PONTIFICIA UNIVERSIDAD CATÓLICA DEL PERÚ

ESCUELA DE POSGRADO

# EMPIRICAL MODELING OF LATIN AMERICAN STOCK MARKETS RETURNS AND VOLATILITY USING MARKOV-SWITCHING GARCH MODELS

Tesis para optar el grado de Magíster en Economía con especialización en Finanzas y Mercados de Capitales que presenta

Miguel Ataurima Arellano

Dirigido por

Gabriel Rodríguez

San Miguel, 2016

# Empirical Modeling of Latin American Stock Markets Returns and Volatility using Markov-Switching GARCH Models

Miguel Ataurima Arellano
Pontificia Universidad Católica del Perú

August 12th, 2016

**Abstract**

Using a sample of weekly frequency of the stock markets returns series, we estimate a set of Markov-Switching-Generalized Autoregressive Conditional Heterocedasticity (MS-GARCH) models to a set of Latin American countries (Argentina, Brazil, Chile, Colombia, Mexico and Peru) with an approach based on both the Monte Carlo Expectation-Maximization (MCEM) and Monte Carlo Maximum Likelihood (MCML) algorithms suggested by Augustyniak (2014). The estimates are compared with a standard GARCH, MS and other models. The results show that the volatility persistence is captured differently in the MS and MS-GARCH models. The estimated parameters with a standard GARCH model exacerbates the volatility in almost double compared to MS-GARCH model. There is different behavior of the coefficients and the variance according the two regimes (high and low volatility) by each model in the Latin American stock markets. There are common episodes related to global international crises and also domestic events producing the different behavior in the volatility of each time series.

**JEL Classification:** C22, C52, C53.

**Keywords:** MS-GARCH Models, GARCH Models, Returns, Volatility, Latin-American Stock market.

# Contents

# 1 Introduction

Analyzing the characteristics of the stock market returns and volatility from Latin-American countries has been inspired by the crucial role they play in the crisis, for instance the global financial crisis of 2008-2009. An important ingredient during a crisis is the possibility of modeling and estimate volatility under a reasonable level of accuracy. Stock returns exhibit sudden jumps due not only to structural breaks in the real economy, but also to changes in expectations about the future, stemming from different information or dissimilar preferences. The real volatility is affected by shocks, that never persist for a long time, rendering its behavior mean-reverting. It follows that, a good stock return volatility model should entail a different way of treating shocks.

Time series of stock market returns have four typical stylized facts, according to Franses and Van Dijk (2000): *i*) large returns occur more often than expected (leptokurtosis or fat tails), which implies that the kurtosis is much larger than 3, or the tails of the distributions are fatter than the tails of the Normal distribution; *ii*) large stock market returns are often negative (negative skewness), which implies that the left tail of the distribution is fatter than the right tail, or that large negative returns tend to occur more often than large positive ones; *iii*) large returns tend to occur in clusters, which implies that relatively volatile periods, characterized by large price changes (large returns) alternate with quieter periods in which prices remain more or less stable (small returns); *iv*) large volatility often follows large negative stock market returns, which implies that periods of large volatility tend to be triggered by a large negative return (this stylized fact is also called “leverage effect”). These features of stock market returns require nonlinear models, simply because linear models would not be able to generate data with these features[1].

Most popular and widely nonlinear financial models used for modeling of volatility are generalized autoregressive conditional heteroskedasticity (GARCH), Engle (1982), Bollerslev (1986); and regime change models such as Markov Switching models (MS), Hamilton (1989), and Threshold Autoregressive models (TAR), Tong (1983, 1993). Due to the popularity presenting GARCH models by allowing explicitly modeling the volatility; and the ability of the MS models to model the distribution of returns under the regime type (or state of the economy) that this driving first past the unobservable Markov chain; it is interesting to combine and consider a single MS-GARCH model, which can be understood as a GARCH model in which the parameters depend on unobservable regime (periods of high or low volatility of returns on financial assets)[2].

As the exact calculation of the likelihood of MS-GARCH models is unfeasible in practice, because its estimate is dependent on the path, they have emerged in the literature various alternative methods to estimate it. In this paper we use the method presented by Augustyniak (2014), who manages to estimate the maximum likelihood estimator (MLE) of the MS-GARCH model using Monte Carlo Expectation-Maximization (MCEM) and Monte Carlo Maximum Likelihood (MCML) algorithms; and also get an approximation of the asymptotic standard errors of the MLE.

The objective of this research is to estimate the MS-GARCH parameters of the volatility of stock returns of the following Latin American stock markets: Argentina, Brazil, Colombia, Chile, Mexico and Peru; in order to distinguish episodes of high and low volatility that crossed each economy with more accurately, and recognize some common behavior

[1] For a review of stylized facts in stock market of Peru, see Humala and Rodríguez (2013).

[2] Lamoureux and Lastrapes (1990) justify this compact model, while Mikosch and Starica (2004) show that high persistence observed in the variance of financial returns can be explained by parameters time-varying GARCH.

pattern during financial turmoils. All these MS-GARCH models are compared with standard GARCH models in terms of their ability to estimate volatility, compared with MS models in terms of their ability to capture the volatility persistence and compared with other model in terms of maximum likelihood. The estimating performances of the competing models are evaluated using weekly frequency time of Latin American stock market returns.

The results shows that the fit of the MS-GARCH model in Latin-American countries is superior to standard GARCH, Gray and MS model according to the BIC, allowing capture the jumps in the long-term average value of volatility without exacerbating volatility. The temporal correlations between countries show that after the international financial crisis, correlations tend to be positive, revealing a kind of positive interdependence during episodes of financial turmoil.

The remaining of the paper is organized as follows. Section 2 presents the literature review. Section 3 presents the methodology to estimate the standard GARCH models, MS-GARCH models, the path dependence problem. Section 4 describes and analyzes the data and shows empirical results of the models. Conclusions are presented in Section 5. In the Appendix, we present the MCEM-MCML algorithm proposed by Augustyniak (2014).

## 2 Literature Review

So far in the literature, many volatility models have been put forward, but the most successful is Engle (1982) who formally introduces an autoregressive conditional heteroskedasticity model (ARCH) to explain the dynamic of inflation in the United Kingdom, on the basis of which a series of extensions are developed. For instance, Bollerslev (1986) presents a generalization of the ARCH (GARCH) process by allowing past conditional variances to be incorporated as regressors within the current conditional variance equation. GARCH models are popular because of their ability to capture many of the typical stylized facts of financial time series, such as time-varying volatility, persistence and volatility clustering.

MS-GARCH models begins with Hamilton and Susmel (1994) who are the first to apply simultaneously the seminal idea of endogenous regime-switching parameters by Hamilton (1988) into an ARCH specification to account for the possible presence of structural breaks. Nevertheless, they use an ARCH specification instead of a GARCH to overcome the problem of infinite path-dependence, i.e. to avoid the conditional variance at time $t$ depending on the entire sample path Hamilton and Susmel (1994) noted that the estimation is path dependent model is a challenging task because exact computation of the likelihood is infeasible in practice. Given this impasse, Gray (1996) , Dueker (1997), Klaassen (2002), Haas et al. (2004), among others, propose variants of the MS-GARCH model to avoid the problem of path dependency with maximum likelihood; while others suggested alternative estimation methods.

The first to suggest a method where the conditional distribution of returns is independent of the regime path was Gray (1996). He suggests to integrate out the unobserved regime path in the GARCH equation using the conditional expectation of the past variance. His model can be regarded as the first MS-GARCH.

Following this line, Klaassen (2002), generalize the regime-switching ARCH models of Cai (1994) and Hamilton and Susmel (1994) allowing a GARCH dynamics and compute multi-period-ahead volatility forecasts through a first-order recursive procedure allowing to use all available information, instead of using only part of it like Gray (1996). He used data on the three major U.S. dollar exchange rates, revealing that the variance dynamics differ across regimes, getting a better fit with his model.

As an empirical application, Moore and Wang (2007) investigated the volatility in stock markets for five new European Union (EU) member states of the Czech Republic, Hungary, Poland, Slovenia and Slovakia over the sample period 1994–2005 using a Markov switching model. Their model detects that there are two or three volatility states for emerging stock markets. The results reveal a tendency of emerging stock markets to move from the high volatility regime in the earlier period of transition into the low volatility regime as they move into the EU. They finding that joining to the EU cause signs of a stabilization of the emerging stock markets by a reduction of its volatility.

Considering the Markov Switching GARCH(1,1) model with time varying transition probabilities, Kramer (2008) derive sufficient conditions for the square of the process to display long memory and provides some additional intuition for the empirical observation that estimated GARCH parameters often sum to almost one.

Interested in distinguishing between two processes, one being a regime-dependent stationary process and the other being a non-stationary IGARCH process, Liang and Yongcheol (2008) develops an optimal test procedure designed to have a maximal power to detect MS-GARCH mechanisms. They consider the case in which the conditional variance follows an IGARCH process under the null whilst it is globally stationary under the alternative and find strong evidence in favor of MS-GARCH models in an application to the weekly stock return data for five East Asian emerging markets.

Taking up interest in dependence on the path, Francq et al. (2008), are the first to propose an estimation method without changing the MS-GARCH model. They used the generalized method of moments (GMM) with which they avoid addressing the problem of dependence on the path to not be based on the likelihood[3]. On the other hand, Bauwens et al. (2010) develop MS-GARCH models wherein the conditional mean and variance switch in time by a hidden Markov chain from one GARCH process to another. They provide sufficient conditions for geometric ergodicity and existence of moments of the process. They are the first to estimate the MS-ARCH model using Bayesian MCMC techniques. As in Francq et al. (2008), this alternative estimation is based on the failure to obtain the maximum likelihood estimator (MLE) MS-GARCH model because the dependence of the path makes calculating the likelihood is unworkable in practice.

Another empirical application of Markov Switching approach is developed by Rim and Khemiri (2012). Its aim was to examine the relationship between exchange rates and determinants underlying microstructural. To this end, he used a MSEGARCH (1,1) model that ensures, by construction, a non-negative conditional variance and the ability to capture asymmetry in volatility and compares it against a MS-GARCH (1,1). Both models estimated using the EM algorithm of Hamilton (1990, 1994). He finds that the MSEGARCH model is best fits to intraday data and a positive correlation between "trading volume" price Deutsche Mark (DM)/$ prices as well as a positive effect of order flow on returns.

Augustyniak (2014) proposes a method for the MLE of the MS-GARCH model based on the Monte Carlo expectation-maximization (MCEM) algorithm, Wei and Tanner (1990), and the Monte Carlo maximum likelihood (MCML) method of Geyer (1994, 1996). The proposed algorithm is based on simulations from the posterior distribution of the state vector and incorporates the technique of increasing data of Tanner and Wong (1987)[4]. Likewise, he proposes a method of estimating the asymptotic variance matrix and covari-

[3] They use a technique based on analytical expressions derived in Francq and Zakoian (2005). Incurring in problems of identifiability, robustes and bias they have not been able to get their asymptotic standard errors GMM estimators due to numerical difficulties

[4] This method is the frequentist version of the Bayesian MCMC technique used by Bauwens et al. (2010).

ance matrix of the MLE. Practical implementation of the proposed model is discussed and its effectiveness is demonstrated in simulation and empirical results. He uses daily and weekly percentage log-returns on the S&P 500 price index.

# 3 Methodology

Let us consider a stock market index $p_t$ and its corresponding rate of return $r_t$, $r_t = 100 \times [\log(p_t) - \log(p_{t-1})]$, where the index $t$ denotes the weekly closing observations.

## 3.1 The Generalized ARCH (GARCH) Model

The GARCH$(1,1)$ model for the series of returns $r_t$ can be written as

$$\begin{aligned} r_t &= \mu + \epsilon_t = \mu + \sigma_t \eta_t, \\ \sigma_t^2 &= \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2. \end{aligned}$$

where $\omega > 0$, $\alpha \geq 0$ and $\beta \geq 0$ to ensure a positive conditional variance $\sigma_t^2$, $\alpha + \beta < 1$ to ensure that unconditional variance $\text{var}(\epsilon_t) = \omega / (\alpha + \beta)$ is defined[5], and $\eta_t \sim i.i.d.\ \mathbb{N}(0,1)$.

## 3.2 The Markov Regime-Switching GARCH (MS-GARCH) Model

Following Bauwens et al. (2010) and Francq et al. (2001), the MS-GARCH model can be defined by the following equations:

$$r_t = \mu_{S_t} + \sigma_t (S_{1:t}) \eta_t, \tag{1}$$

$$\sigma_t^2 (S_{1:t}) = \omega_{S_t} + \alpha_{S_t} \epsilon_{t-1}^2 (S_{t-1}) + \beta_{S_t} \sigma_{t-1}^2 (S_{1:t-1}), \tag{2}$$

$$\epsilon_{t-1} (S_{t-1}) = r_{t-1} - \mu_{S_{t-1}}. \tag{3}$$

The vector $(r_1, \ldots, r_T)$ represents the observations to be modeled and $\eta_t \sim i.i.d.\mathbb{N}(0,1)$. At each time point, the conditional mean of the observation $r_t$ is $\mu_{S_t} = E[r_t | S_t]$ and the conditional variance is $\sigma_t^2 = \text{var}(r_t | r_{1:t-1}, S_{1:t})$, where $r_{1:t-1}$ and $S_{1:t}$ are shorthand for the vectors $(r_1, \ldots, r_{t-1})$ and $(S_1, \ldots, S_t)$, respectively. The process $\{S_t\}$ is an unobserved ergodic time-homogeneous Markov chain process with N-dimensional discrete state space (i.e., $S_t$ can take integer values from 1 to $N$). The $N \times N$ transition matrix of the Markov chain is defined by the transition probabilities $\{p_{ij} = \Pr[S_t = j | S_{t-1} = i]\}_{i,j=1}^N$. The vector $\theta = (\{\mu_i, \omega_i, \alpha_i, \beta_i\}_{i=1}^N, \{p_{ij}\}_{i,j=1}^N)$ denotes the parameters of the model. To ensure positivity of the variance, the following constraints are required: $\omega_i > 0$, $\alpha_i \geq 0$ and $\beta_i \geq 0, \quad i = 1, \ldots, N$. Since $\sum_{j=1}^N p_{ij} = 1$, for $i = 1, \ldots, N$, $\theta$ contains $(4N + N(N-1))$ free parameters. Conditions for stationarity and the existence of moments are studied by Bauwens et al. (2010), Francq et al. (2001) and Francq and Zakoian (2005).

