0.01) [−0.011,0.019]	0.00 (0.01) [−0.012,0.018]	0.01 (0.01) [−0.008,0.021]	0.01 (0.01) [−0.008,0.021]	−0.00 (0.01) [−0.022,0.019]	0.00 (0.01) [−0.011,0.020]	0.01 (0.00) [−0.004,0.032]
μ_h	−0.33 (0.08) [−0.483,0.168]	−0.24 (0.17) [−0.502,0.069]	−0.35 (0.09) [−0.542,−0.168]	−0.36 (0.08) [−0.517,−0.191]	−0.33 (0.08) [−0.480,−0.170]	−0.32 (0.08) [−0.471,−0.169]	−0.310 (0.08) [−0.478,−0.141]
ϕ_{h_1}	0.97 (0.00) [0.956,0.975]	0.94 (0.00) [0.938,0.948]	0.98 (0.00) [0.971,0.981]	0.97 (0.00) [0.968,0.978]	0.97 (0.01) [0.952,0.975]	0.96 (0.01) [0.952,0.974]	0.97 (0.00) [0.963,0982]
ω_h^2	0.05 (0.01) [0.042,0.070]	0.04 (0.00) [0.006,0.054]	0.04 (0.00) [0.040,0.042]	0.04 (0.00) [0.039,0.041]	0.06 (0.01) [0.042,0.077]	0.06 (0.01) [0.043,0.072]	0.04 (0.00) [0.033,0.059]
ϕ_{h_2}	-	0.03 (0.01) [0.038,0.041]	-	-	-	-	-
κ	-	-	-	0.01 (0.00) [0.006,0.021]	-	-	-
μ_k	-	-	-	−0.35 (0.07) [−0.503,−0.253]	-	-	-
σ_k^2	-	-	-	7.78 (1.93) [5.057,10.577]	-	-	-
λ	-	-	-	-	0.01 (0.01) [−0.016,0.040]	-	-
ψ	-	-	-	-	-	−0.02 (0.01) [−0.045,−0.001]	-
ν	-	-	12.60 (1.07) [10.684,14.901]	-	-	-	-
ρ	-	-	-	-	-	-	0.20 (0.032) [0.149,0.265]
$LogL$	−11327.8 (0.12)	−11332.4 (0.18)	−11199.1 (0.69)	−11327.9 (0.57)	−11334.2 (0.13)	−11329.6 (0.19)	−11320.3 (0.23)
$Q(20)$	0.281	0.056	0.141	0.283	0.328	0.362	0.325
$Q_2(20)$	0.000	0.176	0.191	0.001	0.000	0.000	0.000

Notes: $Q(20)$ and $Q_2(20)$ are the p-values of the Ljung-Box and McLeod-Li statistics where the null hypotheses are no autocorrelation in the standarized residuals and standarized squared residuals, respectively

Table 4. Copper: Parameter Posterior Means, Standard Deviations (in parentheses) and 95% Credibility Intervals (in brackets) for the GARCH Models

	GARCH	GARCH-2	GARCH-t	GARCH-J	GARCH-M	GARCH-MA	GARCH-L
μ	0.013 (0.014) [−0.016,0.040]	0.013 (0.014) [−0.015,0.040]	0.016 (0.013) [−0.010,0.042]	0.032 (0.014) [0.004,0.060]	−0.058 (0.019) [−0.096,−0.020]	0.013 (0.015) [−0.016,0.041]	0.007 (0.014) [−0.020,0.035]
α_o	0.033 (0.000) [0.032,0.033]	0.034 (0.000) [0.034,0.035]	0.008 (0.000) [0.007,0.008]	0.003 (0.000) [0.003,0.004]	0.033 (0.000) [0.033,0.034]	0.033 (0.000) [0.032,0.033]	0.047 (0.000) [0.046,0.047]
α_1	0.058 (0.000) [0.057,0.058]	0.060 (0.000) [0.059,0.061]	0.032 (0.001) [0.031,0.033]	0.040 (0.001) [0.039,0.041]	0.058 (0.000) [0.057,0.058]	0.058 (0.000) [0.057,0.059]	0.057 (0.001) [0.056,0.059]
β_1	0.930 (0.000) [0.929,0.930]	0.874 (0.001) [0.872,0.875]	0.951 (0.001) [0.950,0.953]	0.948 (0.000) [0.948,0.949]	0.929 (0.000) [0.928,0.930]	0.930 (0.000) [0.929,0.931]	0.916 (0.000) [0.915,0.917]
β_2	-	0.053 (0.001) [0.051,0.054]	-	-	-	-	-
κ	-	-	-	0.099 (0.001) [0.095,0.100]	-	-	-
μ_k	-	-	-	−0.326 (0.119) [−0.561,−0.095]	-	-	-
σ_k^2	-	-	-	4.518 (0.362) [3.849,5.272]	-	-	-
λ	-	-	-	-	0.040 (0.007) [0.026,0.055]	-	-
ψ	-	-	-	-	-	−0.005 (0.012) [−0.028,0.018]	-
ν	-	-	6.385 (0.043) [6.302,6.467]	-	-	-	-
δ	-	-	-	-	-	-	0.017 (0.002) [0.013,0.020]
$LogL$	−15081.1 (0.23)	−15081.8 (0.11)	−14844.0 (0.09)	−14864.8 (0.19)	−15088.8 (0.34)	−15085.2 (0.09)	−15097.4 (0.10)
$Q(20)$	0.6003	0.6002	0.6784	0.6988	0.5167	0.5537	0.5498
$Q_2(20)$	0.7914	0.8148	0.1337	0.4436	0.8198	0.7902	0.9013