## 3.3 Estimation of the MS-GARCH Model

The MS-GARCH model specified by equations (1)-(3) presents difficulties in its estimation because the conditional variance $t$ depends on the complete path $S_{1:t}$. To simplify notation we denote $\sigma_t^2 (S_{1:t})$ as $\sigma_t^2$, $r_{1:T}$ and $S_{1:T}$ as $R$ and $S$ respectively, and let $f(p)$ represents a probability density function. We can calculate the likelihood of the observations,

[5] If $\alpha + \beta = 1$ we are facing a unit root in the variance, also called "Non-stationary in variance" or "Integrated GARCH (IGARCH)". Whereas if $\alpha + \beta > 1$ the conditional variance forecast will tend to infinity as the forecast horizon increases, Brooks (2014).

$f(r|\theta)$, integrating all the possible paths regime, obtaining $f(r|\theta) = \sum_S f(r, S|\theta) = \sum_S f(r|S, \theta)\, p(S|\theta) = \sum_S \left[\prod_{t=1}^{T} \sigma_t^{-1} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{r_t - \mu_{S_t}}{\sigma_t}\right)^2\right)\right] p(S|\theta)$. For a large $T$, the sum grows rapidly in $N^T$ terms and consequently becomes unfeasible its calculation; however, an accurate estimate of the log-likelihood is obtained by Bauwens et al. (2010) by writing $\log f(r|\theta) = \log(r_1|\theta) + \sum_{t=1}^{T-1} \log f(r_{t+1}|r_{1:t}, \theta)$ and estimating $f(r_{t+1}|r_{1:t}, \theta)$, $t = 1 \ldots, T-1$, sequentially with the aid of particle filters. Simulation log-likelihood is difficult to maximize with standard optimization routines because these filter is not a continuous function of $\theta$.

Given this deficiency, Gray (1996) proposes replacing the equations (2) and (3) in the MS-GARCH model with:

$$\sigma_t^2 = \omega_{S_t} + \alpha_{S_t}\epsilon_{t-1}^2 + \beta_{S_t} h_{t-1},$$
$$\epsilon_{t-1} = r_{t-1} - E\left[r_{t-1}|r_{1:t-2}\right].$$

where $h_{t-1} = \text{var}(r_{t-1}|r_{1:t-2})$ has the effect of collapsing all of the possible conditional variances at time $t-1$ into a single value that does not depend on the regime path, allowing the conditional distribution of $r_t$, $f(r_t|r_{1:t-1}, S_{1:t}, \theta)$, becomes independent of $S_{1:t-1}$ and maximum likelihood estimation is tractable, Hamilton (2008). However, Augustyniak (2014) shows that the method of Gray does not generate consistent estimators for the MS-GARCH.

The Expectation-Maximization (EM) algorithm is a technique designed to obtain the MLE of the observed data likelihood through an iterative procedure that does not require the computation of the likelihood. Instead, considering $\widetilde{\theta}$ as a given value of the parameters, we must be able to calculate and maximize $Q\left(\theta|\widetilde{\theta}\right) = E\left[\log f(r, S|\theta)\,|r, \widetilde{\theta}\right] = \sum_S \log f(r, S|\theta)\, p\left(S|r, \widetilde{\theta}\right)$. McCulloch (1997) suggests to combine the EM algorithm with a Newton-Raphson method or switch to a faster method after a few EM iterations. He proposed the MCEM algorithm with the MCML approach, Geyer (1994, 1996). The MCML method does not work well unless $\theta^*$ is in a close neighborhood of the MLE, due MCML algorithm makes use of importance sampling to directly maximize the log-likelihood, Cappé et al. (2005)

In this research we use the algorithm proposed by Augustyniak (2014), which turns out to be a hybrid from MCEM and MCML algorithms. First, iterations of the MCEM algorithm can be performed to get a good estimate, $\theta^*$, of the MLE. This estimate is then used to generate the importance sample in the MCML algorithm. Both algorithms complement each other: the MCEM algorithm addresses the flaw of the MCML algorithm relating to the choice of $\theta^*$, while the MCML method replaces many potential MCEM iterations with a single iteration, leading to a faster convergence.

See the Appendix for more details of the MCEM-MCML algorithm.

### 3.4 Model Specification

In order to estimate the MLE MS-GARCH model, the MCEM-MCML algorithm used as starting points the approximations of Gray, Dueker (1997) or Klaassen (2002) models . To initialize the Gibbs sampler, it takes the smoothed inference model states Gray (Hamilton, 1994) as its first state vector; and for generating the first Markov chain $S_1$, it assumes that the initial state $S_0$ is given and fixed rather than requiring be estimated.

Like the automated strategies for increasing the size of the sample through the MCEM–MCML algorithm, require a certain amount of manual adjustments, and do not guarantee to be used with high reliability, Augustyniak (2014) proposes two simulations schedules. Simulation schedule 1 ($m_1 = 500$, $m_2 = 1000$, $m_3 = 2500$, $m_4 = 5000$, $m^* = 10000$), which allows a quick estimate; and simulation schedule 2 ($m_{1...10} = 500$, $m_{11...28} = 1000$, $m_{29} = 2500$, $m_{30} = 5000$, $m^* = 40000$), which puts more emphasis on precision and is more robust with respect to the choice of starting points. Because accuracy gains are preferred with empirical data, then schedule simulation 2 will be used in this research.

Unconstrained estimation of MS-GARCH models with empirical data can lead to parameters being estimated on the boundary of the parameter space and result in slow convergence of the MCEM–MCML algorithm. For example, Bauwens et al. (2010) and Francq et al. (2008) fitted the MS-GARCH model to the daily S&P 500 data: Bauwens et al. (2010) used the constraint $\alpha_1 = \beta_1 = 0$ in the estimation process while Francq et al. (2008) reported an estimated value of $\alpha_1$ very close to zero. To obtain convergence in the interior of the parameter space, Augustyniak (2014) fitted a constrained MS-GARCH model by imposing $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$ in the estimation process. For weekly data, both the constrained and unconstrained versions were estimated but due to slow convergence simulation schedule 2, he concluding that the estimate of the unrestricted version was not effective.

We perform unconstrained estimation of MS-GARCH model for the countries of Latin America, finding problems similar to those reported by Augustyniak (2014), $\alpha_1$ estimates very close to zero and changes in sign and magnitude at the value of the conditional variance parameter, $\omega$, contradicting the stylized facts of financial returns. In light of these results, we chose to carry out the constrained estimation of MS-GARCH model under the imposition $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$, obtaining consistent results with empirical evidence.

For these reasons, we estimate the following constrained MS-GARCH model:

$$r_t = \mu_{S_t} + \sigma_t\left(S_{1:t}\right)\eta_t, \tag{4}$$

$$\sigma_t^2\left(S_{1:t}\right) = \omega_{S_t} + \alpha\epsilon_{t-1}^2\left(S_{t-1}\right) + \beta\sigma_{t-1}^2\left(S_{1:t-1}\right), \tag{5}$$

$$\epsilon_{t-1}\left(S_{t-1}\right) = r_{t-1} - \mu_{S_{t-1}}. \tag{6}$$

# 4 Empirical Evidence

## 4.1 Data and Preliminary Statistics

The weekly stock market returns series are constructed with diary stock market index data of Argentina, Brazil, Chile, Colombia, Mexico and Peru, obtained from Bloomberg Financial Data. Weekly data is from Wednesdays to the following Wednesdays to avoid most holidays[6]. Weekly data are used due to the presence of more noise with higher frequencies, such as daily data, which makes it more difficult to isolate cyclical variations and hence obscures the analysis of driving moments of switching behavior. See for instance Moore and Wang (2007).

The weekly returns are constructed as the first difference of logarithmic stock index multiplied by 100, $r_t = 100 \times [\log(p_t) - \log(p_{t-1})]$, where $p_t$ is the stock index. The volatil-

[6] Given the omission of data on a Wednesday from a specific week, we decided to choose some other "feasible day" in that week. The criterion for this choice was based on the construction of a ranking of missing data (from lowest to highest) on each day of the week throughout the daily series, selecting as the "first feasible" the day of the week with fewer omissions; if it did not exist, we selected the following day in the ranking of omissions as a feasible second day; and so on. In this way we build weekly series with no missing data.

ity series are constructed as the squared of stock rate returns. It has covered 800 weekly observations between 2000:05:01 and 2015:29:04 for Argentina, including 805 weekly observations between 2000:05:01 and 2015:03:06 for Brazil; 805 weekly observations ranging between 2000:05:01 and 2015:03:06 for Chile; 726 weekly observations ranging between 2001:11:07 and 2015:03:06 for Colombia; 805 weekly observations ranging between 2000:05:01 and 2015:03:06 for Mexico; and included 800 weekly observations between 2000:05:01 and 2015:29:04 for Peru.

The descriptive statistics of returns and volatility of stock indices of the countries of the region under study, Argentina, Brazil, Chile, Colombia, Mexico and Peru, are shown in Table 1. In the first panel shows statistical for stock returns. The average is close to zero in all cases. The highest standard deviation possesses Argentina, followed by Brazil and Colombia. The asymmetry coefficient is negative for all countries in the region, presenting the greatest magnitude Chile and Peru the least. All series display positive skewness and excess of kurtosis, a well-known stylized fact of the presence of an asymmetric distribution with heavy tails of stock markets returns. Stationary in time series is checked by applying the Augmented Dickey Fuller (ADF) test. The results fail to reject the null of a unit root in the logarithmic stock index series, but overwhelmingly reject to the null for the first difference of logarithmic stock index returns[7]. In the second panel, statistical series for the volatilities of returns are shown, with Brazil exhibiting the greatest volatility, followed by Argentina, Peru, Colombia, Chile and Mexico. The Figures of the stock returns of the countries of the region and their respective volatilities are presented in Figures 1 and 2, respectively.

## 4.2 Results

We performed the fit of the constrained MS-GARCH model parameters of each country using three rivals models, the GARCH model, the MS model and the Gray model. The results obtained are shown in Table 2. The MS model is a particular case of the MS-GARCH when $\alpha = 0$ and $\beta = 0$. The preferred model is one with the lowest BIC; nevertheless, to ensure that our MS-GARCH model (a nonlinear model) is preferred to its rivals, we adopted the Davies (1987) upper bound test. Applying this test, we obtained the rejection of the null in all cases, i.e. the MS-GARCH model is prefered among its rivals[8].

As the Gray's model can not generate consistent estimators for the MS-GARCH model, the log-likelihood model MS-GARCH evaluated in the MLE model Gray usually found below that obtained by the GARCH model. See Augustyniak (2014). Also, using the MLE asymptotic standard errors we determined the levels of significance at 1%, 5% and 10% specifying them by the letters $a$, $b$ and $c$ respectively.

This study considers two persistent regimes based on the financial stylized facts of stock market returns. The first persistent regime is the "low volatility regime", characterized by a positive average of returns; and the second is the "high volatility regime", characterized by a negative average of returns.

The results shown in Table 2 reveal that conditional mean of returns in both the MS and the MS-GARCH model, is positive in the low volatility regime ($\mu_1$), and negative in the high volatility regime ($\mu_2$). Likewise, we observed in the MS-GARCH models that

[7] Results are available upon request.

[8] The Davies test use the complete set of information, and is less computationally intensive, to obtain an upper bound for the significance level of the LR statistics under the null hypothesis consisting of the model with lower number of states. For more details see Appendix A of Garcia and Perron (1996).

magnitude in absolute value of the conditional mean of the returns of high volatility is higher than the returns of the regime of low volatility, except for Peru where the conditional media of the regime of high volatility is not significant.

Regarding the long-term average volatility ($\omega$) of the two regimes, we note that in all cases the MS model overestimates their value relative to MS-GARCH model; and that the long-term average volatility of high volatility regime ($\omega_2$) is always positive and higher than in the low volatility regime ($\omega_1$).

Table 2 also shows that in the GARCH models of all countries, the estimated value of the impact of past shocks to current volatility ($\alpha$) is exacerbated regarding the value estimated by the MS-GARCH models; the opposite happens with the estimated value of the weight of lagged variance ($\beta$). Likewise, it is verified in all cases that $\alpha + \beta < 1$, i.e. there is presence of stationarity in the unconditional variance of returns. This result is statistically verified after applying an IGARCH test to each series, verifying the absence of integrated processes of order one, due to the rejection of the null hypothesis of presence of unit root in all cases[9].

The persistence of high volatility regimes ($p_{22}$) estimated by the MS-GARCH model is always less than the estimated by the MS model. The persistence of high volatility regime estimated by the MS-GARCH respect to estimated by the MS model, is about half for Argentina, Brazil and Colombia; about a third for Chile and Mexico; and approximately two-thirds for Peru. For all countries, the estimated persistence of both regimes by MS-GARCH models turn out to be lower than those estimated by the MS and Gray models. So, Table 2 shows that the persistence of high volatility regime estimated by the MS-GARCH respect to estimated by the MS model, is about half for Argentina, Brazil and Colombia; about a third for Chile and Mexico; and approximately two-thirds for Peru. Also, the persistence of high volatility is much lower than the persistence of low volatility under the MS-GARCH model compared with the Gray and MS models. For example, the persistence of high and low volatility estimates for Brazil for the MS-GARCH are 0.857 and 0.422, respectively, while those reported by Gray model are 0.947 and 0.662, and those reported by the MS model are 0.961 and 0.903.

Finally, for each country we get the smoothed probabilities of being in the regime of high volatility using the MS and MS-GARCH models (see Figures 3-8). At first glance, we show that the constrained MS-GARCH model refines the detection of "episodes of high volatility" (measured in weeks) that the MS model infers. As we can see, the incorporation of dynamic GARCH into the MS model reduce significantly the persistence of the high volatility regime, i.e. $p_{22}$ reduce drastically the average values of the long-term conditional variance ($\omega_1$ and $\omega_2$). While in an MS model, persistence in volatility is explained by the persistence of the regime (i.e., long periods of high volatility can occur only when the returns remain in the regime of high volatility) in an MS-GARCH, persistence is best explained due to the incorporation of the dynamics of GARCH component, where the role of the MS process is to allow jumps between regimes, as it is document in the econometric literature. See Eraker et al. (2003).

In the light of these findings, the MS-GARCH model appear to be more consistent with the stylized facts of financial series that its rivals MS, Gray and GARCH models. In the next paragraphs we will discuss some episodes of high volatility, exemplifying the differences between the MS and the MS-GARCH models, comparing inferences about some stylized facts.

[9] Results available upon request.

### 4.2.1 Argentina

During the period from December 2001 through March 2002, the Argentine government introduced capital controls on the local stock market (named the *Corralito's* restrictions) as well as to prevent a speculative attack to the local currency. Then, through the stock market, Argentine investors purchased stocks in the Buenos Aires Stock Exchange using their frozen bank deposits, converted them into American Depositary Receipts (ADRs) in U.S. stock markets, and finally sold the ADRs and deposited the proceeds in the U.S. banking system, causing consequently the Argentine stock market boomed shown by an increase in the MERVAL index. As we can see in Figure 3, the MS model infers that returns experienced the regime of high volatility since the week of 7/11/01 until the week of 7/10/02, totaling a single episode of 53 weeks; however, the MS-GARCH model, during the same period, specifies that the regime of high volatility occurred in three episodes, the first since the week of 7/11/01 until the week of 7/18/01 (2 weeks), the second since the week of 8/29/01 until the week of 10/3/01 (6 weeks), and the last since the week of 5/22/02 until the week of 6/5/02 (3 weeks), totaling 11 weeks of high volatility.

With regard to episodes of high volatility generated by the international financial crisis 2007-2008, the MS model infers that the returns entering the high volatility regime a single episode of 22 weeks since the week of 08/27/08 until the week of 01/21/09; while the MS-GARCH model infers 5 episodes, the first one week in the week of 02/28/07, second since the week of 08/01/07 until the week of 08/15/07 (3 weeks), the third since the week of 01/16/08 until the week of 01/23/08 (two weeks), the fourth in the week of 07/09/08 (1 week) and fifth since the week of 08/06/08 until the week of 10/22/08 (12 weeks) totaling 19 weeks high volatility.

Also, during the last week of July 2014, Argentina suffered a political and financial crisis that began with a default sparking a wave of speculation against the peso. The MS model infers that returns experienced the regime of high volatility since the week of 7/23/14 until the week of 12/17/14, totaling a single episode of 22 weeks; however, during the same period, the MS-GARCH model reveals the presence of four episodes of high volatility, the first in the week of July 23 (one week), the second in the week of August 6 (1 week) the third since the week of 10/8/14 until the week of 10/15/14 (2 weeks), and the fourth in the week of 12/10/14 (one week), totaling four weeks of high volatility.

### 4.2.2 Brazil

As we can see in Figure 4, with regard to episodes of high volatility generated by the international financial crisis 2007-2008, the MS model infers that the returns entering the high volatility regime in 3 episodes, first since the week of 08/01/07 until the week of 08/22/07 (4 weeks), second since the week of 22/21/07 until the week of 2/20/08 (14 weeks), and third since the week of 6/4/08 until the week of 1/21/09 (34 weeks) totaling 52 weeks high volatility; while the MS-GARCH model infers 7 episodes, the first one week in the week of 02/28/07, second since the week of 08/01/07 until the week of 08/15/07 (3 weeks), the third since the week of 01/16/08 until the week of 01/23/08 (2 weeks), the fourth in the week of 07/09/08 (1 week) and fifth since the week of 08/06/08 until the week of 10/22/08 (12 weeks) totaling 30 weeks high volatility.

In August 2013, in international markets there is evidence of some accommodation of prices of commodities, as well as greater volatility and tendency of appreciation of the United States dollar. Risks to the global financial stability remained high, like those associated with the deleveraging process taking place in major economic blocs and with the steep slope of the yield curve of relevant mature economies. The MS model infers that

Brazil's stock returns experienced a single episode of high volatility during the week of June 12, 2013; however, the MS-GARCH model specifies that the regime of high volatility occurred in three episodes, the first during the week of January 30, 2013 (1 week), the second during the week of April 17, 2013 (1 week), and the third from 29 May to 19 June 2013 (4 weeks), totaling six weeks of high volatility.

The political crisis that Brazil at the beginning of September 2014 due to mismanagement of economic policy and the loss of investor confidence led to a jump in volatility of its main stock market index. The MS model infers that stock returns enter the regime of high volatility since the week of September 10 and return to the regime of low volatility during the week of December 10, totaling a single episode of 14 weeks of high volatility; however, during the same year, the MS-GARCH model reveals the presence of two episodes of high volatility, the first between the weeks of September 10 to October 1 (4 weeks) and the second during the week of 22 October (one week), for a total of five weeks of high volatility.

### 4.2.3 Chile

During the years 2007-2008, where the international financial crisis unfolded, Chile experienced fewer episodes of high volatility compared to their counterparts in the region. As we can see in Figure 5, the MS model infers that returns experienced the regime of high volatility in 3 episodes, first since the week of 1/24/07 until the week of 4/4/07 (11 weeks), second since the week of 8/8/07 until the week of 3/12/08 (32 weeks), and third since the week of 6/25/08 until the week of 12/10/08 (25 weeks) totaling 68 weeks high volatility; while the MS-GARCH model infers 5 episodes, the first one week in the week of 8/15/07 (1 week), second in the week of 11/7/07 (1 weeks), third in the week of 1/9/08 (1 weeks), fourth in the week of 07/02/08 (1 week) and fifth since the week of 10/01/08 until the week of 10/08/08 (2 weeks) totaling 6 weeks high volatility.

In October 2008, significant increases in risk premiums and capital outflows from portfolio of Chile. The volatility in the foreign exchange and stock markets reached highs record. The Chilean peso depreciated and part of this depreciation responding to the global appreciation of the dollar, which occurred due to changes in portfolio to US Treasuries, seeking lower risk and higher liquidity. Also pension funds in Chile contributed to stress the exchange market through substantial changes in hedging positions. The MS model infers that returns experienced the regime of high volatility since the week of 6/25/08 until the week of 12/10/08, totaling a single episode of 25 weeks; however, the MS-GARCH, during the same period, specifies two episodes of high volatility, the first in the week of 7/2/08 (1 week), and the second since the week of 10/1/08 until the week of 10/8/08 (2 weeks), totaling 3 weeks of high volatility.

During 2013, the withdrawal of monetary stimulus in the US, the economic slowdown in China and uncertainty about a possible tax reform in Chile, caused sharp fall IPSA join. The MS model infers that the returns entering the high volatility regime in two episodes, the first since the week of 5/29/13 until the week of 10/2/13 (19 weeks); and the second in the week of 11/13/13 (one week), totaling 20 weeks of high volatility; however, during the same year, the MS-GARCH evidence only 1 episode of high volatility in the week of 6/12/13 (1 week).

### 4.2.4 Colombia

Colombian financial system experienced a depreciation during the period March-May 2006, due declines in the value of its marketable securities. This phenomenon was associated to perceived uncertainty in international financial markets and even Colombia led the fall of investments in mutual funds in Latin America in late May 2006 with a decrease of 18.3%. As we can see in Figure 6, the MS model infers that returns experienced the regime of high volatility in two episodes, the first since the week of 2/1/06 until the week of 2/22/06 (4 weeks) and the second since 5/17/06 until the week of 7/12/06 (9 weeks ), totaling 13 weeks of high volatility; meanwhile, the MS-GARCH model, during the same period, specifies three episodes of high volatility, the first in the week of 2/8/06 (1 week), the second since the week of 5/10/06 until the week of 5/17/06 (2 weeks), and the third in the week of 6/14/06 (1 week), totaling 4 weeks of high volatility.

With regard to episodes of high volatility generated by the international financial crisis 2007-2008, the MS model infers that the returns entering the high volatility regime in 3 episodes, first since the week of 16/01/08 until the week of 30/01/08 (3 weeks), and second since the week of 17/09/08 until the week of 29/10/08 (7 weeks), totaling 10 weeks high volatility; while the MS-GARCH model infers 6 episodes, first in the week of 28/02/07 (1 week), second in the week of 30/05/07 (1 week), third in the week of 15/08/07 (1 week), fourth since the week of 09/01/08 until the week of 16/01/08 (2 weeks), fifth in the week of 03/09/08 (1 week), and sixth in the week of 08/10/08 (1 week), totaling 7 weeks high volatility.

### 4.2.5 Mexico

As a result of the bursting of the bubble companies ".com", the terrorist attacks of 11 September 2001 and the risk of deflation by including world trade in countries with low production costs, the stock index of the Mexican stock market went through episodes of high volatility during 2002. As we can see in Figure 7, the MS model infers that returns experienced the regime of high volatility since the week of 5/29/01 and return to low volatility in the week of 12/11/01, totaling 29 weeks of high volatility; however, during the same year, the MS-GARCH model reveals the presence of three episodes of high volatility, each one with 1-week duration, the first in the week of 5/29/01, the second in the week of 6/26/01 and the third in the week of 11/13/01, totaling 3 weeks of high volatility.

Likewise, during September-October 2008, the bankruptcy of Lehman Brothers led to a sharp increase in global risk perceptions and increased uncertainty about the quality of some assets held by financial institutions. The particular characteristics of the Mexican economy have led to these shocks have had a particularly negative effect. Mexico's foreign trade is highly concentrated to the United States, particularly with regard to the export of manufactured goods. Therefore, the decline of US economic activity has had a particularly adverse effect on the economy of Mexico. In this context, capital flows to Mexico's economy contracted sharply affecting the exchange rate and the stock market. With regard to financial contagion, rising liquidity and capital in international markets and tight credit policies of banks in the world affected financing conditions. The MS model infers that returns experienced the regime of high volatility in a single episode, since the week of 9/3/08 until the week of 9/11/09 (63 weeks); meanwhile, the MS-GARCH model, during the same period, identifies two episodes of high volatility, the first since the week of 8/27/08 until the week of 10/8/08 (7 weeks), and the second in the week of 10/28/09 (1 week), totaling 8 weeks of high volatility.

The international financial crisis of 2007-2008 generated several episodes of high volatility in the Mexican stock market, the MS model infers that the returns entering the high volatility regime in 3 episodes, first since the week of 28/02/07 until the week of 21/03/07

(4 weeks), second since the week of 25/07/07 until the week of 05/03/08 (33 weeks), and third since the week of 03/09/08 until the week of 11/11/09 (63 weeks), totaling 100 weeks high volatility; while the MS-GARCH model infers 7 episodes, first in the week of 28/02/07 (1 week), second in the week of 27/06/07 (1 week), third in the week of 01/08/07 (1 week), fourth in the week of 15/08/07 (1 week), fifth in the week of 07/11/07 (1 week), sixth in the week of 30/04/08 (1 week), and seventh since the week of 27/08/08 until the week of 08/10/08 (7 weeks), totaling 13 weeks high volatility.

### 4.2.6 Peru

With regard to episodes of high volatility generated by the international financial crisis 2007-2008, as we can see in Figure 8, the MS model infers that the returns entering the high volatility regime in 3 episodes, first since the week of 28/03/07 until the week of 13/06/07 (12 weeks), second since the week of 15/08/07 until the week of 05/03/08 (30 weeks), and third since the week of 02/07/08 until the week of 03/12/08 (23 weeks), totaling 65 weeks high volatility; while the MS-GARCH model infers 3 episodes too, first since the week of 21/03/07 until the week of 09/05/07 (8 weeks), second since the week of 24/10/07 until the week of 05/12/07 (7 weeks), and third since the week of 02/07/08 until the week of 22/10/08 (17 weeks), totaling 32 weeks high volatility

The accentuation of political uncertainty for the victory of nationalist candidate Ollanta Humala in 2011 generated a negative performance in the Lima Stock Exchange, bag mainly mining, due to nervousness of foreign mining investors before the election proposals to implement a new tax on the productive sector, as well as various speculative manifestations given in various segments. The MS model infers that returns experienced the regime of high volatility since the week of 3/16/11 and return to low volatility in the week of 10/12/11, totaling one episode of 31 weeks of high volatility; likewise, the MS-GARCH model reveals the presence of one episode of high volatility since the week of 3/9/11 until the week of 4/27/11, totaling 8 weeks of high volatility.

In 2013, Lima Stock Exchange recorded a loss of 23.63%, due to: the fall in international prices of raw materials (due to lower growth in China and the debt crisis in the United States and eurozone), the falling prices of domestic shares due to slower growth of the Peruvian economy, liquidation of positions in mining papers from the Administrators of Pension Funds and investment in foreign markets, and the political noise (Executive ads attempting to buy the assets in Peru of the Spanish Repsol, and changes in the cabinet) influencing the investment decisions of investors. The MS model infers that returns experienced the regime of high volatility in two episodes, the first in the week of 4/17/13, and the second since the week of 6/12/13 until the week of 8/14/13 (10 weeks); meanwhile, the MS-GARCH model identifies a single episode of high volatility, since the week of 4/10/13 until the week of 4/24/13 (3 weeks).