Notes: $Q(20)$ and $Q_2(20)$ are the p-values of the Ljung-Box and McLeod-Li statistics where the null hypotheses are no autocorrelation in the standarized residuals and standarized squared residuals, respectively.

Table 5. Copper: Parameter Posterior Means, Standard Deviations (in parentheses) and 95% Credibility Intervals (in brackets) for the SV Models

	SV	SV-2	SV-t	SV-J	SV-M	SV-MA	SV-L
μ	0.02 (0.01) [−0.009,0.042]	0.02 (0.01) [−0.008,0.044]	0.02 (0.01) [−0.007,0.043]	0.02 (0.01) [−0.011,0.043]	0.02 (0.02) [−0.022,0.058]	0.02 (0.01) [−0.009,0.041]	0.01 (0.01) [−0.017,0.036]
μ_h	0.60 (0.11) [0.390,0.808]	0.59 (0.12) [0.387,0.788]	0.41 (0.13) [0.139,0.662]	0.55 (0.08) [0.397,0.713]	0.60 (0.11) [0.388,0.815]	0.60 (0.11) [0.387,0.809]	0.60 (0.11) [0.390,0.815]
ϕ_{h_1}	0.99 (0.00) [0.979,0.990]	0.84 (0.08) [0.689,0.98]	0.99 (0.01) [0.969,0.994]	0.97 (0.00) [0.968,0.978]	0.99 (0.00) [0.980,0.991]	0.99 (0.00) [0.980,0.990]	0.99 (0.00) [0.981,0.990]
ω_h^2	0.02 (0.00) [0.013,0.023]	0.04 (0.01) [−0.019,0.285]	0.02 (0.01) [0.007,0.040]	0.04 (0.00) [0.037,0.039]	0.02 (0.00) [0.013,0.022]	0.02 (0.00) [0.013,0.023]	0.02 (0.00) [0.013,0.022]
ϕ_{h_2}	-	0.13 (0.08) [0.024,0.049]	-	-	-	-	-
κ	-	-	-	0.04 (0.02) [0.0130,0.0830]	-	-	-
μ_k	-	-	-	−0.07 (0.20) [−0.4029,0.4312]	-	-	-
σ_k^2	-	-	-	2.04 (0.85) [0.893,4.160]	-	-	-
λ	-	-	-	-	−0.00 (0.01) [−0.021,0.019]	-	-
ψ	-	-	-	-	-	−0.02 (0.01) [−0.046,−0.002]	-
ν	-	-	10.52 (2.38) [7.775,16.597]	-	-	-	-
ρ	-	-	-	-	-	-	−0.12 (0.05) [−0.220,−0.022]
$LogL$	−14893.6 (0.03)	−14890.9 (0.15)	−14839.0 (0.02)	−14920.4 (0.40)	−14900.4 (0.06)	−14895.2 (0.07)	−14893.9 (0.04)
$Q(20)$	0.282	0.227	0.345	0.156	0.301	0.492	0.313
$Q_2(20)$	0.063	0.022	0.519	0.012	0.083	0.047	0.066

Notes: $Q(20)$ and $Q_2(20)$ are the p-values of the Ljung-Box and McLeod-Li statistics where the null hypotheses are no autocorrelation in the standarized residuals and standarized squared residuals, respectively

Table 6. Oil: Parameter Posterior Means, Standard Deviations (in parentheses) and 95% Credibility Intervals (in brackets) for the GARCH Models

	GARCH	GARCH-2	GARCH-t	GARCH-J	GARCH-M	GARCH-MA	GARCH-L
μ	0.033 (0.001) [−0.005,0.071]	0.034 (0.019) [−0.004,0.072]	0.035 (0.019) [−0.001,0.071]	0.056 (0.021) [0.015,0.097]	−0.023 (0.025) [−0.071,0.025]	0.034 (0.020) [−0.005,0.073]	0.022 (0.020) [−0.015,0.06]
α_o	0.052 (0.001) [0.051,0.053]	0.054 (0.001) [0.053,0.056]	0.024 (0.001) [0.022,0.026]	0.017 (0.001) [0.015,0.020]	0.052 (0.001) [0.051,0.054]	0.052 (0.001) [0.051,0.054]	0.066 (0.001) [0.065,0.067]
α_1	0.078 (0.001) [0.077,0.079]	0.081 (0.001) [0.080,0.082]	0.045 (0.001) [0.044,0.047]	0.055 (0.001) [0.053,0.058]	0.077 (0.001) [0.076,0.079]	0.078 (0.001) [0.077,0.079]	0.070 (0.001) [0.068,0.072]
β_1	0.914 (0.001) [0.913,0.916]	0.859 (0.001) [0.857,0.861]	0.934 (0.001) [0.931,0.936]	0.930 (0.001) [0.928,0.932]	0.915 (0.001) [0.913,0.916]	0.914 (0.001) [0.913,0.916]	0.906 (0.001) [0.905,0.907]
β_2	-	0.052 (0.001) [0.051,0.054]	-	-	-	-	-
κ	-	-	-	0.093 (0.007) [0.075,0.100]	-	-	-
μ_k	-	-	-	−0.547 (0.188) [−0.920,−0.186]	-	-	-
σ_k^2	-	-	-	7.902 (0.951) [6.270,10.034]	-	-	-
λ	-	-	-	-	0.018 (0.005) [0.009,0.027]	-	-
ψ	-	-	-	-	-	0.006 (0.013) [−0.019,0.032]	-
ν	-	-	7.632 (0.061) [7.514,7.747]	-	-	-	-
δ	-	-	-	-	-	-	0.027 (0.002) [0.023,0.031]
$LogL$	−15767.3 (0.10)	−15768.6 (0.19)	−15620.1 (0.08)	−15655.3 (0.05)	−15775.5 (0.15)	−15770.9 (0.09)	−15774.5 (0.15)
$Q(20)$	0.7394	0.7373	0.7653	0.7923	0.7273	0.6871	0.7149
$Q_2(20)$	0.3779	0.3626	0.2612	0.2889	0.4668	0.3761	0.4777

Notes: $Q(20)$ and $Q_2(20)$ are the p-values of the Ljung-Box and McLeod-Li statistics where the null hypotheses are no autocorrelation in the standarized residuals and standarized squared residuals, respectively

Table 7. Oil: Parameter Posterior Means, Standard Deviations (in parentheses) and 95% Credibility Intervals (in brackets) for the SV Models