### 4.2.7 Latin American stock market

Figure 9 shows a bar graph of occurrences of the regime of high volatility experienced by countries in the time horizon of study. In each of the 7 panels, we took account of the countries experiencing the regime of high volatility, temporarily distributed in horizons of two years for each panel, starting from the first week of January 2001 and ending on the last week of December 2014. Inspecting the smoothed transition probabilities inferred by the MS-GARCH model, we can see that only for one week, all stock markets experienced high volatility regime. This week is corresponding to Wednesday October 8, 2008, in which the international financial crisis began belongs.

We can also identify groups of 5 and 4 countries who have experienced the regime of high volatility simultaneously. The regime of high volatility experienced by 5 countries simultaneously, was given in three weeks, first in the week of 8/15/07 (Argentina, Brazil, Chile, Colombia and Mexico), second in the week of 9/3/08 (Argentina, Brazil, Colombia, Mexico and Peru) and third in the week of 10/1/08 (Argentina, Brazil, Colombia, Mexico and Peru). The regime of high volatility experienced by 4 countries simultaneously, was given in the following weeks: first in the week of 9/12/01 (Argentina, Brazil, Chile and Mexico), second, third and fourth in the weeks of 3/16/05, 5/17/06 and 2/28/07 (Argentina, Brazil, Colombia and Mexico), fifth in the weeks of 8/27/08, six since the week of 9/10/08 until the week of 9/24/08 (Argentina, Brazil, Mexico and Peru); and finally in the week of 8/10/11 (Argentina, Brazil, Colombia and Mexico).

We can also count the number of weeks each country experienced high volatility at the time horizon of this research. In total there are 183 weeks experienced regime of high volatility at least by one country. Considering this result, the number of weeks in which countries exhibited high volatility, from highest to lowest, were Brazil (102 weeks), Argentina (69 weeks), Peru (55 weeks), Mexico (37 weeks), Colombia (32 weeks) and Chile (12 weeks).

On the other hand, we can also count the number of episodes, groups of uninterrupted weeks, where the regime of high volatility was experienced. Thus, the country who experienced a greater number of episodes of high volatility was Brazil (37 episodes) followed by Mexico (27 episodes), Colombia (26 episodes), Argentina (23 episodes), Chile (11 episodes) and Peru (9 episodes).

Considering the duration of episodes of high volatility, measured between Wednesday of each week, the biggest episode owns Peru since the week of 7/2/08 until the week of 10/22/08 (17 weeks), followed by Brazil since the week of 7/23/08 until the week of 10/22/08 (14 weeks), Argentina since the week of 8/6/08 until the week of 10/22/08 (12 weeks), Mexico since the week of 8/27/08 until the week of 10/8/08 (7 weeks), and Chile since the week of 10/1/08 until the week of 10/8/08 (2 weeks); while Colombia experienced 6 episodes, the longest lasting in the region, first since the week of 10/10/01 until the week of 10/17/01 (2 weeks), second since the week of 11/14/01 until the week of 11/21/01, third since the week of 5/5/04 until the week of 5/12/04, fourth since the week of 5/10/6 until the week of 5/17/06, fifth since the week of 1/9/04 until the week of 1/16/08, and since the week of 12/3/14 until the week of 12/19/14.

Figure 9 also shows bunches of uninterrupted weeks, in which one or more countries experienced the regime of high volatility. For example, we find one of these bunches since the week of 6/4/08 until the week of 10/22/08 (21 weeks), with a peak in 10/8/08, with all countries experiencing high volatility regime. This episode is in line with the stylized fact related to failures of massive financial institutions in the United States on September 16, 2008, due primarily to exposure of securities of packaged Subprime lending subprime loans and credit default swaps, which quickly devolved into a global crisis resulting in a number of bank failures in Europe and sharp reductions in the value of stocks and commodities worldwide.

We found another bunch since the week of 7/13/11 until the week of 8/10/11 (5 weeks), the peak is on the last date with Argentina, Brazil, Colombia and Mexico experiencing high volatility regime. This episode is in line with the stylized fact related to a sharp drop in stock prices on 8/8/11 (Black Monday) on the stock markets of the United States, Middle East, Europe and Asia, due to the United States debt-ceiling crisis in 2011, which caused the reduction of its category from AAA to AA+ on 8/6/11, 2011, and fears of contagion from the sovereign debt crisis in Spain and Italy.

All of these joint experiences of the regime of high volatility prompted us to perform the calculation of correlations between the smoothed probabilities of being in the regime 2 using windows of one year (52 weeks), year and a half (78 weeks) and two years (104 weeks). The respective Figures are 10, 11 and 12. Each of these Figures contains six panels, one per country. In each panel they have been drawn correlations of each country versus others, and identified (with a black dot) corresponding to the maximum correlation experienced in the common study horizon week. For example, in the first panel, the correlations of Argentina (MERVAL) versus Brazil (IBOV), Chile (IPSA), Colombia (IGBC), Mexico (MEXBOL) and Peru (IGBVL) is displayed, experiencing the highest correlations for weeks corresponding to 7/29/09 (0.955), 12/13/06 (0.743), 5/2/07 (0.787), 6/13/07 (0.933) and 5/7/14 (0.967) respectively. As we can see, as the window is becoming larger, correlations tend to be more stable over time. Likewise, we can see a co-movement (correlations greater than 0.5) in all windows ending in mid-2010, demonstrating the presence of systematic effects on Latin American stock markets of the international financial crisis. Also, in most panels it can be seen that after the international financial crisis, at the end of the period of analysis, correlations tend to be positive, revealing a kind of positive interdependence during episodes of financial turmoil.

This evidence of co-movement of the Latin-American stock market indices on periods of financial turbulence after the post-international financial crisis, it remains as a possible extension to this research.

# 5 Conclusions

In this research we study the volatility of stock returns of the following Latin American stock markets: Argentina, Brazil, Colombia, Chile, Mexico and Peru, estimating the parameters of the MS-GARCH model in order to distinguish episodes of high and low volatility that crossed each economy with more accurately, and recognize some common behavior pattern during financial turmoils. The estimates are compared with a standard GARCH, MS and Gray models.

The MS-GARCH models are those models that, in order to better capture the persistence of the volatility of stock returns, incorporate the dynamics of GARCH models. This enhancement allows to refine the capture of the regimes of high (positive returns) and low (negative returns) volatility of the stock markets returns in the time horizon of the study, allowing regime changes have the ability to jump in the value of the long-term volatility. Thus, the empirical results obtained using these models for stock markets studied provide evidence of the existence of a more persistent high volatility regimes rather than in low volatility regimes. To estimate the parameters of the MS-GARCH through the MLE models, we adopted the methodology suggested by Augustyniak (2014), in order to address the problem of path dependence faced in these models.

All these MS-GARCH models are compared with standard GARCH models in terms of their ability to estimate volatility, compared with MS models and Gray model in terms of their ability to capture the volatility persistence in terms of maximum likelihood. The estimating performances of the competing models are evaluated using weekly frequency time of Latin American stock market returns. All time series analyzed shows that the standard GARCH model exacerbates the volatility in almost double compared MS-GARCH model. According to Bayesian information criterion (BIC), the best model for estimating the stock market return is the MS-GARCH, second best is the MS model, the third is a standard GARCH model and the last is the model of Gray (only used for comparative terms). In Peru according in terms of maximum likelihood the best model for estimating the stock market returns is an MS-GARCH, the second is the model of Gray, the third is a standard GARCH the latter is a MS model. The persistence of negative returns-high volatility (regime two) is higher in Colombia and Peru than Mexico and Brazil. Chile is the country with a lower persistence of being in regime two.

Likewise, the temporal correlations between countries show that after the international financial crisis, at the end of the period of analysis, correlations tend to be positive, revealing a kind of positive interdependence during episodes of financial turmoil. This evidence of co-movement of the Latin-American stock market indices on periods of financial turbulence after the post-international financial crisis, remains as a possible extension to this research.

## Appendix: The MCEM-MCML Algorithm

Given an initial guess $\theta^{(0)}$, the algorithm started at $r=1$ produces a sequence of iterates $\{\theta(r)\} \geq 1$ allowing us to compute the MLE of model (1)–(3):

1. Simulate $m_r$ samples of the state vector $S$ from $p\left(S|r,\theta^{(r-1)}\right)$ using a single-move Gibbs sampler. The states are simulated sequentially for $t=1,...,T$ based on the following full conditional distribution:

$$p\left(S_t|S_{1:t-1}^{(i)}, S_{t+1:T}^{(i-1)}, r, \theta^{(r-1)}\right) \propto p_{S_{t-1}^{(i)}} p_{S_{t+1}^{(i-1)}} \Pi_{j=t}^{T} \sigma_j^{-1} \exp\left[-\frac{1}{2}\left(\frac{r_j-\mu_{S_j}}{\sigma_j}\right)^2\right]. \quad (7)$$

   To ease notation, the expression $\sigma_j(S_{1:t})$ was reduced to $\sigma_j$. In the context of (7), $\sigma_j$ represents $\sigma_j(S_{1:t-1}^{(i)}, S_t, S_{t+1:j}^{(i-1)})$.It is straightforward to sample $S_t$ from (7) since $S_t$ can only take integer values from 1 to $N$. However, it should be noted that it is not possible to compute (7) numerically for each value of $S_t$ since this will result in underflow. To avoid underflow, we can calculate the ratios of these expressions and then recover the probabilities for $S_t = 1,...,N$ from them. The $m_r$ simulations of the state vector $S$ that are obtained are denoted by $\{S^{(i)}\}_{i=1}^{m_r}$. These draws form a Markov chain with $p(S|r,\theta^{(r-1)})$ as its stationary distribution (see Frühwirth-Schnatter (2006)).

2. Monte Carlo E-step: Calculate $\widehat{Q}\left(\theta|\theta^{(r-1)}\right)$ ,an approximation of the conventional E-step $Q\left(\theta|\theta^{(r-1)}\right)$ ,where

$$\begin{aligned}
\widehat{Q}\left(\theta|\theta^{(r-1)}\right) &= \frac{1}{m_r}\sum_{i=1}^{m_r} \log[f\left(r, S^{(i)}|\theta\right)] && (8)\\
&= -\frac{T\log(2\pi)}{2} - \frac{1}{2m_r}\sum_{t=1}^{T}\sum_{i=1}^{m_r}\left[\log\left(\sigma_t^{(i)}\right)^2 + \frac{\left(r_j-\mu_{S_j^{(i)}}\right)^2}{\left(\sigma_t^{(i)}\right)^2}\right] \\
&\quad + \frac{1}{m_r}\sum_{t=1}^{T}\sum_{i=1}^{m_r}\log\left(p_{S_{t-1}^{(i)},}\, S_t^{(i)}\right) \\
&= \text{term 1} + \text{term 2}. && (9)
\end{aligned}$$

   In the previous expressions, $\sigma_t^{(i)}$ is shorthand for $\sigma_t(S_{1:t}^{(i)})$.

3. M-step: Perform the following maximization:

$$\theta^{(r)} = \arg\max_{\theta} \widehat{Q}\left(\theta|\theta^{(r-1)}\right)$$

   This optimization can be split into two independent steps since terms 1 and 2 of (9) involve different subsets of the parameters. Term 1 includes the mean and GARCH parameters while term 2 only contains transition probabilities. Maximization of term 1 must be performed numerically and is similar to a standard GARCH optimization to calculate the MLE. To improve the performance of that optimization, the gradient of term 1 with respect to the mean and GARCH parameters should be provided to the optimization routine. Maximization of term 2 can be done analytically. Term 2 is at its maximum when the transition probabilities takes the values:

$$p_{jk} = \frac{f_{jk}}{\sum\limits_{l=1}^{N} f_{jl}}, \qquad j,k = 1,...,N,$$

where $f_{jk}$ denotes the total number of transitions from state $j$ to state $k$ in all of the $m_r$ simulated state vectors[10].

4. Apply a decision rule to determine whether to switch to the MCML algorithm. If the decision is to switch, go to the step 5 and set $\theta^* = \theta^{(r)}$.Otherwise, add 1 to $r$ and go to step 1.

5. Simulate $m^*$ samples of the state vector $S$ from $p(S|r,\theta^*)$ using the single-move Gibbs sampler described in step 1 of the algorithm to obtain the importance sample $\left\{S^{(i)}\right\}_{i=1}^{m^*}$.

6. MCML-step: Perform the following maximization to obtain the MLE:

$$\hat{\theta} = \underset{\theta}{\arg\max} \left[\log \sum_{i=1}^{m^*} w^{(i)}_{\theta|\theta^*}\right]$$

In contrast to the M-step, this optimization cannot be split in two steps[11].

Using importance sampling, the final sample, $\left\{S^{(i)}, \bar{w}^{(i)}_{\widehat{\theta}|\theta}\right\}_{i=1}^{m^*}$, from $p\left(S|r,\widehat{\theta}\right)$, where $\bar{w}^{(i)}_{\widehat{\theta}|\theta} = w^{(i)}_{\widehat{\theta}|\theta^*} / \sum\limits_{i=1}^{m^*} w^{(i)}_{\widehat{\theta}|\theta^*}$, $i = 1,...,m^*$. This sample can be used to obtain an estimate of the smoothed inference of the state at time $t$, $p\left(S_t = j|r,\widehat{\theta}\right)$, $j = 1,...,N$, with $\sum\limits_{i=1}^{m^*} \bar{w}^{(i)}_{\widehat{\theta}|\theta^*} I_{\left\{S_t^{(i)}=j\right\}}$ or to compute the asymptotic variance-covariance matrix of the MLE.

The MCEM-MCML algorithm requires a set of initial values, which can influence the convergence. For this purpose, Augustyniak (2014) uses the resulting estimates of the basic MS model, GARCH model and standard model of Gray, setting a value to the initial state $S_0$ for generating Markov chain.

Likewise, considering that work with empirical data require that the algorithm MCEM-MCML put more emphasis on precision and become more robust with respect to the choice of the initial values, Augustyniak (2014) recommends the use of a schedule that increases the sample size along MCEM-MCML algorithm, rather than implementing automated schedules for each iteration.

[10] A proof of this result is in Appendix B of Augustyniak (2014).
[11] Appendix C of Augustyniak (2014) provides some details related to its implementation.

## References

Augustyniak, M. (2014). Maximum likelihood estimation of the markov-switching garch model. *Computational Statistics & Data Analysis 76*(C), 61–75.

Bauwens, L., A. Preminger, and J. V. K. Rombouts (2010). Theory and inference for a markov switching garch model. *Econometrics Journal 13*(2), 218–244.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics 31*(3), 307–327.

Brooks, C. (2014). *Introductory Econometrics for Finance.* Cambridge University Press.

Cai, J. (1994). A markov model of switching-regime arch. *Journal of Business & Economic Statistics 12*(3), 309–316.

Cappé, O., E. Moulines, and T. Ryden (2005). *Inference in Hidden Markov Models (Springer Series in Statistics).* Secaucus, NJ, USA: Springer-Verlag New York, Inc.

Dueker, M. J. (1997). Markov switching in garch processes and mean-reverting stock-market volatility. *Journal of Business & Economic Statistics 15*(1), 26–34.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation. *Econometrica 50*(4), 987–1007.

Eraker, B., M. Johannes, and N. Polson (2003). The impact of jumps in volatility and returns. *Journal of Finance 58*(3), 1269–1300.

Francq, C., M. Roussignol, and J.-M. Zakoian (2001). Conditional heteroskedasticity driven by hidden markov chains. *Journal of Time Series Analysis 22*(2), 197–220.

Francq, C., Zakoian, and Jean-Michel (2008). Deriving the autocovariances of powers of markov-switching garch models, with applications to statistical inference. *Computational Statistics & Data Analysis 52*(6), 3027–3046.

Francq, C. and J.-M. Zakoian (2005). The -structures of standard and switching-regime {GARCH} models. *Stochastic Processes and their Applications 115*(9), 1557 – 1582.

Franses, P. H. and D. Van Dijk (2000). *Non-linear time series models in empirical finance.* Cambridge University Press.

Frühwirth-Schnatter, S. (2006). *Finite Mixture and Markov Switching Models.* Springer Series in Statistics. Springer New York.

Garcia, R. and P. Perron (1996). An Analysis of the Real Interest Rate under Regime Shifts. *The Review of Economics and Statistics 78*(1), 111–25.

Geyer, C. (1994). On the convergence of monte carlo maximum likelihood calculations. *Journal of the Royal Statistical Society 56*(B), 261–274.

Geyer, C. (1996). *Estimation and Optimization of Functions*, pp. 241–258. Chapman and Hall.

Gray, S. F. (1996). Modeling the conditional distribution of interest rates as a regime-switching process. *Journal of Financial Economics 42*(1), 27–62.

Haas, M., S. Mittnik, and M. S. Paolella (2004). A new approach to markov-switching garch models. *Journal of Financial Econometrics 2*(4), 493–530.

Hamilton, J. D. (1988). Rational-expectations econometric analysis of changes in regime : An investigation of the term structure of interest rates. *Journal of Economic Dynamics and Control 12*(2-3), 385–423.

Hamilton, J. D. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica 57*(2), 357–84.

Hamilton, J. D. (1990). Analysis of time series subject to changes in regime. *Journal of Econometrics 45*(1-2), 39–70.

Hamilton, J. D. (1994). *Time series analysis.* Princenton University Press.

Hamilton, J. D. (2008). Regime switching models. In S. N. Durlauf and L. E. Blume (Eds.), *The New Palgrave Dictionary of Economics.* Basingstoke: Palgrave Macmillan.

Hamilton, J. D. and R. Susmel (1994). Autoregressive conditional heteroskedasticity and changes in regime. *Journal of Econometrics 64*(1-2), 307–333.

Humala, A. and G. Rodríguez (2013). Some stylized facts of return in the foreign exchange and stock markets in Peru. *Studies in Economics and Finance 30*(2), 139–158.

Klaassen, F. (2002). Improving garch volatility forecasts with regime-switching garch. *Empirical Economics 27*(2), 363–394.

Kramer, W. (2008). Long memory with markov-switching garch. *Economics Letters 99*(2), 390–392.

Lamoureux, C. G. and W. D. Lastrapes (1990). Persistence in variance, structural change, and the garch model. *Journal of Business & Economic Statistics 8*(2), 225–34.

Liang, H. and S. Yongcheol (2008). Optimal test for markov switching garch models. *Studies in Nonlinear Dynamics & Econometrics 12*(3), 1–27.

McCulloch, C. E. (1997). Maximum likelihood algorithms for generalized linear mixed models. *Journal of the American Statistical Association 92*(437), 162–170.

Mikosch, T. and C. Starica (2004). Nonstationarities in financial time series, the long-range dependence, and the igarch effects. *The Review of Economics and Statistics 86*(1), 378–390.

Moore, T. and P. Wang (2007). Volatility in stock returns for new eu member states: Markov regime switching model. *International Review of Financial Analysis 16*(3), 282–292.

Rim and Khemiri (2012). Volume and volatility in foreign exchange market microstructure: a markov switching approach. *Applied Financial Economics 22*(14), 1121–1133.

Tanner, M. A. and W. H. Wong (1987). The calculation of posterior distributions by data augmentation. *Journal of the American Statistical Association 82*(398), 528–540.

Tong, H. (1983). *Threshold models in non-linear time series analysis.* Lecture notes in statistics. Springer-Verlag.

Tong, H. (1993). *Non-linear Time Series: A Dynamical System Approach*. Dynamical System Approach. Clarendon Press.

Wei, G. and M. Tanner (1990). A monte carlo implementation of the em algorithm and the poor man's data augmentation algorithms. *Journal of the American Statistical Association 85*, 699–704.

Table 1. Descriptive Statistics for Latin American Stock Markets Returns

| Country | Mean | sd | Maximum | Minimum | Skewness | Kurtosis |
|---|---|---|---|---|---|---|
| | | | Returns | | | |
| Argentina | 0.3884 | 4.7104 | 21.2396 | -23.1362 | -0.4649 | 5.0633 |
| Brazil | 0.1441 | 3.6971 | 16.1700 | -25.4903 | -0.4760 | 6.0501 |
| Chile | 0.1587 | 2.4670 | 10.1573 | -21.5317 | -1.0333 | 11.0876 |
| Colombia | 0.3205 | 3.1065 | 14.3729 | -22.4525 | -0.8615 | 9.8286 |
| Mexico | 0.2283 | 2.9548 | 11.6942 | -19.4440 | -0.5658 | 6.6148 |
| Peru | 0.2497 | 3.6067 | 22.8661 | -18.5938 | -0.2664 | 8.6979 |
| | | | Volatility | | | |
| Argentina | 22.3107 | 43.9975 | 535.2840 | 0.0000 | 5.6660 | 49.4219 |
| Brazil | 13.6721 | 30.4888 | 649.7552 | 0.0000 | 12.5928 | 243.2389 |
| Chile | 6.1038 | 19.0778 | 463.6153 | 0.0000 | 17.7825 | 414.0088 |
| Colombia | 9.7398 | 28.1417 | 504.1136 | 0.0000 | 10.4008 | 153.0567 |
| Mexico | 8.7724 | 20.3959 | 378.0707 | 0.0000 | 9.1279 | 141.5508 |
| Peru | 13.0541 | 35.9400 | 522.8599 | 0.0000 | 7.1471 | 73.9478 |

Table 2. Estimated Parameters for weekly Latin American Stock Markets Returns

| Model | $\mu_1$ | $\mu_2$ | $\omega_1$ | $\omega_2$ | $\alpha$ | $\beta$ | $p_{11}$ | $p_{22}$ | BIC |
|---|---|---|---|---|---|---|---|---|---|
| | | | | Argentina | | | | | |
| GARCH | $0.4655^a$ | | $1.5564^a$ | | $0.1194^a$ | $0.8079^a$ | | | 2321.470 |
| MS | $0.7107^a$ | -0.8355 | $13.2432^a$ | $54.1271^a$ | | | $0.9858^a$ | $0.9463^a$ | 2317.549 |
| Gray | $0.5437^a$ | $-10.7190^a$ | 0.0000 | 0.0005 | $0.1237^a$ | $0.8225^a$ | $0.9913^a$ | 0.0000 | 2385.240 |
| MS-GARCH | $0.9420^a$ | $-3.2911^a$ | 0.0011 | $5.0116^a$ | $0.0521^c$ | $0.9071^a$ | $0.9192^a$ | $0.5544^a$ | 2302.766 |
| | | | | Brazil | | | | | |
| GARCH | $0.2269^c$ | | $1.1284^b$ | | $0.0960^a$ | $0.8224^a$ | | | 2172.799 |
| MS | $0.5277^a$ | $-0.8236^b$ | $8.3040^a$ | $25.8365^a$ | | | $0.9614^a$ | $0.9028^a$ | 2165.095 |
| Gray | $0.4650^a$ | -1.9469 | 0.0000 | $11.8396^b$ | 0.0219 | $0.8066^a$ | $0.9474^a$ | $0.6625^a$ | 2216.450 |
| MS-GARCH | $0.9110^a$ | $-2.5763^a$ | 0.0993 | $5.3889^a$ | $0.0250^c$ | $0.8745^a$ | $0.8576^a$ | $0.4224^a$ | 2157.950 |
| | | | | Chile | | | | | |
| GARCH | $0.2065^a$ | | $0.1094^c$ | | $0.0847^a$ | $0.9018^a$ | | | 1824.542 |
| MS | $0.3588^a$ | -0.2561 | $2.5162^a$ | $13.2096^a$ | | | $0.9528^a$ | $0.9024^a$ | 1795.893 |
| Gray | $0.2916^a$ | -3.1276 | 0.0000 | $22.5475^b$ | $0.1518^a$ | $0.7000^a$ | $0.9751^a$ | 0.0276 | 1867.967 |
| MS-GARCH | $0.3217^a$ | $-1.8813^a$ | $0.3236^a$ | $9.3289^a$ | 0.0000 | $0.8328^a$ | $0.9473^a$ | $0.2815^a$ | 1786.731 |
| | | | | Colombia | | | | | |
| GARCH | $0.3282^a$ | | $1.6675^a$ | | $0.1979^a$ | $0.6262^a$ | | | 1789.305 |
| MS | $0.4565^a$ | -0.9994 | $5.4868^a$ | $47.9785^a$ | | | $0.9750^a$ | $0.7580^a$ | 1766.539 |
| Gray | $0.3957^a$ | -1.7655 | 0.0000 | $26.6445^b$ | $0.1229^b$ | $0.6093^a$ | $0.9377^a$ | 0.2509 | 1833.626 |
| MS-GARCH | $0.5098^a$ | $-0.9892^a$ | $0.6462^a$ | $10.2839^a$ | $0.1040^a$ | $0.6383^a$ | $0.8817^a$ | $0.3841^a$ | 1760.807 |
| | | | | Mexico | | | | | |
| GARCH | $0.3284^a$ | | $0.2177^b$ | | $0.1323^a$ | $0.8482^a$ | | | 1937.977 |
| MS | $0.4047^a$ | -0.1220 | $3.8913^a$ | $18.1237^a$ | | | $0.9799^a$ | $0.9602^a$ | 1936.484 |
| Gray | $0.3989^a$ | -2.5017 | 0.0000 | 6.1426 | $0.1069^a$ | $0.8281^a$ | $0.9798^a$ | 0.4065 | 1941.229 |
| MS-GARCH | $0.6639^a$ | $-1.7675^a$ | 0.0000 | $3.0723^a$ | $0.0693^a$ | $0.8541^a$ | $0.8640^a$ | $0.3427^a$ | 1917.090 |
| | | | | Peru | | | | | |
| GARCH | $0.3123^a$ | | $0.4246^a$ | | $0.1680^a$ | $0.8102^a$ | | | 2042.911 |
| MS | $0.2942^a$ | 0.0943 | $5.1412^a$ | $40.3401^a$ | | | $0.9770^a$ | $0.9197^a$ | 2027.092 |
| Gray | $0.8187^a$ | $-1.0518^a$ | 0.0000 | 0.1252 | $0.2147^a$ | $0.7492^a$ | $0.9419^a$ | $0.8828^a$ | 2069.192 |
| MS-GARCH | $0.3978^a$ | -1.0106 | $0.1669^a$ | $8.5172^a$ | 0.0000 | $0.9401^a$ | $0.9778^a$ | $0.7010^a$ | 2010.583 |

$a$, $b$, $c$ denote significance level at 1%, 5% and 10% respectively.