	SV	SV-2	SV-t	SV-J	SV-M	SV-MA	SV-L
μ	0.04 (0.02) [−0.000,0.074]	0.04 (0.02) [0.003,0.081]	0.04 (0.02) [0.000,0.074]	0.05 (0.02) [0.011,0.096]	0.07 (0.03) [0.009,0.122]	0.04 (0.02) [0.000,0.072]	0.02 (0.02) [−0.021,0.054]
μ_h	1.20 (0.11) [0.985,1.415]	1.18 (0.18) [0.852,1.467]	1.05 (0.13) [0.784,1.310]	1.11 (0.14) [0.838,1.370]	1.21 (0.11) [0.984,1.422]	1.20 (0.11) [0.987,1.420]	1.20 (0.11) [0.980,1.421]
ϕ_{h_1}	0.98 (0.00) [0.977,0.989]	0.84 (0.08) [0.675,0.972]	0.99 (0.00) [0.985,0.993]	0.99 (0.00) [0.983,0.993]	0.98 (0.00) [0.979,0.989]	0.98 (0.00) [0.978,0.989]	0.99 (0.00) [0.979,0.990]
ω_h^2	0.02 (0.00) [0.015,0.027]	0.04 (0.01) [0.000,0.302]	0.01 (0.00) [0.009,0.016]	0.02 (0.00) [0.011,0.020]	0.02 (0.00) [0.014,0.026]	0.02 (0.00) [0.015,0.025]	0.02 (0.00) [0.014,0.025]
ϕ_{h_2}	-	0.14 (0.08) [0.029,0.050]	-	-	-	-	-
κ	-	-	-	0.05 (0.02) [0.0132,0.0959]	-	-	-
μ_k	-	-	-	−0.73 (0.36) [−1.3286,−0.1462]	-	-	-
σ_k^2	-	-	-	8.65 (4.44) [3.402,18.930]	-	-	-
λ	-	-	-	-	−0.01 (0.01) [−0.026,0.004]	-	-
ψ	-	-	-	-	-	−0.01 (0.01) [−0.036,0.011]	-
ν	-	-	12.58 (1.80) [9.685,16.597]	-	-	-	-
ρ	-	-	-	-	-	-	−0.25 (0.05) [−0.355,−0.143]
$LogL$	−15625.0 (0.02)	−15623.8 (0.15)	−15598.3 (0.01)	−15628.7 (0.42)	−15631.0 (0.09)	−15628.4 (0.05)	−15616.5 (0.03)
$Q(20)$	0.569	0.531	0.626	0.373	0.568	0.586	0.582
$Q_2(20)$	0.129	0.107	0.214	0.201	0.102	0.134	0.159

Notes: $Q(20)$ and $Q_2(20)$ are the p-values of the Ljung-Box and McLeod-Li statistics where the null hypotheses are no autocorrelation in the standarized residuals and standarized squared residuals, respectively

Table 8. Natural gas: Parameter Posterior Means, Standard Deviations (in parentheses) and 95% Credibility Intervals (in brackets) for the GARCH Models

	GARCH	GARCH-2	GARCH-t	GARCH-J	GARCH-M	GARCH-MA	GARCH-L
μ	0.017 (0.035) [−0.051,0.089]	0.017 (0.035) [−0.051,0.085]	0.018 (0.035) [−0.049,0.086]	−0.009 (0.044) [−0.096,0.076]	0.024 (0.055) [−0.085,0.131]	0.018 (0.036) [−0.051,0.088]	0.044 (0.035) [−0.025,0.111]
α_o	0.216 (0.004) [0.208,0.225]	0.226 (0.004) [0.218,0.234]	0.164 (0.006) [0.152,0.175]	0.148 (0.005) [0.140,0.159]	0.215 (0.004) [0.207,0.224]	0.216 (0.004) [0.208,0.224]	0.225 (0.004) [0.217,0.233]
α_1	0.078 (0.001) [0.077,0.080]	0.081 (0.001) [0.080,0.083]	0.059 (0.001) [0.057,0.061]	0.069 (0.001) [0.067,0.071]	0.078 (0.001) [0.076,0.080]	0.078 (0.001) [0.077,0.080]	0.097 (0.001) [0.095,0.099]
β_1	0.902 (0.001) [0.900,0.904]	0.846 (0.001) [0.843,0.848]	0.908 (0.002) [0.905,0.912]	0.904 (0.001) [0.902,0.905]	0.902 (0.001) [0.900,0.904]	0.902 (0.001) [0.900,0.904]	0.902 (0.001) [0.900,0.903]
β_2	-	0.052 (0.001) [0.051,0.054]	-	-	-	-	-
κ	-	-	-	0.095 (0.006) [0.079,0.100]	-	-	-
μ_k	-	-	-	0.454 (0.354) [−0.249,1.142]	-	-	-
σ_k^2	-	-	-	13.009 (1.655) [10.012,16.669]	-	-	-
λ	-	-	-	-	−0.001 (0.006) [−0.012,0.010]	-	-
ψ	-	-	-	-	-	0.000 (0.014) [−0.028,0.027]	-
ν	-	-	10.554 (0.130) [10.305,10.800]	-	-	-	-
δ	-	-	-	-	-	-	−0.040 (0.001) [−0.042,−0.037]
$LogL$	−14525.0 (0.05)	−14526.7 (0.09)	−14477.9 (0.14)	−14493.5 (0.08)	−14530.5 (0.23)	−14528.7 (0.16)	−14520.5 (0.21)
$Q(20)$	0.0217	0.0212	0.0200	0.0234	0.0217	0.0163	0.0222
$Q_2(20)$	0.0009	0.0006	0.0005	0.0015	0.0009	0.0009	0.0006

Notes: $Q(20)$ and $Q_2(20)$ are the p-values of the Ljung-Box and McLeod-Li statistics where the null hypotheses are no autocorrelation in the standarized residuals and standarized squared residuals, respectively

Table 9. Natural gas: Parameter Posterior Means, Standard Deviations (in parentheses) and 95% Credibility Intervals (in brackets) for the SV Models

	SV	SV-2	SV-t	SV-J	SV-M	SV-MA	SV-L
μ	0.02 (0.03) [−0.048,0.086]	0.01 (0.04) [−0.056,0.082]	0.02 (0.03) [−0.049,0.086]	−0.00 (0.05) [−0.105,0.100]	0.05 (0.07) [−0.083,0.175]	0.02 (0.03) [−0.049,0.085]	0.04 (0.04) [−0.047,0.113]
μ_h	2.05 (0.08) [1.890,2.206]	2.04 (0.11) [1.857,2.208]	2.00 (0.09) [1.833,2.169]	2.01 (0.09) [1.833,2.178]	2.05 (0.08) [1.892,2.205]	2.05 (0.08) [1.891,2.212]	2.05 (0.08) [1.903,2.214]
ϕ_{h_1}	0.98 (0.00) [0.966,0.983]	0.92 (0.08) [0.750,1.053]	0.98 (0.00) [0.969,0.984]	0.98 (0.00) [0.967,0.984]	0.97 (0.00) [0.965,0.982]	0.98 (0.00) [0.967,0.983]	0.96 (0.03) [0.892,0.983]
ω_h^2	0.02 (0.00) [0.015,0.027]	0.03 (0.01) [0.022,0.045]	0.02 (0.00) [0.013,0.023]	0.02 (0.00) [0.015,0.027]	0.02 (0.00) [0.016,0.027]	0.02 (0.00) [0.014,0.025]	0.04 (0.03) [0.015,0.103]
ϕ_{h_2}	-	0.04 (0.08) [−0.106,0.213]	-	-	-	-	-
κ	-	-	-	0.05 (0.03) [0.0047,0.0974]	-	-	-
μ_k	-	-	-	0.44 (0.85) [−1.4073,2.0453]	-	-	-
σ_k^2	-	-	-	6.18 (5.06) [1.271,18.502]	-	-	-
λ	-	-	-	-	−0.00 (0.01) [−0.019,0.011]	-	-
ψ	-	-	-	-	-	−0.01 (0.01) [−0.035,0.015]	-
ν	-	-	48.95 (22.80) [19.830,95.881]	-	-	-	-
ρ	-	-	-	-	-	-	0.18 (0.11) [−0.040,0.322]
$LogL$	−14467.8 (0.03)	−14465.7 (0.14)	−14465.7 (0.02)	−14471.5 (0.13)	−14474.8 (0.04)	−14471.5 (0.01)	−14463.4 (0.07)
$Q(20)$	0.027	0.022	0.024	0.035	0.027	0.022	0.035
$Q_2(20)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Notes: $Q(20)$ and $Q_2(20)$ are the p-values of the Ljung-Box and McLeod-Li statistics where the null hypotheses are no autocorrelation in the standarized residuals and standarized squared residuals, respectively

Table 10. Robustness Check: Non-Informative Priors Estimates for the GARCH Models