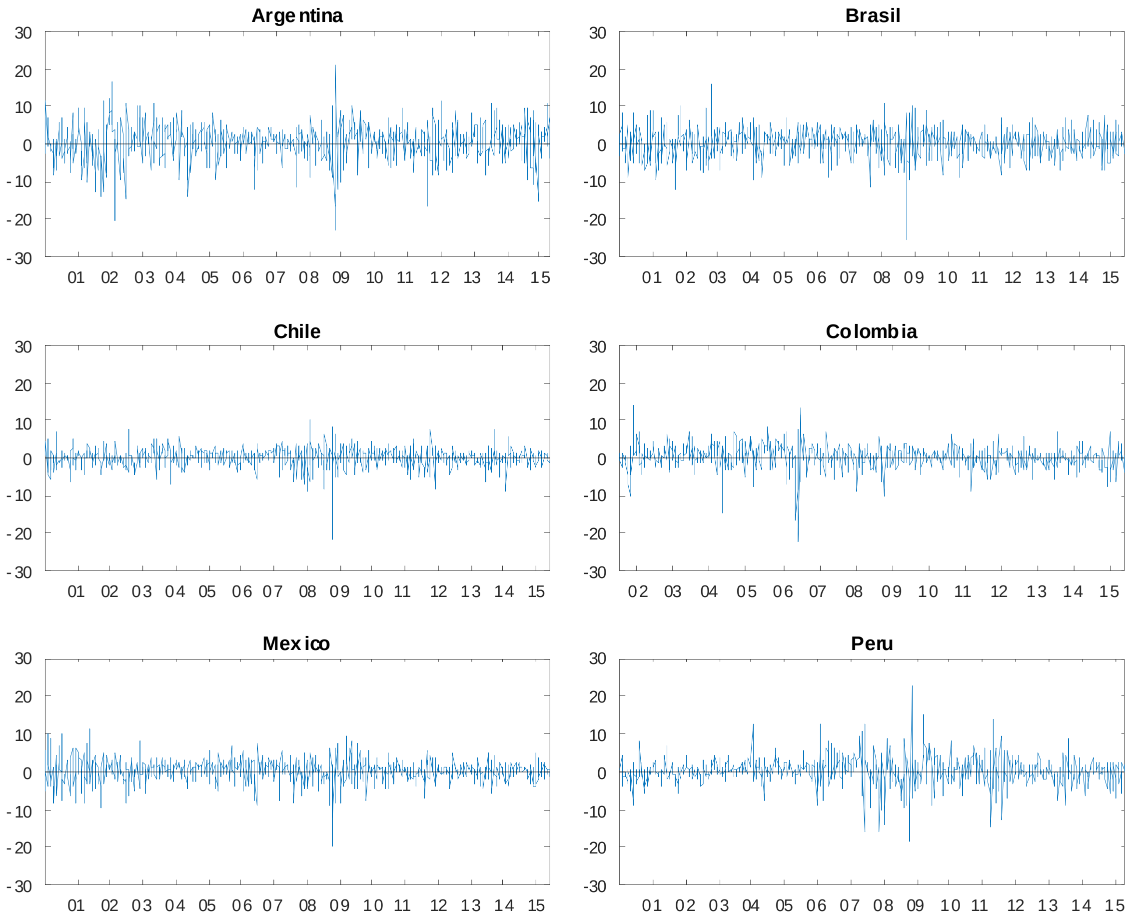


**Figure 1.** Latin American Stock Markets Returns

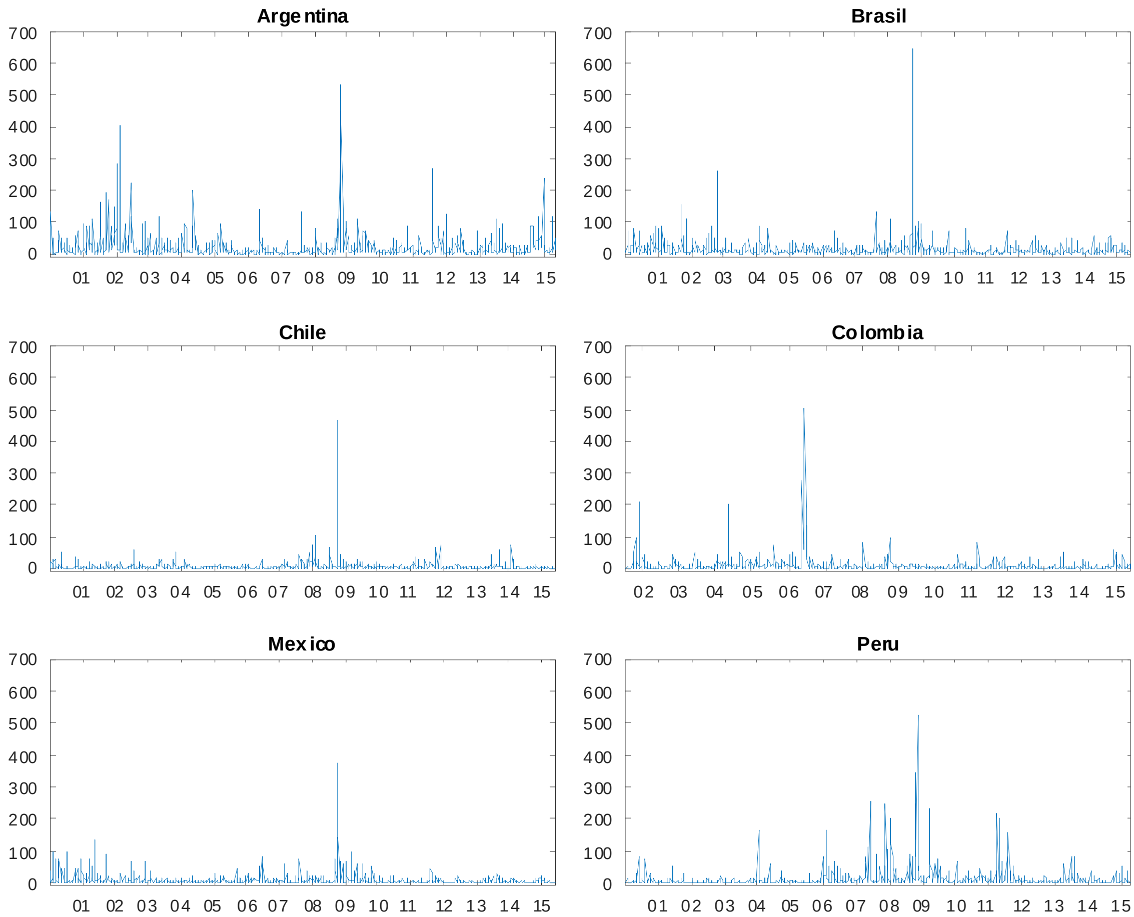


**Figure 2.** Squared of Latin American Stock Markets Returns

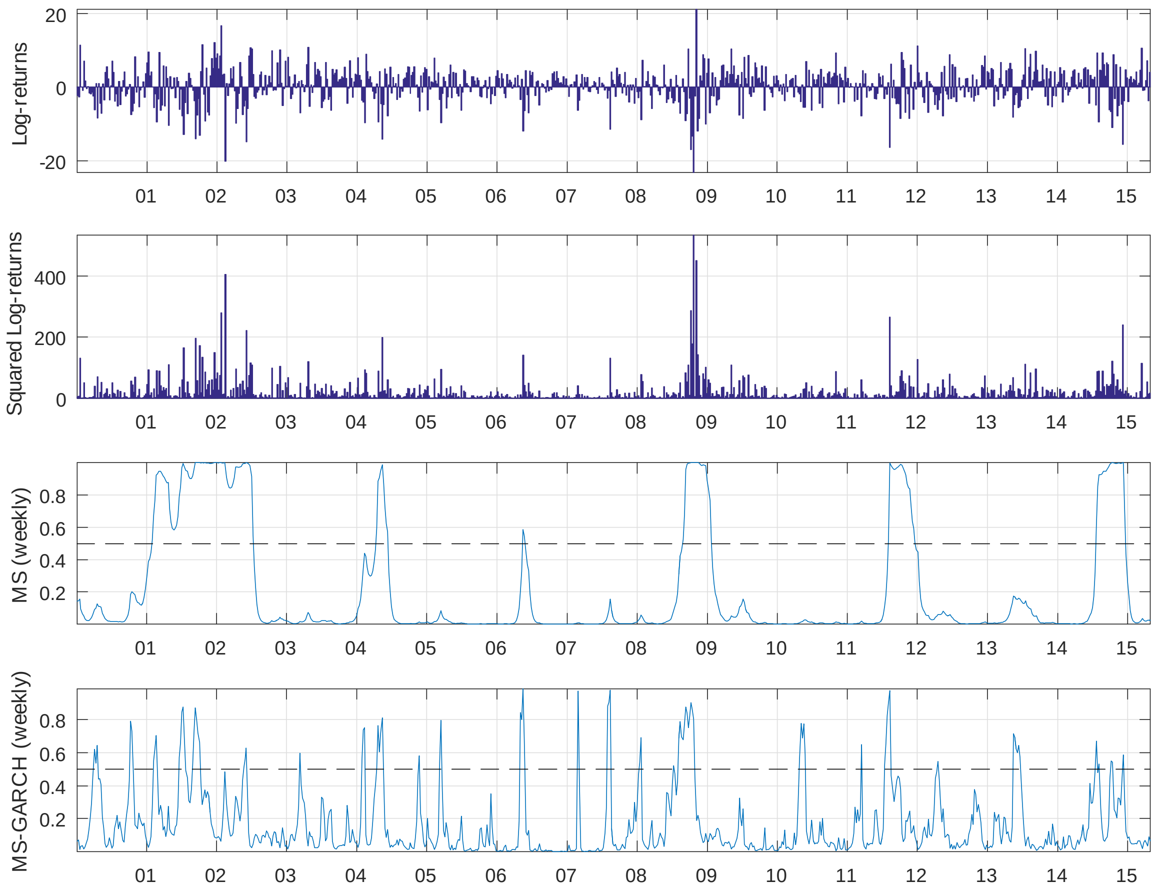


**Figure 3.** Argentina: Smoothed Probabilities of beign in regime two (high volatility)

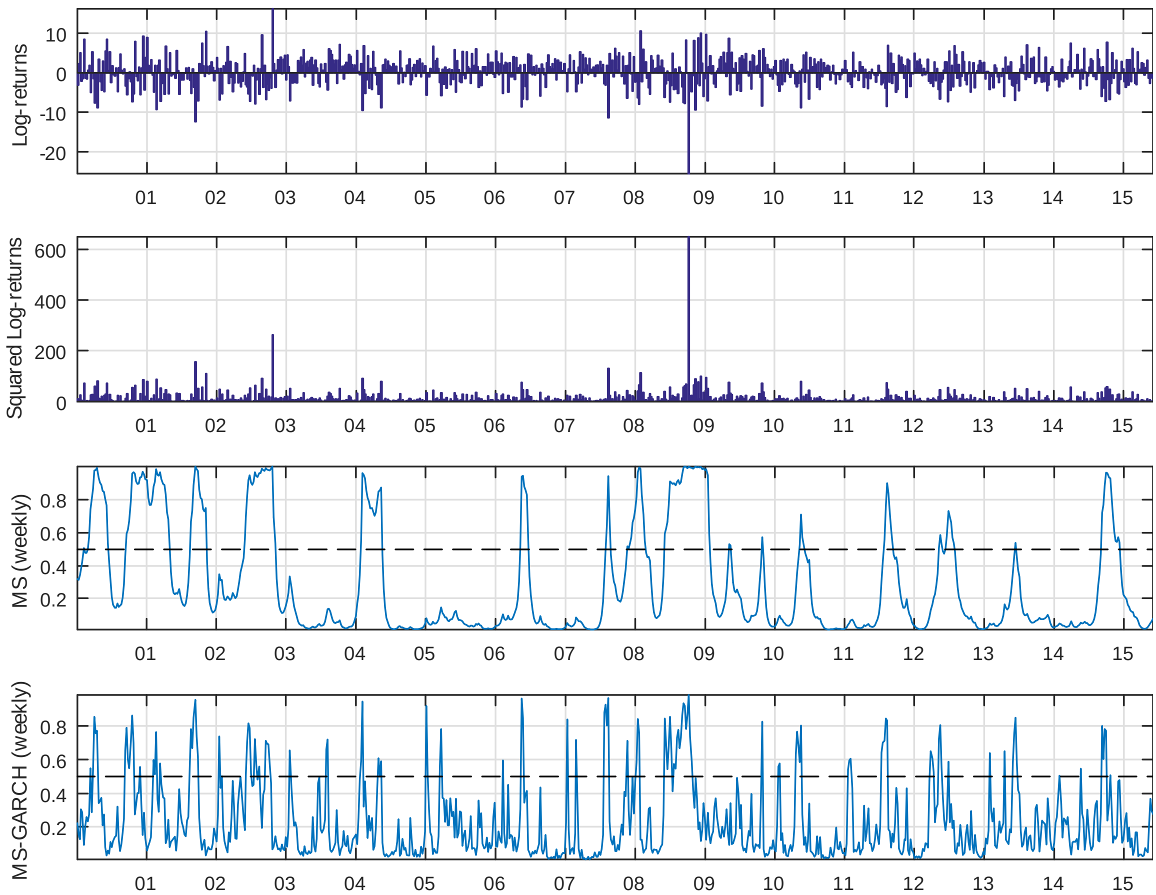


**Figure 4.** Brazil: Smoothed Probabilities of beign in regime two (high volatility)

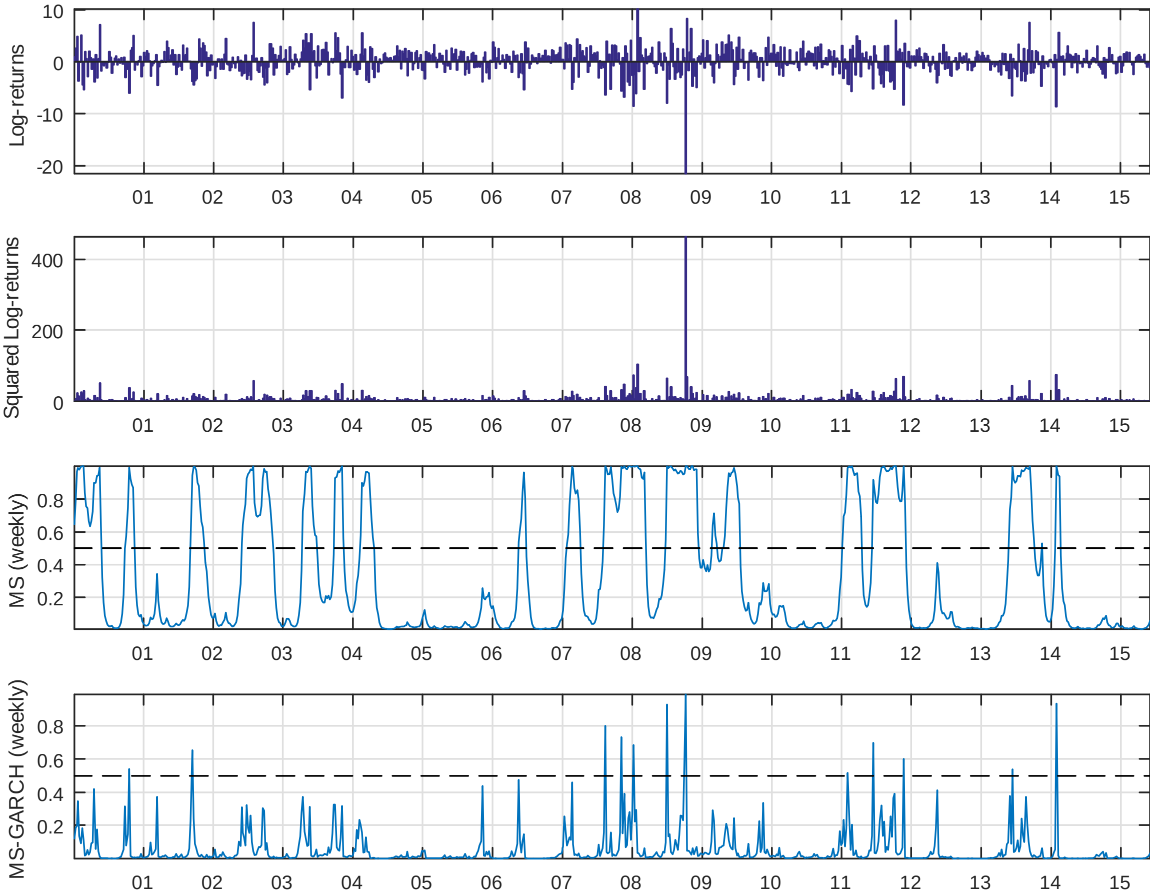


**Figure 5.** Chile: Smoothed Probabilities of beign in regime two (high volatility)

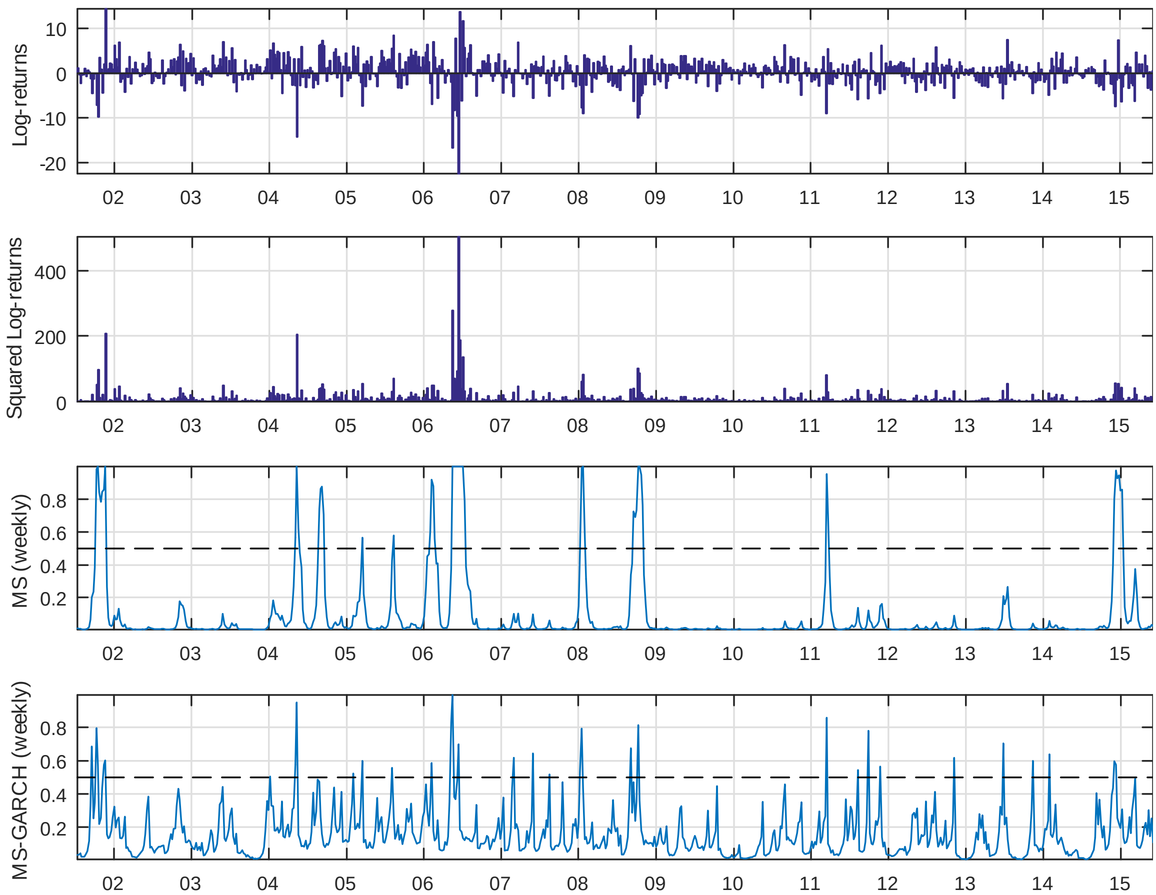