	Gold		Copper		Oil		Natural Gas	
	GARCH-t	GARCH-J	GARCH-t	GARCH-J	GARCH-t	GARCH-J	GARCH-t	GARCH-J
μ	0.007 (0.008) [−0.008,0.022]	0.012 (0.008) [−0.004,0.028]	0.016 (0.013) [−0.010,0.042]	0.032 (0.015) [0.004,0.061]	0.034 (0.019) [−0.003,0.071]	0.057 (0.021) [0.013,0.098]	0.019 (0.035) [−0.049,0.086]	−0.008 (0.043) [−0.094,0.075]
α_o	0.002 (0.000) [0.002,0.002]	0.003 (0.000) [0.002,0.003]	0.007 (0.000) [0.007,0.008]	0.002 (0.000) [0.002,0.003]	0.024 (0.001) [0.022,0.026]	0.016 (0.001) [0.014,0.019]	0.163 (0.006) [0.152,0.175]	0.146 (0.005) [0.138,0.157]
α_1	0.024 (0.000) [0.023,0.025]	0.031 (0.000) [0.030,0.032]	0.031 (0.001) [0.030,0.033]	0.040 (0.001) [0.039,0.041]	0.045 (0.001) [0.044,0.047]	0.055 (0.001) [0.053,0.058]	0.059 (0.001) [0.056,0.061]	0.069 (0.001) [0.067,0.071]
β_1	0.954 (0.001) [0.953,0.956]	0.950 (0.000) [0.949,0.951]	0.952 (0.001) [0.950,0.954]	0.949 (0.000) [0.949,0.950]	0.934 (0.001) [0.932,0.936]	0.931 (0.001) [0.929,0.933]	0.909 (0.002) [0.905,0.913]	0.904 (0.001) [0.902,0.906]
κ	-	0.100 (0.000) [0.099,0.100]	-	0.100 (0.001) [0.096,0.100]	-	0.093 (0.007) [0.077,0.100]	-	0.094 (0.006) [0.079,0.100]
μ_k	-	−0.146 (0.085) [−0.312,0.021]	-	−0.327 (0.120) [−0.562,−0.092]	-	−0.548 (0.188) [−0.922,−0.171]	-	0.446 (0.351) [−0.253,1.122]
σ_k^2	-	3.646 (0.240) [3.203,4.149]	-	4.480 (0.368) [3.788,5.226]	-	7.861 (0.929) [6.306,9.981]	-	13.135 (1.731) [10.145,16.870]
ν	3.943 (0.019) [3.907,3.980]	-	6.375 (0.043) [6.293,6.457]	-	7.621 (0.061) [7.500,7.737]	-	10.542 (0.128) [10.297,10.784]	-
$LogL$	−11159.0 (0.23)	−11251.1 (0.16)	−14849.0 (0.09)	−14871.9 (0.20)	−15625.0 (0.17)	−15633.6 (0.13)	−14483.3 (0.10)	−14502.6 (0.24)
$Q(20)$	21.873	21.52	16.596	16.198	15.194	14.690	35.049	34.442
$Q_2(20)$	8.479	5.9046	27.8532	21.226	23.675	23.00	47.487	44.056

Notes: $Q(20)$ and $Q_2(20)$ are the p-values of the Ljung-Box and McLeod-Li statistics where the null hypotheses are no autocorrelation in the standarized residuals and standarized squared residuals, respectively