**Figure 6.** Colombia: Smoothed Probabilities of beign in regime two (high volatility)

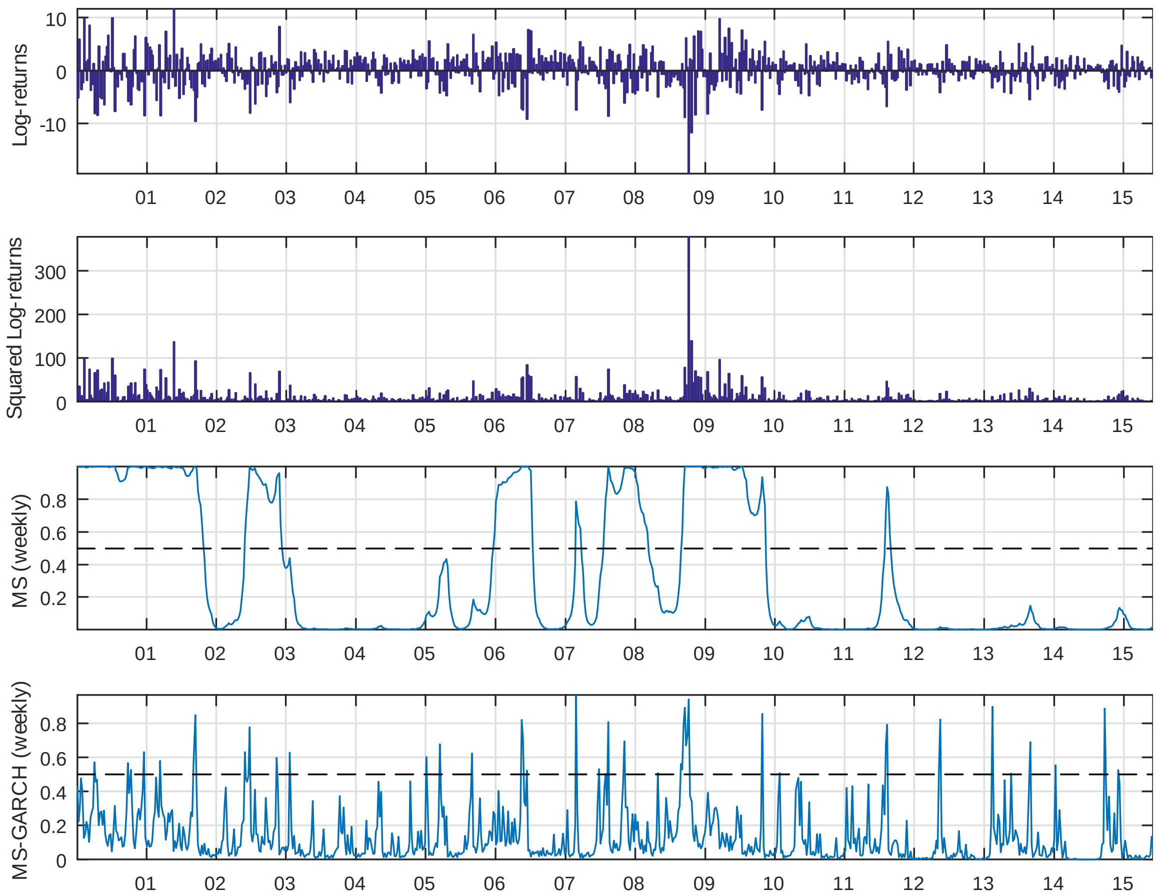


**Figure 7.** Mexico: Smoothed Probabilities of beign in regime two (high volatility)

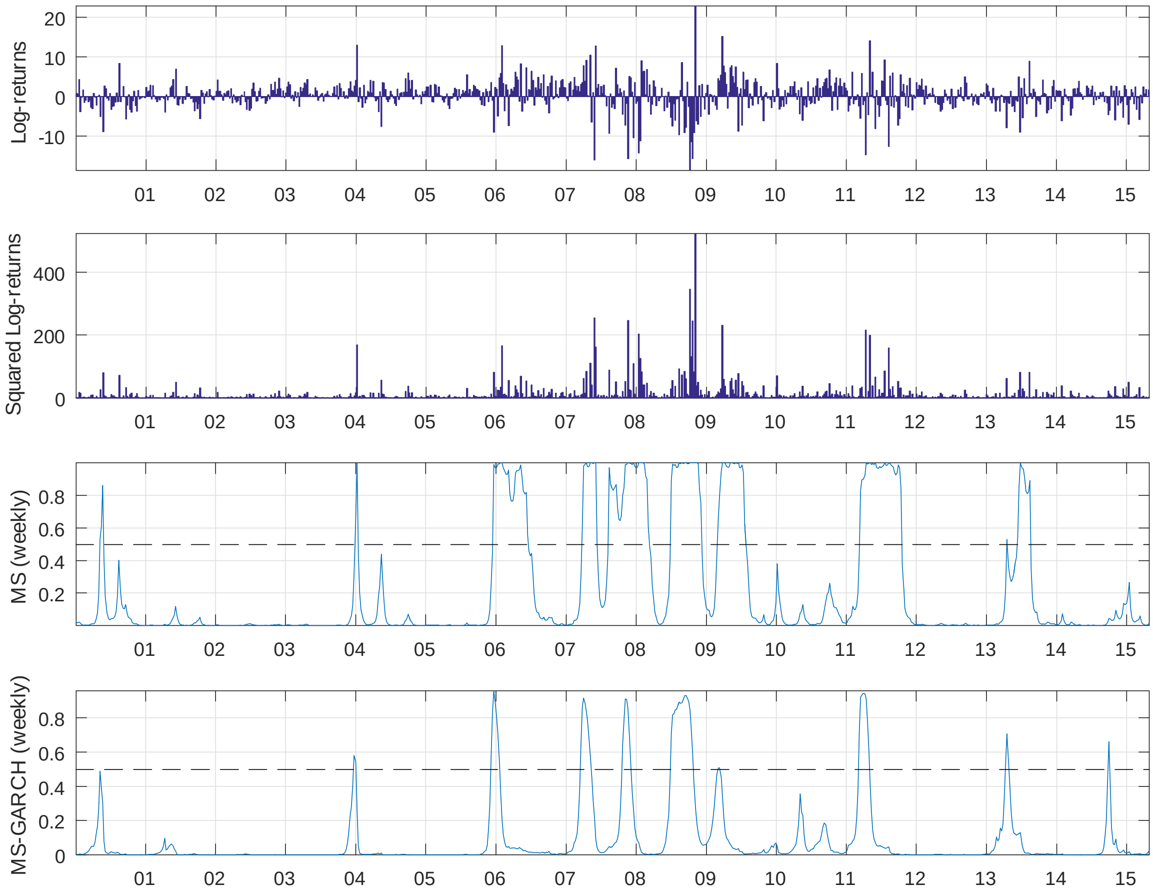


**Figure 8.** Peru: Smoothed Probabilities of beign in regime two (high volatility)

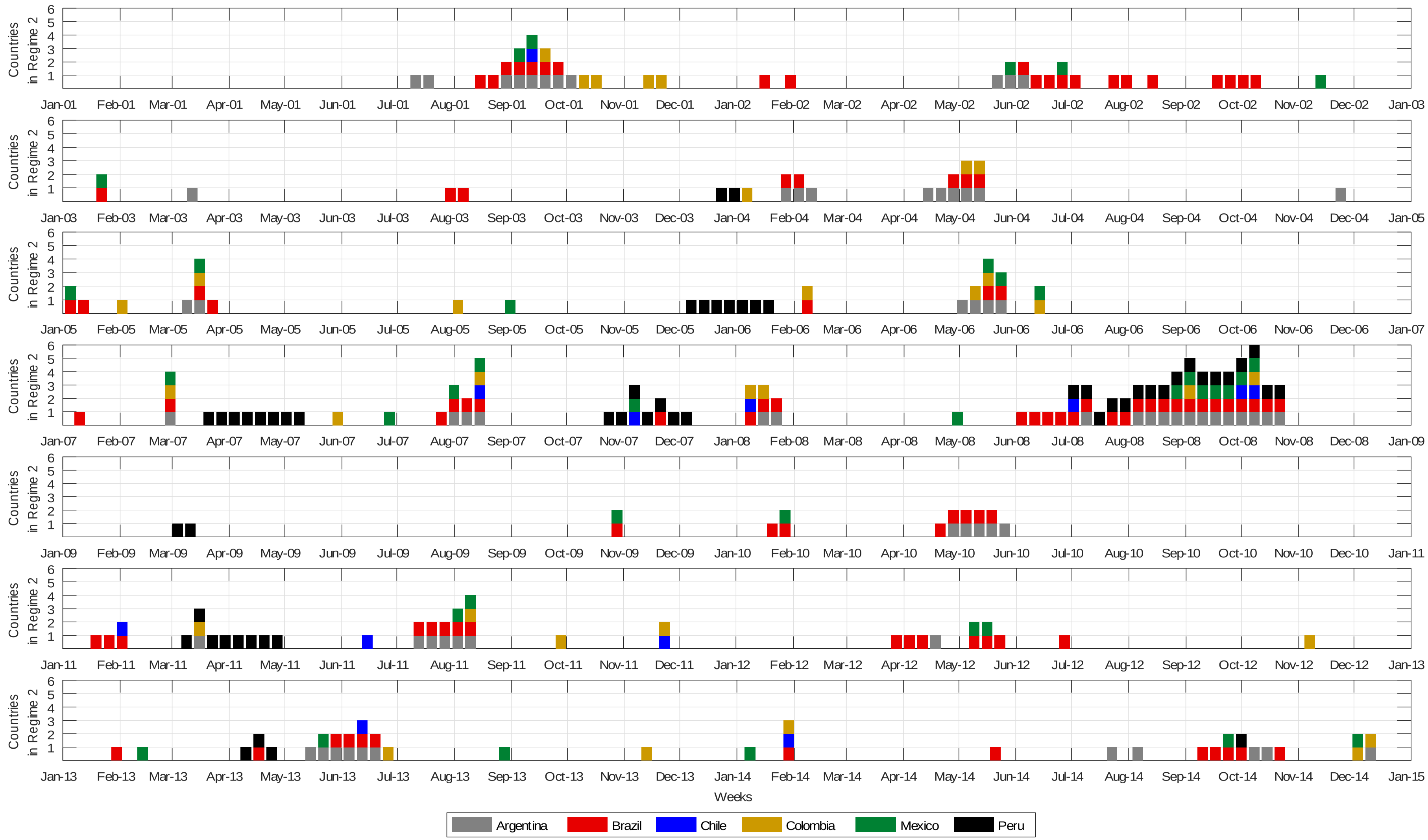


**Figure 9.** Weekly mapping of Latin American countries facing the regime 2. As shown in the second panel, the MS-GARCH shows that in the second week of October 2008 all the countries studied faced high volatility, this due to crash widespread than was given on October 10 in all world stock markets.