Table 11. Robustness Check: Non-Informative Priors Estimates for the SV Models

	Gold		Copper		Oil		Natural Gas	
	SV-t	SV-L	SV-t	SV-L	SV-t	SV-L	SV-t	SV-L
μ	0.009	0.014	0.018	0.011	0.037	0.016	0.018	0.044
	(0.007)	(0.008)	(0.013)	(0.014)	(0.019)	(0.019)	(0.035)	(0.035)
	[−0.006,0.024]	[−0.002,0.029]	[−0.008,0.044]	[−0.016,0.038]	[−0.001,0.074]	[−0.022,0.053]	[−0.050,0.088]	[−0.025,0.113]
μ_h	−0.383	−0.314	0.392	0.612	1.044	1.208	2.004	2.059
	(0.081)	(0.083)	(0.151)	(0.109)	(0.142)	(0.116)	(0.087)	(0.086)
	[−0.541,−0.223]	[−0.477,−0.152]	[0.093,0.691]	[0.401,0.829]	[0.993,1.424]	[0.980,1.440]	[1.833,2.175]	[1.893,2.230]
ϕ_{h_1}	0.966	0.970	0.993	0.987	0.991	0.986	0.978	0.977
	(0.003)	(0.004)	(0.002)	(0.003)	(0.002)	(0.002)	(0.004)	(0.004)
	[0.961,0.972]	[0.963,0.977]	[0.989,0.996]	[0.980,0.991]	[0.987,0.995]	[0.981,0.991]	[0.970,0.986]	[0.969,0.985]
ω_h^2	0.063	0.049	0.008	0.017	0.011	0.018	0.018	0.019
	(0.001)	(0.005)	(0.001)	(0.003)	(0.002)	(0.002)	(0.003)	(0.003)
	[0.061,0.065]	[0.039,0.058]	[0.006,0.011]	[0.012,0.023]	[0.007,0.015]	[0.014,0.023]	[0.013,0.023]	[0.014,0.026]
ν	12.677	-	9.120	-	12.088	-	45.121	-
	(1.107)		(0.906)		(1.701)		(20.571)	
	[10.748,15.084]		[7.537,11.086]		[9.403,16.047]		[18.446,92.466]	
ρ	-	0.196	-	−0.103	-	−0.266	-	0.231
		(0.032)		(0.054)		(0.055)		(0.058)
		[0.123,0.255]		[−0.216,−0.008]		[−0.364,−0.163]		[0.114,0.348]
$LogL$	−11222.7	−11325.5	−14839.5	−14898.0	−15600.4	−15621.0	−14469.9	−14468.2
	(1.07)	(0.17)	(0.01)	(0.02)	(0.01)	(0.02)	(0.01)	(0.01)
$Q(20)$	21.280	22.180	20.843	22.698	17.372	17.742	34.304	33.962
$Q_2(20)$	28.485	56.579	16.696	29.954	25.881	25.449	49.150	51.158

Notes: $Q(20)$ and $Q_2(20)$ are the p-values of the Ljung-Box and McLeod-Li statistics where the null hypotheses are no autocorrelation in the standarized residuals and standarized squared residuals, respectively

6 Figures

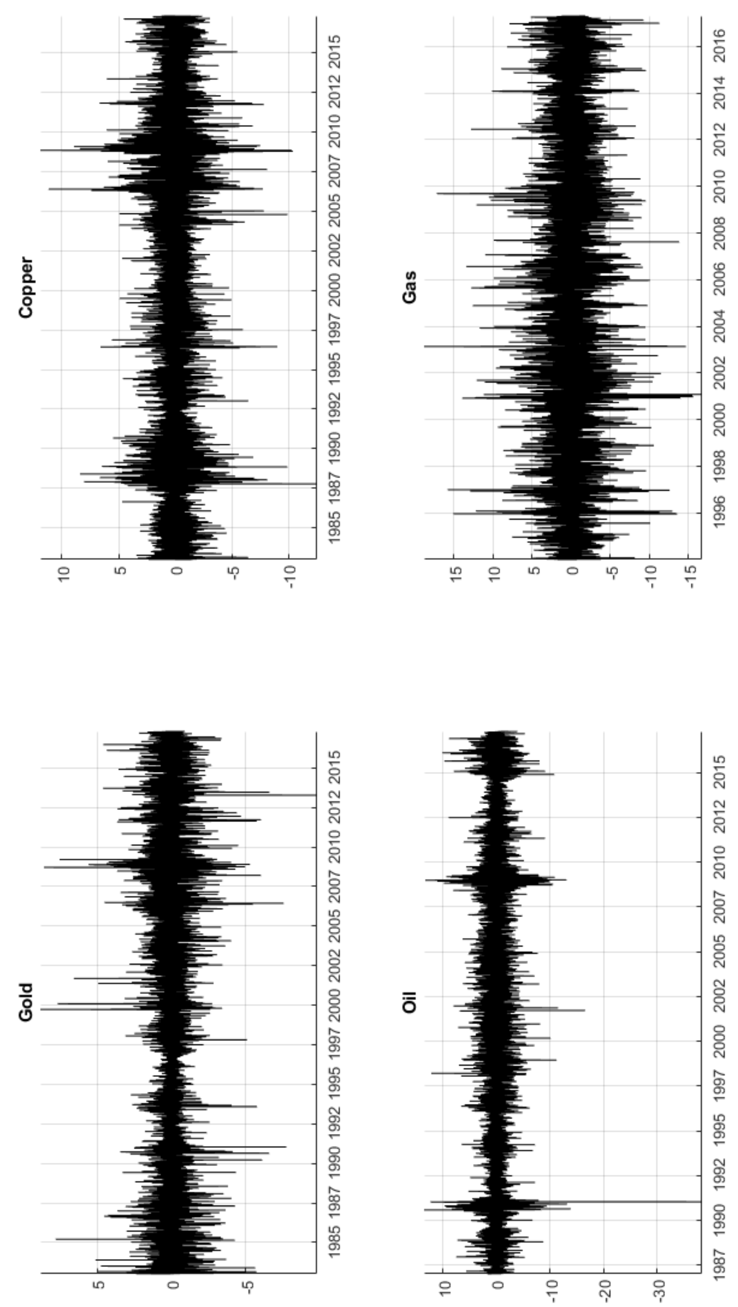

Figure 1. Commodity Daily Returns

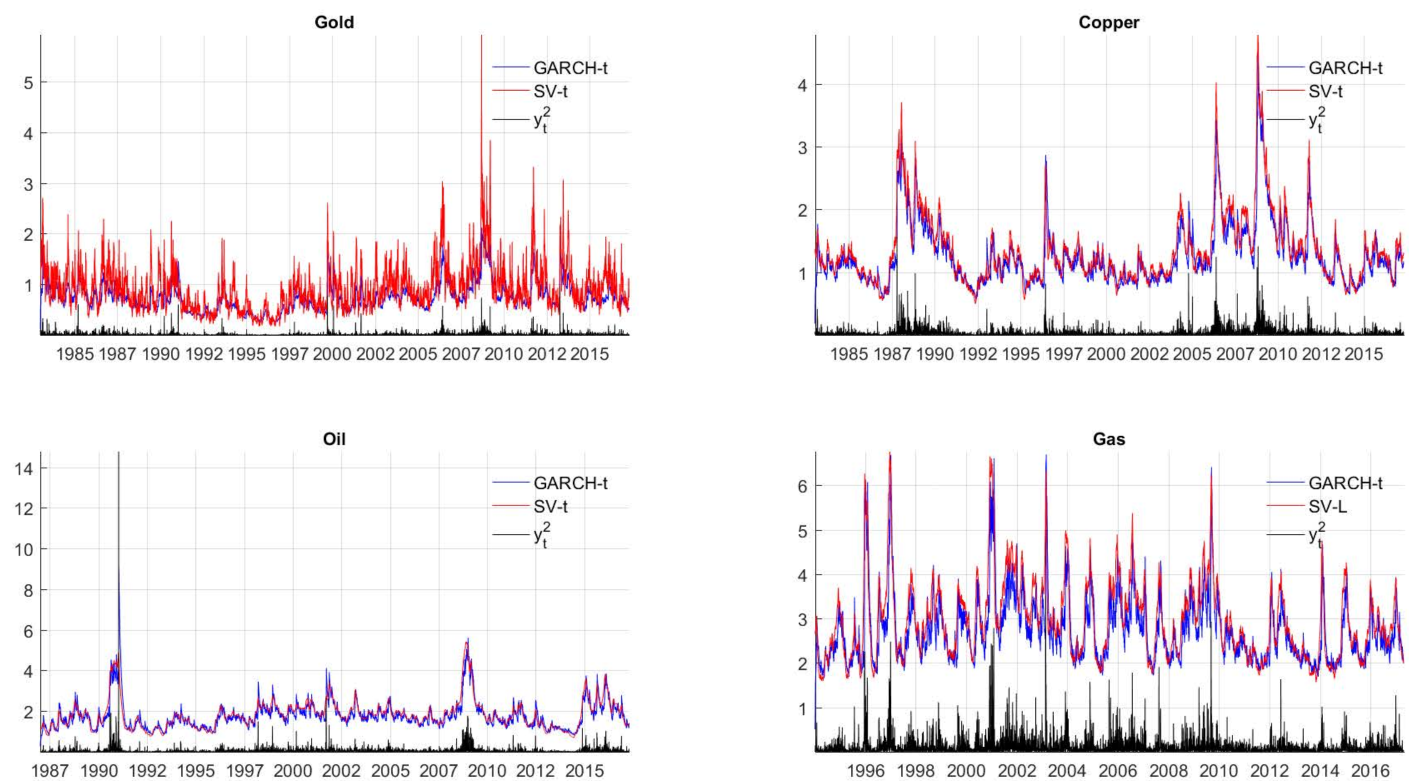

Figure 2. GARCH and SV Estimated Volatilities and Squared Returns