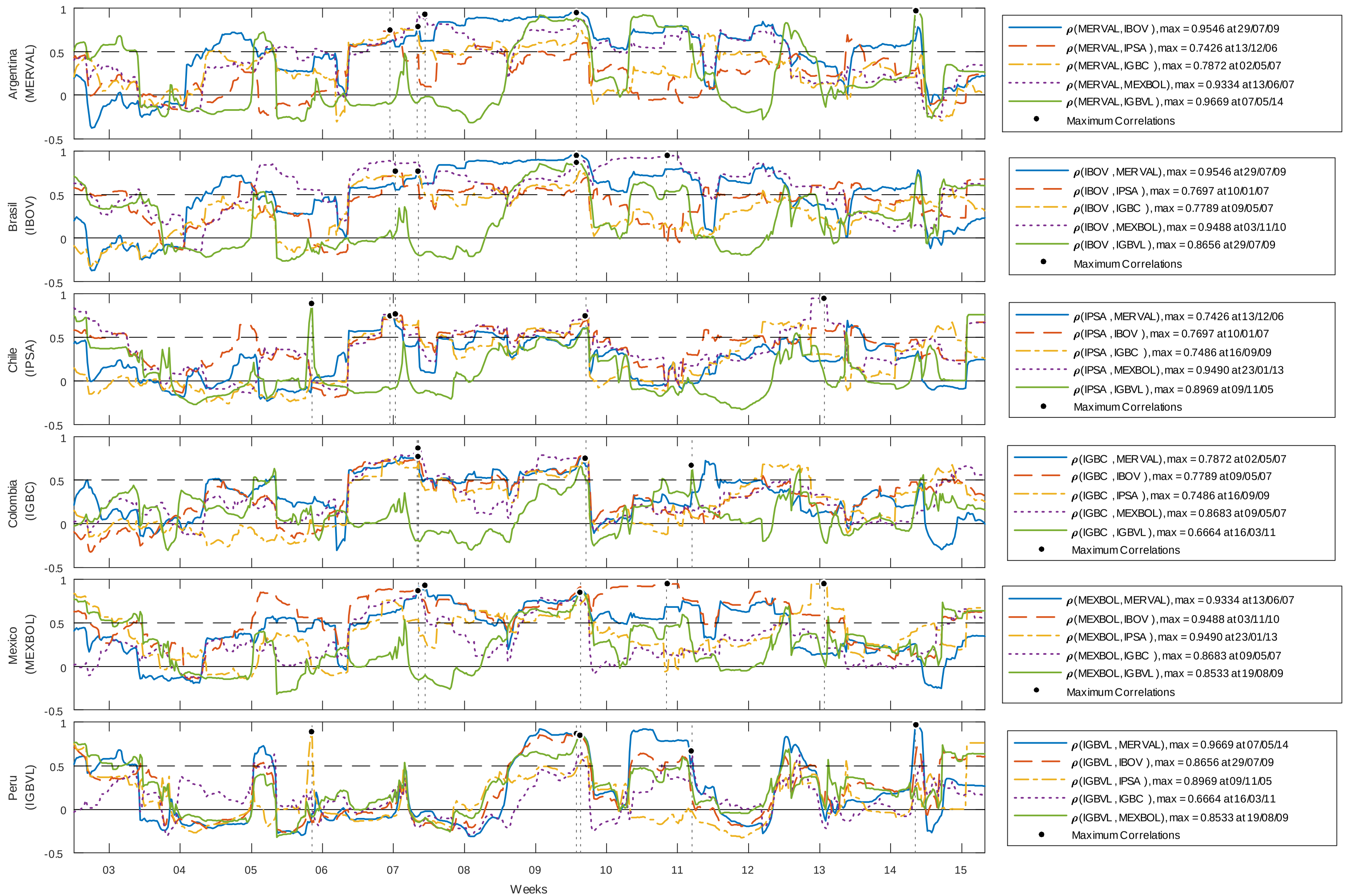


**Figure 10.** Stock Market Correlations between Smoothed Probabilities of negative returns - high volatility using a window of 52 weeks

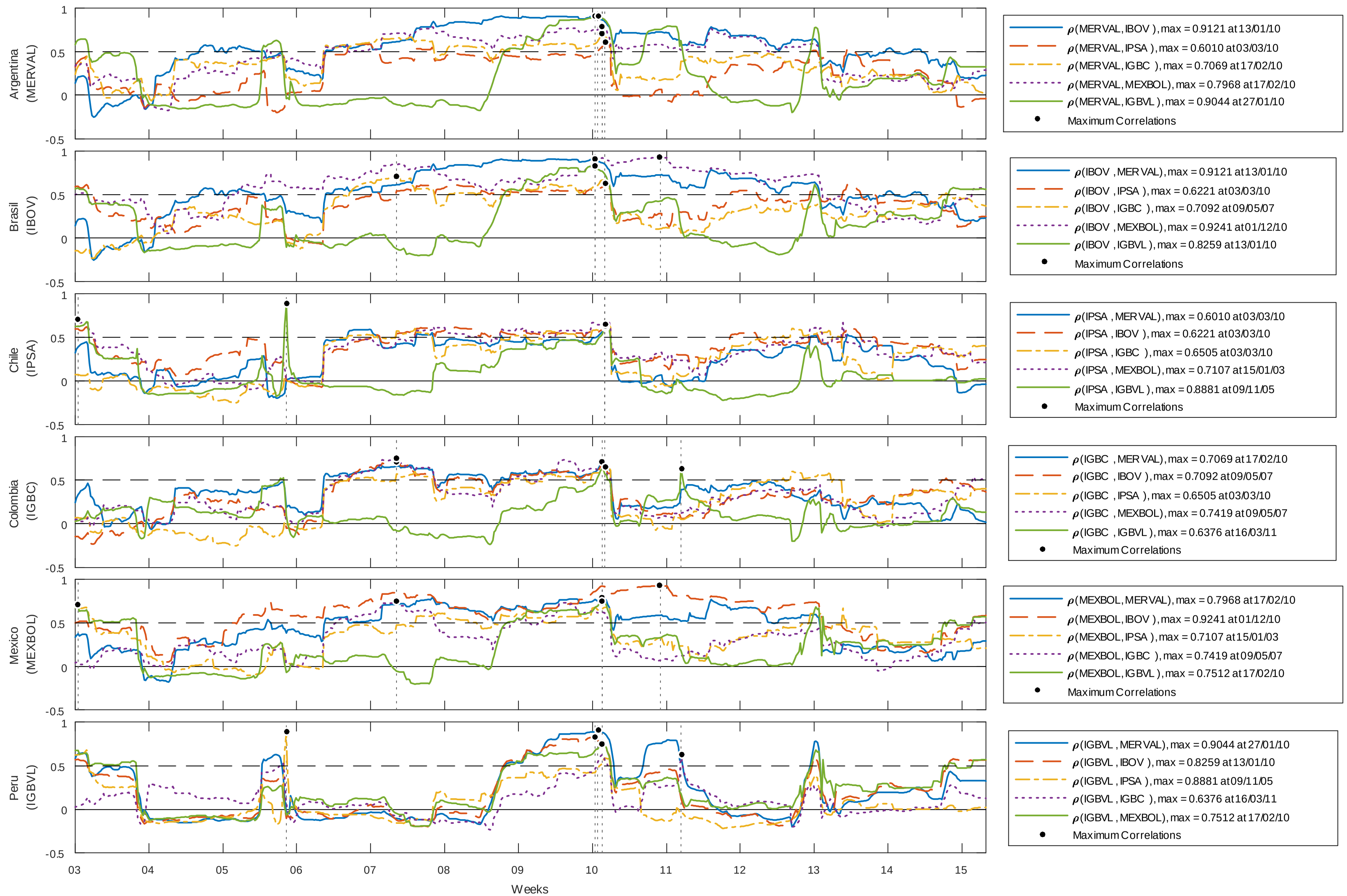


**Figure 11.** Stock Market Correlations between Smoothed Probabilities of negative returns - high volatility using a window of 78 weeks

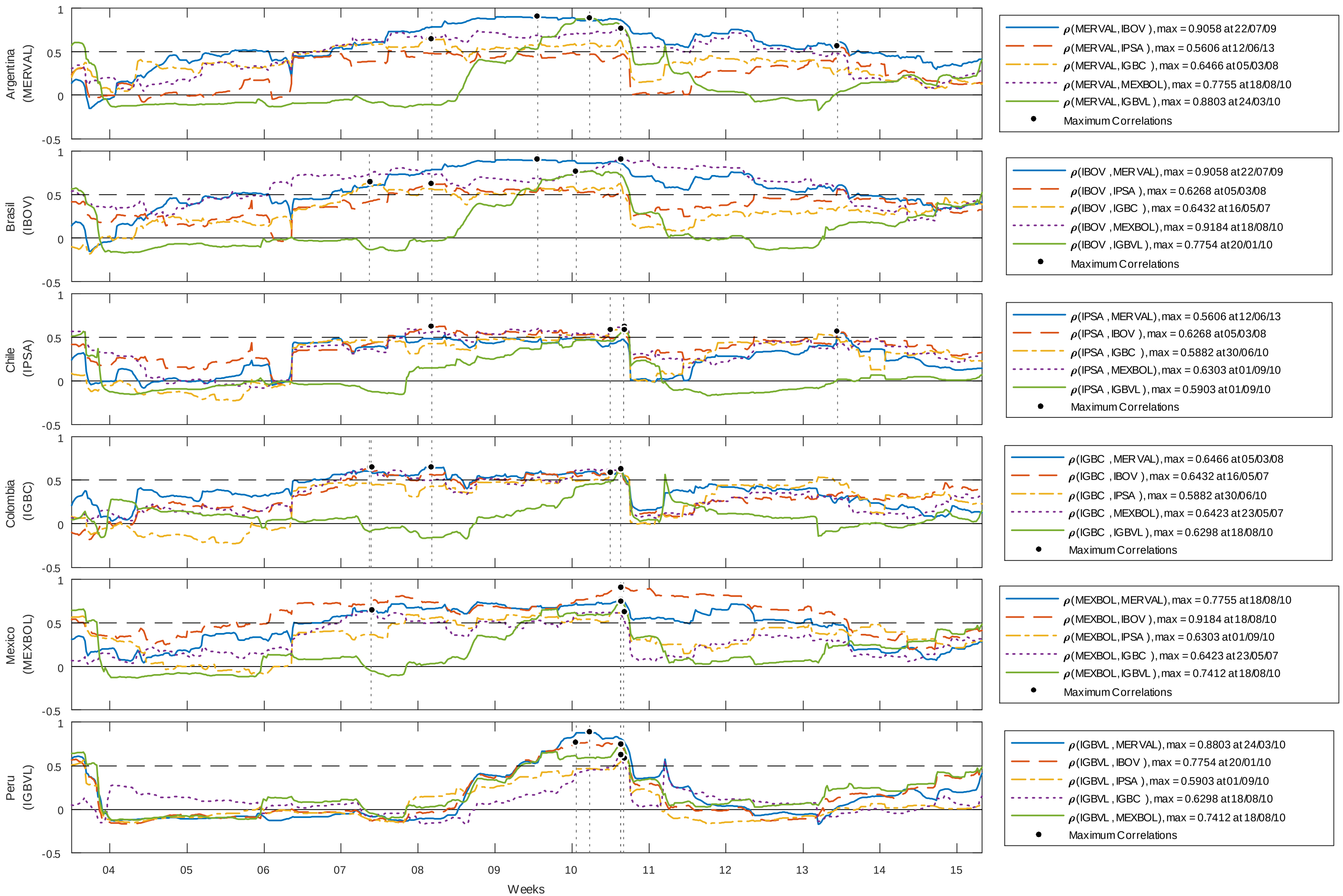


**Figure 12.** Stock Market Correlations between Smoothed Probabilities of negative returns - high volatility using a window of 104 weeks